

Consumers as Financiers: Crowdfunding, Initial Coin Offerings and Consumer Surplus*

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Abstract

Product market competition prevents a firm from extracting full surplus from consumers. Hence, traditional bank funding based on cashflow (NPV) leads to underinvestment relative to social welfare. Crowdfunding mitigates this problem by letting consumers fund the firm directly. A resale market gives consumers an option to sell their claims to future consumers, but future consumers cannot commit to pay for their surplus; underinvestment persists. Speculative premia in the resale market can remedy the problem, but also can cause overinvestment. We provide testable implications on the efficiency of crowdfunding with an active resale market, as in the case of initial coin offerings.

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In the normal course of business, firms maximize shareholder value by choosing projects or actions that generate positive net present value. We examine the efficiency of this decision rule when shareholders may also consume the underlying good. Our study is motivated by the rise of the multimultibillion-dollar global crowdfunding industry that has emerged as an alternative to traditional funding schemes such as bank loans or venture capital ([WorldBank \(2013\)](#)). Crowdfunding is a way for firms to raise money from a large number of small investors. In the United States the Jumpstart Our Business Startups (JOBS) act was passed in 2012 to promote crowdfunding for small businesses. More recently, a new type of crowdfunding has emerged in the form of initial coin offerings (ICOs). The innovation of ICOs is that the digital assets they issue facilitate a liquid resale market. ICOs have raised over 18 billion dollars, and the number is growing rapidly ([Howell, Niessner and Yermack \(2018\)](#)). In this paper we study the efficiency of crowdfunding with and without a resale market. In particular, how does crowdfunding differ from traditional bank (or venture capital) funding? and how does a resale market for financial claims affect productive efficiency?

The premise of our paper is that the extent of consumer surplus a firm can extract depends on the competitiveness of the product market. This affects the efficiency of bank funding decisions which depend on cashflow potential. If the product market is competitive, a firm's cashflows are less than the firm's total consumer surplus. Hence, bank funding leads to inefficient underinvestment absent other frictions. Crowdfunding mitigates this problem by letting consumers fund the firm directly. It serves as a commitment device, which allows consumers to internalize their surplus. Building on this insight, we explore the implications of a resale market. While the resale market gives consumers an option to sell their claims to future consumers, future consumers cannot commit to pay for their surplus; underinvestment persists. Speculative premia in the resale market can remedy the problem, but also can cause overinvestment.

To make our argument precise, we present a simple discrete time, infinite horizon model of investment. There are three types of agents: an entrepreneur, a bank, and consumers. A

penniless entrepreneur has a project, which produces a stream of output when funded. The output price depends on the competitiveness of the product market, which we capture with Nash bargaining. The bank funds any project with positive net present value (NPV), i.e. for which the present value of the revenue stream exceeds the required investment. Consumers, however, derive value from consuming the output directly, and so internalize the consumer surplus and are willing to fund a different set of projects. To capture the idea that consumers and the bank might have different time horizons, consumers face an idiosyncratic liquidity shock, which leads them to discount the value of future output more heavily than the bank. This generates a demand for the resale market. Finally, in an extension, we introduce speculators who trade in the resale market based on different beliefs following [Harrison and Kreps \(1978\)](#).

There are four main results. First, the fact that consumers enjoy a large net surplus once the project is funded, can prevent a project with positive total surplus from getting the bank funding in the first place — product market competition is a double-edged sword. If product market competition is fierce, consumers enjoy higher surplus. But this generates low expected cashflows. In this case, bank funding based on an NPV rule results in inefficient underinvestment, even absent other frictions. This problem arises because in the traditional bank funding model, consumers and financiers are distinct and consumers cannot commit to pay higher prices once the initial investment is sunk. Crowdfunding can improve efficiency because it acts as a commitment device.

The second main result is the benefit of an active resale market (such as the case in ICOs). With no resale market, the relative benefit of crowdfunding over bank funding depends on the trade-off between consumer surplus and a liquidity premium. Since consumers face liquidity shocks, they heavily discount the output in the far future, making crowdfunding without resale unsuitable for long-term projects. With an active resale market, in which consumers can sell their claims to future consumers, crowdfunding is always as efficient as bank funding. However, we note that the resale market cannot by itself eliminate the underinvestment problem because future consumers face a similar commitment problem.

The third main result is the nuanced implication of speculation in the resale market. Following the literature, we introduce speculators whose independent valuations for the claim are lower than the consumers' in all states. In spite of this, the premia induced by the speculators' resale option can raise prices in all states above consumers' fundamental valuation. Thus, speculation can redress the inefficient underinvestment caused by the consumers' commitment problem. In short, even though speculation is viewed as harmful, in this context it may improve investment efficiency.

Finally, we present various testable predictions. Due to the consumption benefit enjoyed by consumers, we predict that crowdfunded projects may appear to have lower profitability measured in terms of cash flow than bank funded projects. This does not imply that they are socially inefficient because consumers obtain an unobserved consumption benefit from the project. We suggest that the analysis of any crowdfunded projects should include an estimate of the consumption benefit. Finally, we note that even though there are no portfolio effects in our model (all agents are risk neutral), crowdfunding with resale necessarily induces a positive correlation between consumers' portfolio performance and consumption benefit. This should be taken into account when evaluating the size restrictions on individual investments permitted under the 2012 JOBS act.

Related Literature. A large literature, building on [Diamond \(1984\)](#) considers incentive problems between a potentially informed or ill-behaving entrepreneur and potential funders. Given economies of scale, or co-ordination problems among investors, intermediaries arise as the natural funders of projects. Specifically, allocating monitoring rights to intermediaries can increase the total value of the investment to the funders. [Strausz \(2017\)](#) follows in this tradition by considering the additional benefit of crowdfunding, namely as a way of acquiring information about the eventual payoff of the project if there is demand uncertainty. The benefit of eliciting information in crowdfunding must be balanced against inefficiencies in controlling entrepreneurial moral hazard. Our focus differs from this literature because we abstract from incentive problems and asymmetric information between the entrepreneur and

funder and focus on the extent to which consumers and other funders' valuations for projects differ. As we emphasize, consumers enjoy the benefit of low prices from any product they consume, even though that reduces profitable cash flows. Crowdfunding is a way for them to ensure projects are undertaken even though they cannot commit to pay high prices.

There is a small (but rapidly growing) literature on crowdfunding and ICOs. [Chemla and Tinn \(2018\)](#) considers how reward based crowdfunding provides valuable information to the entrepreneur. Specifically, consumer interest allows entrepreneurs to reduce demand uncertainty, the resulting real option is most valuable for innovative or uncertain goods, which they argue should be crowdfunded. [Atebro et al. \(2017\)](#) empirically and theoretically show how crowdfunding can be interpreted as a form of rational herding in which pledges are made in response to informed investment. They therefore do not view uninformed investment as a particular cause of concern. Further, [Brown and Davies \(2018\)](#) interpret the “all-or-nothing” feature of crowdfunding as a way of providing credible information about an underlying risky project.

As we emphasize in our framework, ICOs permit crowdfunding with resale. Theoretically, ICOs have been examined by [Chod and Lyandres \(2018\)](#), who consider ICOs as a funding method that allows risk averse entrepreneurs to transfer risk to well diversified investors without giving up control rights. By contrast, [Catalini and Gans \(2018\)](#) considers how an ICO may allow an entrepreneur to raise funding for a good that has a fixed value. To compare traditional funders and ICO funders, they assume that under traditional funding, the entrepreneur can commit to charge consumers their valuation of the good (i.e., they receive no surplus), but under an ICO, all purchases are made by token and so the supply of tokens implicitly determines the price of the good. By contrast, we consider the case in which the products' pricing is independent of how the product is funded, and interpret ICOs as a means to permit the resale of claims among consumers, and hence increase their willingness to fund the project. This distinction allows us to provide testable predictions on the cash-flow properties of projects that are crowdfunded with resale compared to those that are funded traditionally.

The benefits of ICOs in mitigating network externalities (i.e., affecting the realization of future demand) are developed by [Li and Mann \(2018\)](#). Empirical evidence on the properties of ICOs are presented in [Momtaz \(2018\)](#), while [Lee, Li and Shin \(2018\)](#) characterize how information and analysis is aggregated in these offerings. A further literature develops frameworks for valuing cryptocurrencies. For example, [Cong, Li and Wang \(2018\)](#) considers how the underlying tokens should be valued in the presence of network effects, as do [Pagnotta and Buraschi \(2018\)](#) and [Sockin and Xiong \(2018\)](#). We note that in our framework, a speculative premium may increase investment at the funding stage as it potentially compensates consumer/investors who would otherwise attach a discount to their share of the project.

1 Basic Model

We present a basic model to show how competition in the product market affects the productive efficiency of the bank’s funding. We describe the setup in [Section 1.1](#), provide the first-best funding outcome in [Section 1.2](#), and determine the efficiency of the bank funding in [Section 1.3](#).

1.1 Setup

Consider the following discrete time, infinite horizon model of investment. There are three types of agents: one penniless entrepreneur, one bank, and a continuum of consumers, whose measure is normalized to one. All agents are risk-neutral and have a zero time discount rate. Both the bank and the consumers have deep pockets and a zero opportunity cost of capital. At $t = 0$, the entrepreneur is endowed with a project, which costs a fixed amount $I > 0$. If the project is funded, it produces a perishable good each period for $t = 1, 2, \dots$, until it fails. The project fails with probability $1 - \delta \in (0, 1)$ at each point in time. The marginal production cost is zero. Consumers derive utility $v > 0$ from consuming the output, while neither the bank nor entrepreneur derives any value from it directly.

There are two markets: a funding market and a product market. In the funding market, the funder (either the bank or the crowd) makes a take-it-or-leave-it offer to the entrepreneur. (The take-it-or-leave-it assumption is for simplicity, and does not affect the tenor of our results.) In the product market, the entrepreneur sells the output to the consumers. To capture the competitiveness of the product market, we assume that the price is determined every period by generalized Nash bargaining. The parameter $\alpha \in [0, 1]$ is the consumers' bargaining power with the entrepreneur. When $\alpha = 1$, the consumer extracts full surplus; when $\alpha = 0$, the entrepreneur extracts full surplus.

1.2 First-Best Funding Outcome

Total surplus from the project is the sum of firm surplus and consumer surplus (Marshall (1890)). Denote by V the expected present value of the consumer value from a project.

$$V := \sum_{\tau=1}^{\infty} \delta^{\tau} v = \frac{\delta}{1-\delta} v. \quad (1)$$

Total surplus net of the initial investment is $V - I$.

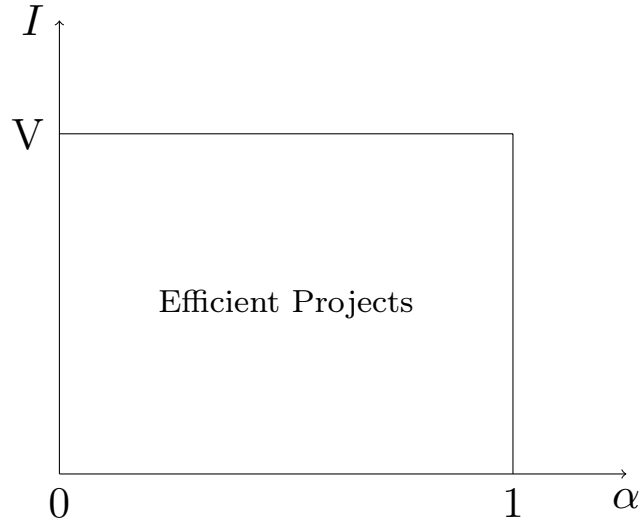


Figure 1: **First-Best Funding Outcome**

Hence, first-best funding outcome is achieved if all projects with positive net total surplus

(i.e. $V > I$) are funded and no project with negative net total surplus (i.e. $V < I$) is funded. Going forward, we refer to productive efficiency as the extent to which the first-best funding outcome is achieved. In Figure 1, the set of efficient projects is independent of α , the competitiveness of the product market.

1.3 Bank Funding Choice

A deep-pocketed bank with a zero cost of capital will fund projects for which the present value of the expected revenue from the project exceeds the required initial investment I , i.e. if the project has positive net present value (NPV).

When the bank funds the entrepreneur, the entrepreneur sells the output in the product market. Recall that the price of the output is determined by generalized Nash bargaining. Both the entrepreneur and the consumers have a zero outside option. If the entrepreneur does not sell the output she receives nothing, and if the consumers do not buy the output they receive nothing. Thus, at each $t = 1, 2, \dots$, the price of the output p is chosen to maximize

$$(v - p)^\alpha p^{1-\alpha}. \quad (2)$$

Since the marginal production cost is zero, it is immediate that the price and thus the revenue each period is

$$p = (1 - \alpha)v. \quad (3)$$

We note that if $\alpha = 0$, then consumers pay their valuation and the monopolist is “perfectly discriminating.” If $\alpha = 1$, the product market price is zero and equal to the marginal cost of production. Let V^b denote the present value of the revenue stream at $t = 0$, which is the bank’s valuation.

$$V^b := \sum_{\tau=1}^{\infty} \delta^\tau p = (1 - \alpha)V. \quad (4)$$

Hence, the bank chooses to fund the project if and only if V^b exceeds the initial investment I , i.e. the net present value is greater than zero or $V^b - I > 0$. Whether the bank funding

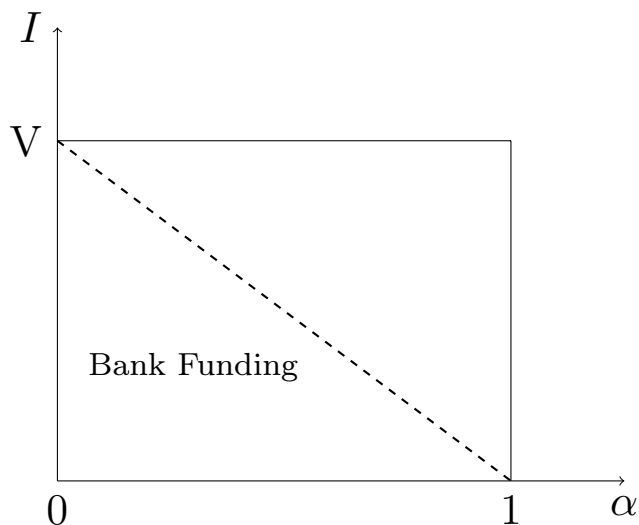


Figure 2: Underinvestment of Bank Funding

achieves the first best depends on the competition in the product market. The following proposition summarizes this idea. (All proofs are in the Appendix.)

Proposition 1. *If and only if $\alpha > 0$, so consumers have bargaining power in the product market, then the bank's valuation V^b (defined in Equation (4)) is strictly less than the total surplus V (defined in Equation (1)). The bank fails to fund projects with positive net surplus and negative NPV, i.e. $I \in [V^b, V)$, and there is inefficient underinvestment.*

Bank funding based on the NPV rule can be inefficient. When the product market is competitive, the cashflow is a fraction of the value that it generates to the consumers. The difference between the total surplus and the bank's valuation is the consumer surplus.

$$V - V^b = \alpha V. \tag{5}$$

Figure 2 provides an illustration of this: the set of projects funded by the bank depends on α . As the product market becomes more competitive, the bank fails to fund efficient projects.

Competition in the product market is, thus, a double-edged sword. On the one hand, with more competition consumers pay lower prices and enjoy higher surplus provided that

the project is funded. On the other hand, competition lowers the cashflows of the project and hence NPV. This prevents the project from getting funding from the bank in the first place.

The key friction is that consumers cannot commit to pay their full valuation for the good once the investment has been sunk. This friction arises because the bank and consumers are distinct in traditional funding. Once the initial investment is sunk, consumers will only pay the market price.

Next, we study whether crowdfunding, which allows consumers to participate directly in funding new projects, can mitigate the bank funding's underinvestment problem.

2 Crowdfunding and Initial Coin Offerings

We introduce a model of crowdfunding and initial coin offerings. We describe the setup in Section 2.1, analyze the consumers' crowdfunding decision without resale in Section 2.2, and study the implications of the initial coin offerings, which we model as crowdfunding with resale in Section 2.3.

2.1 Setup

Suppose the entrepreneur can crowdfund her project directly from consumers. Recall, consumers derive value $v > 0$ from the output each period. In the funding market, consumers make a take-it-or-leave-it offer to the entrepreneur. In exchange for the funding, consumers receive the rights to the entire stream of the output.

We assume that consumers have a shorter time horizon. While the entrepreneur and the bank are infinitely lived, consumers are subject to an idiosyncratic "liquidity" shock. At each point in time, she receives the shock with probability $\lambda \in [0, 1]$. Upon receiving the shock, she consumes everything and dies the next period, at which point new consumers are born so that the measure of consumers is held fixed at one.

If there is an active resale market, then consumers can trade their claims to the stream of

the output. In particular, when one of the consumers who crowdfunds the project receives a liquidity shock, she can sell the claim to other consumers and consume the proceeds before she dies next period. To simplify the price formation mechanism, we assume that sellers in this resale market make a take-it-or-leave-it offer to buyers.¹ Finally, we do not allow short sales.

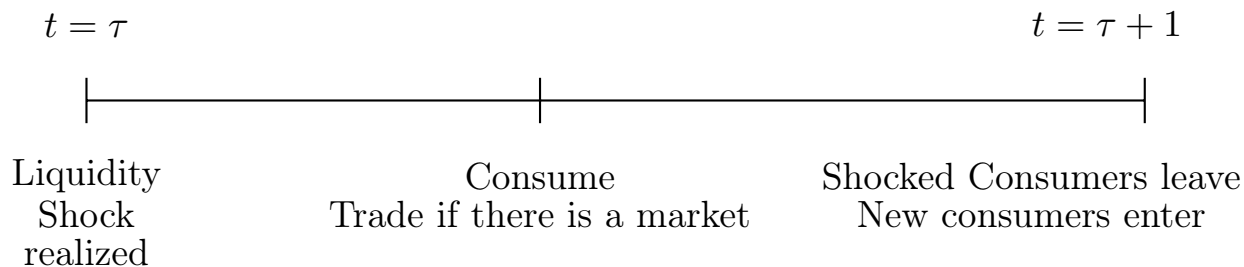


Figure 3: Life Cycle of a Consumer

2.2 Crowdfunding without Resale

Consumers are different from the bank in two aspects. First, consumers take into account the consumer surplus. Second, consumers are subject to an idiosyncratic liquidity shock (λ). Let V^c denote the value of the project to consumers.²

$$V^c := \sum_{\tau=1}^{\infty} \delta^{\tau} (1 - \lambda)^{\tau-1} v = \left(\frac{1 - \delta}{1 - \delta + \delta\lambda} \right) V. \quad (6)$$

Hence, consumers would crowdfund the project if and only if V^c exceeds the initial investment I . The liquidity shock prevents consumers from achieving first best.

Proposition 2. *If and only if $\lambda > 0$, so that consumers face a liquidity shock, then consumers' valuation without resale V^c defined in Equation(6) is strictly less than the first best cutoff V defined in Equation (1) and crowdfunding without resale fails to fund projects with positive net surplus or $I \in [V^c, V)$.*

¹The results would be qualitatively the same as long as sellers have any bargaining power with the buyers so that sellers can partly benefit from the resale market.

²At each $\tau \geq 1$, only $(1 - \lambda)^{\tau-1}$ (not having received the shock until $t = \tau - 1$) fraction of consumers are the ones who crowdfund the project at $t = 0$; the others are born after the project is funded.

Crowdfunding without resale can be inefficient. Although consumers internalize their own surplus, they do not internalize the surplus of future consumers. The difference between V and V^c is the liquidity discount.

$$V - V^c = \left(\frac{\delta\lambda}{1 - \delta + \delta\lambda} \right) V. \quad (7)$$

Consumers require compensation for the liquidity shock, i.e. the project must generate sufficiently large value to justify the initial investment before they die.

We can compare the consumers' valuation V^c with the bank's valuation V^b from Equation (4) to obtain

$$V^c - V^b = \underbrace{\alpha V}_{\text{consumer surplus}} - \underbrace{\left(\frac{\delta\lambda}{1 - \delta + \delta\lambda} \right) V}_{\text{liquidity premium}}. \quad (8)$$

Hence, whether crowdfunding reduces inefficiency relative to the bank depends on the trade-off between consumer surplus and the liquidity discount.

Proposition 3. *If and only if*

$$\alpha > \frac{\delta\lambda}{1 - \delta + \delta\lambda}, \quad (9)$$

so the consumer surplus exceeds the liquidity discount then the crowd funds strictly more projects than the bank. The crowd funds additional projects that have positive net total surplus but negative NPV for $I \in [V^b, V^c)$.

Whether crowdfunding can be more efficient than the bank funding depends on the condition in Equation (9). On the left hand side is the competitiveness of the product market, captured by consumers' bargaining power α . The right hand side increases both in λ , the consumers' liquidity shock and δ , the continuation probability of the project each period. Intuitively, if the product market is more competitive, then crowdfunding supports a larger range of projects because the consumer surplus is large. Conversely, a higher probability of a liquidity shock makes crowdfunding less important because funding consumers do not internalize the surplus of future consumers. This implies that crowdfunding without resale

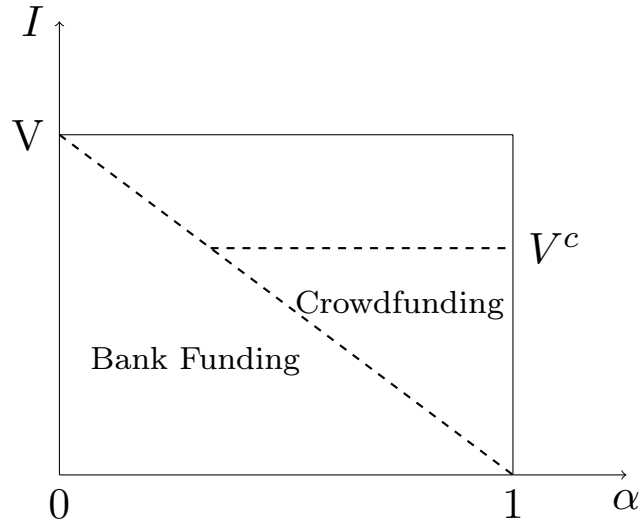


Figure 4: **Trade-off between Bank funding and Crowdfunding**

is not suitable for longer-term projects.

The trade-off between bank funding and crowdfunding is illustrated in Figure 4. As the product market becomes more competitive, bank funding becomes inefficient whereas crowdfunding becomes more important.

One way to interpret the inefficiency is that there is a missing market. Consumers who crowdfund the project at $t = 0$ secure rights to the output of the project indefinitely. When consumers receive a liquidity shock, they cannot enjoy the consumption value from the next period onward, and so the rights to future output have no value to them. However, future consumers value the output and are willing to pay for it. Next, we introduce a market in which consumers can trade the claims for the future output.

2.3 Crowdfunding with Resale: Initial Coin Offerings

The existence of a secondary market raises the consumers' valuation for the project because of the resale option. When consumers receive a liquidity shock, they can sell the claim to future output to other consumers so that they can consume the proceeds before they die. This, of course, would help address the underinvestment problem of crowdfunding without resale: Can crowdfunding with resale achieve first best?

The specific application that we have in mind is Initial Coin Offerings. In ICOs, a new venture raises capital directly from consumers by issuing digital assets, called “tokens”. The tokens make it easier for consumers to retrade their claims.

Recall, the inefficiency in bank funding arises because consumers cannot commit to pay a high price after the investment cost is sunk, even though doing so would induce the bank to fund more projects. The same friction still applies. Unlike the consumers who provide initial funding, new consumers can only commit to pay the prevailing market price p for the output each period. They cannot commit to pay their entire consumer surplus because they enter after the project is funded.

Let P be the price that new consumers can commit to pay for the claim to future output. Since sellers make a take-it-or-leave-it offer, this is the resale price of the claim. Then

$$P := \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} p = \frac{\delta}{1-\delta} p = (1-\alpha) V = V^b. \quad (10)$$

The resale price of the claim is the same as the bank’s valuation of the project. Notice that the consumers’ liquidity shock, λ , does not affect the resale price: Future shocked consumers know they will be able to sell their own claims forward.

Given the resale price of the claim P , the initial consumers’ valuation for the project is now given by

$$V^r := \sum_{\tau=1}^{\infty} \delta^{\tau} (1-\lambda)^{\tau-1} (v + \lambda P). \quad (11)$$

Conditional on not having received a liquidity shock until $t = \tau - 1$, consumers always get v and additionally receive P in exchange of the claim if they get the shock at $t = \tau$. Substituting (10) into (11), we have

$$V^r = \left(1 - \frac{\alpha \delta \lambda}{1 - \delta + \delta \lambda} \right) V. \quad (12)$$

The valuation of consumers with resale V^r coincides with the first-best cutoff V if and only if $\alpha = 0$ or $\delta = 0$. In other words, crowdfunding with resale achieves first best if and

only if either consumers have no bargaining power ($\alpha = 0$), in which case the bank funding alone is efficient or consumers have no liquidity shock ($\lambda = 0$), in which case crowdfunding without resale is efficient.

Even though it does not achieve first best, consumers' valuation with resale, V^r , is at least as high as the consumers' valuation without resale V^c or the bank's valuation V^b . From Equations (4), (6), and (12),

$$V^r = V^c + (1 - \alpha) \underbrace{\left(\frac{\delta\lambda}{1 - \delta + \delta\lambda} \right)}_{\text{liquidity premium}} V = V^b + \left(\frac{1 - \delta}{1 - \delta + \delta\lambda} \right) \underbrace{\alpha V}_{\text{consumer surplus}}. \quad (13)$$

Proposition 4. *Crowdfunding with resale (such as an ICO) is at least as efficient as crowdfunding without resale or as bank funding, i.e. $V^r \geq \max\{V^b, V^c\}$. If and only if either $\lambda > 0$ or $\alpha > 0$, so consumers have either bargaining power, or face any liquidity shock then crowdfunding with resale (ICO) fails to fund all projects with positive net surplus.*

Hence, crowdfunding with resale always improves efficiency (at least weakly) relative to bank funding and crowdfunding without resale. The higher valuations are more efficient since the valuations V^b , V^c , and V^r do not exceed the first-best cutoff V , meaning that the source of inefficiency is underinvestment, rather than overinvestment. Although ICOs can improve efficiency by allowing resale, they do not achieve the first-best funding outcome. The inherent frictions that consumers cannot commit to pay a high price after the investment is sunk and that consumers face a liquidity shock cannot be overcome by simply opening a resale market for the claims.

In Figure 5 we illustrate the benefit of introducing the resale market to crowdfunding. With resale, consumers can always fund more efficient projects than the bank funding. As the product market becomes more competitive, the role of resale becomes limited due to future consumers' lack of commitment.

One possible negative consequence of crowdfunding with resale, and one much touted in the case of ICOs is the possibility that prices do not reflect fundamental values. Specifically,

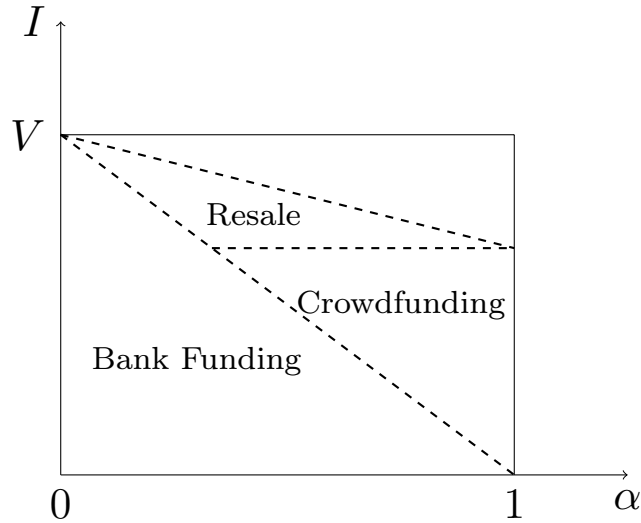


Figure 5: **Benefit of the Resale Market in Crowdfunding**

there are concerns that prices are inflated because of speculation. Next, we study how speculation can affect the efficiency of ICOs.

3 Speculative Premia in Initial Coin Offerings

In this section we introduce investment uncertainty and study the effect of speculation on welfare in crowdfunding with resale. We describe the setup in Section 3.1, provide the benchmark results without speculation in Section 3.2, and show that the speculation can address the underinvestment problem but also causes the overinvestment problem in Section 3.3.

3.1 Setup

To motivate speculation in the resale market driven by differences in beliefs as in [Harrison and Kreps \(1978\)](#), we now introduce uncertainty in the investment. Specifically, at each $t = 1, 2, \dots$, conditional on the project not having failed yet, an aggregate state that can be either high or low or $s_t = s \in \{h, l\}$ is realized and publicly observed. The aggregate state affects the value of the project's output to consumers. When the state is high, consumers

value the output as v i.e. $v(h) = v$, while in the low state, the value is normalized to zero so $v(l) = 0$.

At $t = 1$, the state is high with probability one. From $t = 2$ on, the state evolves according to a Markov chain. The transition matrix Q is

$$Q = \begin{bmatrix} q(h, h) & q(h, l) \\ q(l, h) & q(l, l) \end{bmatrix} = \begin{bmatrix} q_h & 1 - q_h \\ 1 - q_l & q_l \end{bmatrix}, \quad (14)$$

where $q_s \in [0, 1]$ is the conditional probability of remaining in state s given that the project continues next period. We assume that the consumers beliefs are represented by the true transition matrix Q .

We allow a continuum of deep-pocketed and risk-neutral speculators to participate in the resale market. Speculators do not derive any value from consuming the output directly. Akin to the bank, speculators value the output each period at $p = (1 - \alpha)v$, the market price of the output as presented in Equation (3). Speculators believe that the transition matrix, conditional on the project's continuation, is given by

$$Q' = \begin{bmatrix} q'_h & 1 - q'_h \\ 1 - q'_l & q'_l \end{bmatrix}, \quad (15)$$

where Q' may differ from Q . Speculators and consumers agree to disagree. To highlight the effect of speculation, we assume that Q' is such that speculators' valuation for the claim is always lower than the consumers' valuation in each state, i.e. speculators are always more pessimistic than consumers (see Assumption 1 below). This implies that speculators would not participate in the resale market if there were not for the speculative opportunities.

The rest of the model is the same as that in 2.1. Recall that the short sales are not allowed in the resale market. For simplicity, we assume $\lambda = 1$, i.e consumers live for one period only.³ Before we delve into speculation in 3.3, we briefly discuss how the investment

³Note that this assumption will only make crowdfunding with resale less efficient, rather than more efficient.

uncertainty affects different valuations. To do so, we re-derive first best, consumer and bank valuations. We denote these valuations with uncertainty by a tilde.

3.2 Benchmark results

To determine the total surplus with investment uncertainty, denote by V_s for $s \in \{h, l\}$, the conditional expectation of future consumer value from the project given the current state s .

Then for any $t \geq 1$, we have

$$V_s := \mathbb{E} \left\{ \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} v_{\tau} \mid s_t = s \right\}. \quad (16)$$

Recursively, V_h and V_l solve

$$V_h = \frac{\delta q_h}{1 - \delta q_h} v + \frac{\delta(1 - q_h)}{1 - \delta q_h} V_l \quad \text{and} \quad V_l = \frac{\delta(1 - q_l)}{1 - \delta q_l} (v + V_h). \quad (17)$$

The V_h is the the present value of v until the first time that the state changes to l , at which point the value is V_l . The V_l is the present value of $v + V_h$, which is the value at the first chance the state changes to h . It follows that

$$\begin{aligned} V_h &= \frac{q_h + \delta(1 - q_h - q_l)}{1 + \delta(1 - q_h - q_l)} \frac{\delta v}{1 - \delta} \\ V_l &= \frac{1 - q_l}{1 + \delta(1 - q_h - q_l)} \frac{\delta v}{1 - \delta}, \end{aligned} \quad (18)$$

which are well defined since $1 + \delta(1 - q_h - q_l) > 0$.⁴

Since the state at $t = 1$ is high ($s_1 = h$) by assumption, the present value of all consumer value at $t = 0$ is given by

$$\tilde{V} := \delta(v + V_h) = \left(\frac{1 - \delta q_l}{1 + \delta(1 - q_h - q_l)} \right) \frac{\delta v}{1 - \delta}. \quad (19)$$

The first-best is achieved when the project is funded if and only if $\tilde{V} > I$.

⁴Note $1 + \delta(1 - q_h - q_l) \geq q_h + \delta(1 - q_h - q_l)$, with equality if and only if $q_h = 1$; the RHS can be written as $(1 - \delta)q_h + \delta(1 - q_l) \geq 0$, which is strict if $q_h = 1$.

Since the revenue of the project is still a fraction $1 - \alpha$ of the consumer value each period, the linearity of the expectation implies that the bank's valuation with investment uncertainty is simply

$$\tilde{V}^b = (1 - \alpha) \tilde{V}, \quad (20)$$

analogous to (4).

To find the consumers' valuation, recall that in this section, for simplicity, we assume $\lambda = 1$. Without resale, the funding consumers enjoy the output for the next period only. Hence, without resale, the consumer's $t = 0$ valuation is given by

$$\tilde{V}^c = \delta v, \quad (21)$$

which is strictly less than \tilde{V} . It is higher than $(1 - \delta) \tilde{V}$, which we would obtain from substituting $\lambda = 1$ into (6) because of the assumption that the state is high at $t = 1$. With resale, the consumers' valuation increases to

$$\tilde{V}^r = \tilde{V}^c + \delta(1 - \alpha) V_h = \delta(v + (1 - \alpha) V_h), \quad (22)$$

where $(1 - \alpha) \tilde{V}_h$ is the price that consumers at $t = 1$ are willing to pay for the stream of output conditional on the project continuation because only the funding consumers can commit to pay for their surplus. Again, the consumers' valuation with resale \tilde{V}^r is as high as \tilde{V}^c and \tilde{V}^b . It is less than \tilde{V} as long as the consumers have any bargaining power ($\alpha > 0$). The efficiency comparisons in Section 2 remain essentially unchanged.

3.3 Speculation: Heterogeneous Beliefs

Absent speculators, consumers' willingness to pay for the claim in the resale market in each state $s \in \{h, l\}$ is $(1 - \alpha)$ fraction of V_s given by (18). Absent consumers, speculators' valuation for the claim in each state is given similarly except replacing the consumers' beliefs with the speculators' beliefs q'_h and q'_l . As described earlier, we assume that speculators are

always more pessimistic than consumers, i.e. speculators only participate for speculation.

Assumption 1. *The conditional transition matrices Q and Q' are given so that the speculators' valuation for the claim is lower than the consumers' valuation for the claim in both states, i.e.*

$$\frac{q_h + \delta(1 - q_h - q_l)}{1 + \delta(1 - q_h - q_l)} > \frac{q'_h + \delta(1 - q'_h - q'_l)}{1 + \delta(1 - q'_h - q'_l)} \text{ and } \frac{1 - q_l}{1 + \delta(1 - q_h - q_l)} > \frac{1 - q'_l}{1 + \delta(1 - q'_h - q'_l)}. \quad (23)$$

To illustrate how speculation affects the resale price, consider when the speculator, whose valuation is lower than consumers in both states, would buy the claim in low state. The speculator would pay more for the claim than the consumers if the option to sell it back to the consumers in high state is sufficiently valuable. The fact that they can sell it at $(1 - \alpha)V_h$ to consumers in high state, makes speculators willing to pay

$$P'_l = \frac{\delta(1 - q'_l)}{1 - \delta q'_l} (1 - \alpha)(v + V_h), \quad (24)$$

in the low state. This is strictly greater than the consumers' valuation in the low state $((1 - \alpha)V_l)$ if and only if $q'_l < q_l$. As long as speculators believe that the state will change to h more quickly than consumers do, they are willing to pay more for the claim in the low state than the consumers.

In turn, the high price that speculators are paying in low state increases the consumers' willingness to pay for the claim in high state.

$$P_h = \frac{\delta q_h}{1 - \delta q_h} (1 - \alpha)v + \frac{\delta(1 - q_h)}{(1 - \alpha)} P'_l. \quad (25)$$

Proceeding iteratively, one can construct prices in both states that reflect speculator's resale options.

Proposition 5. *If and only if either $q_l > q'_l$ or $q_h < q'_h$ but not both, there is a speculative premium in the resale market and the equilibrium resale price of the claim is higher than the*

consumers' valuation and the speculator's valuation in each state.

If $q_l > q'_l$, the equilibrium resale price for the claim in each state is given by

$$\begin{aligned} P_h^* &= \frac{q_h + \delta (1 - q_h - q'_l)}{1 + \delta (1 - q_h - q'_l)} \frac{\delta (1 - \alpha) v}{1 - \delta} \\ P_l^* &= \frac{1 - q'_l}{1 + \delta (1 - q_h - q'_l)} \frac{\delta (1 - \alpha) v}{1 - \delta}. \end{aligned} \tag{26}$$

If $q_h < q'_h$, the equilibrium resale price for the claim in each state is given by

$$\begin{aligned} P_h^* &= \frac{q'_h + \delta (1 - q'_h - q_l)}{1 + \delta (1 - q'_h - q_l)} \frac{\delta (1 - \alpha) v}{1 - \delta} \\ P_l^* &= \frac{1 - q_l}{1 + \delta (1 - q'_h - q_l)} \frac{\delta (1 - \alpha) v}{1 - \delta}. \end{aligned} \tag{27}$$

As in [Harrison and Kreps \(1978\)](#), the mere presence of a speculator, whose valuation is lower than the consumers in each state, can lead to higher prices for the claim in both states. This is because heterogeneous beliefs between the consumers and the speculator presents speculative opportunities. With the short-sale constraint, traders must buy the claim first to take advantage of the opportunities. The price of the claim rises to reflect the value of speculative opportunities, or “speculative premium.”

To see this more clearly, consider the following example with $\delta = \alpha = v = q_h = q_l = .5$, $q'_h = .1$ and $q'_l = .4$. Then the consumers' valuation of the claim is .125 in each state and the speculator's valuation is .07 and .12 in high and low states respectively. In each state, the consumers have a higher valuation for the claim than the speculator does. Notice that the condition in [Proposition 5](#) is satisfied since $q_l = .5 > q'_l = .4$ and $q_h = .5 > q'_h = .1$.⁵ Therefore, the equilibrium price is given by .13 and .14 and the speculative premium is .005 and .015 in high and low states respectively.

This speculative premium affects consumers' crowdfunding decision. Since the funding consumers can resell the claim at a higher price, they are willing to fund the project with

⁵In fact, [Assumption 1](#) that the speculator's valuation is lower than the consumers' in both states ensures that the two inequalities in [Proposition 5](#) never hold simultaneously.

a higher required investment. The consumers' valuation with the speculative resale market \tilde{V}^s is therefore

$$\tilde{V}^s = \tilde{V}^c + \delta P_h^* = \delta(v + P_h^*). \quad (28)$$

To determine the efficiency implication of speculation, we compare \tilde{V}^s with the first-best cutoff \tilde{V} (19) and obtain

$$\frac{\tilde{V} - \tilde{V}^s}{\delta} = \underbrace{\alpha V_h}_{\text{consumer surplus}} - \underbrace{(P_h^* - P_h)}_{\text{speculative premium}} \quad (29)$$

Whether speculation is efficient or not depends on the trade-off between consumer surplus and speculative premium.

Proposition 6. *Suppose either $q_l > q_l'$ or $q_h < q_h'$ but not both. Define*

$$\alpha_{min} := 1 - \frac{P_h}{P_h^*}, \quad (30)$$

where $P_h = (1 - \alpha)V_h$ and V_h is given by (18) and P_h^* is given as in Proposition 5. Then $\alpha_{min} \in (0, 1)$; speculation mitigates the underinvestment problem (i.e. $\tilde{V}^s \leq \tilde{V}$) if $\alpha \geq \alpha_{min}$ and the speculation causes the overinvestment problem (i.e. $\tilde{V}^s > \tilde{V}$) if $\alpha < \alpha_{min}$.

Some speculative premium improves productive efficiency. As the market becomes more competitive with the consumers' bargaining power α increasing, the underinvestment problem from the lack of commitment of futures consumers becomes more severe. Speculative premium, by raising the price at which initial consumers can sell the claim, mitigates the underinvestment problem.

However, as the market becomes less competitive so that the underinvestment problem is milder, the speculative premium can deteriorate efficiency. Too much speculative premium overshoots the efficient valuation and encourages the initial consumers to fund the project even when the net total surplus is negative. Speculative premium can cause overinvestment problem.

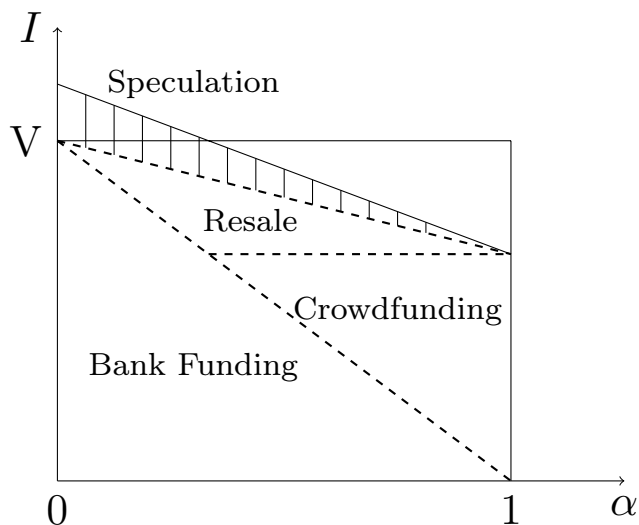


Figure 6: **Efficient and Inefficient Speculation**

In Figure 6 speculative premium raises the required investment I of the project, for which initial consumers are willing to provide funding. As the product market becomes more, both the bank funding and the resale market become ineffective. Hence, speculative premium can improve efficiency. As the product market becomes less competitive, both the bank funding and crowdfunding with resale can effectively fund efficient projects. Hence, speculative premium leads to overinvestment problem, funding inefficient projects with negative net total surplus.

Recall the simple example from earlier with $\delta = v = q_h = q_l = .5$, $q'_h = .1$ and $q'_l = .4$. In this case, the ratio of P_h to P_h^* is given by $.07/.13 = .54$, which is independent of α . Hence, the speculative premium from heterogeneous beliefs mitigates underinvestment if $\alpha \geq .54$ but overshoots the efficient level and causes overinvestment if $\alpha < .54$.

4 Discussion and Implications

In this section, we discuss product market competition in Section 4.1, present testable implications in Section 4.2, and provide policy implications in Section 4.3.

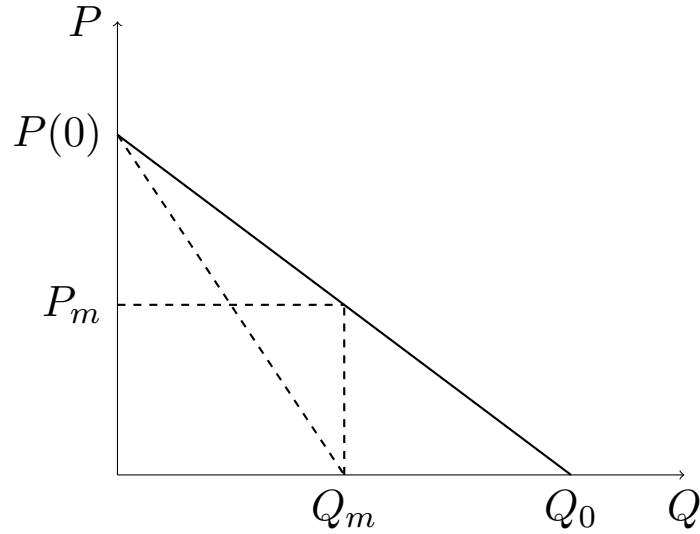


Figure 7: **Graphical Interpretation of the Model**

4.1 Product Market Competition

The key idea of our paper is that the entrepreneur may not extract full surplus depending on the competitiveness of the product market. Although we use a generalized Nash bargaining setup for tractability, the setup can be interpreted more broadly.

Figure 7 presents a simple, downward sloping inverse demand curve and the graphical solution to the monopolist’s problem. Faced with a zero marginal cost of production, he will produce up to the point that the marginal revenue equals zero, which implies a monopolist quantity and price denoted by P_m and Q_m . The profit that the monopolist can extract from the market is simply $P_m Q_m$, or the square under the demand curve. This also corresponds to the maximal amount that a monopolist would be willing to pay (i.e., investment) to serve the market. Now consider the consumer surplus. With a zero economic cost of production, the gross consumer surplus is the entire area under the inverse demand curve or the triangle, given by $P(0)$, Q_0 and the origin – with an area of $\frac{P(0)Q_0}{2}$. This area corresponds to the “ v ” of our model. This is also the maximal amount that a central planner would pay to fund a project.

Now consider the ratio of the two areas, that is

$$\frac{P_m Q_M}{\frac{P(0)Q_0}{2}}. \quad (31)$$

This is the monetary profit as a fraction of the consumer surplus. The ratio of the two corresponds to our measure $(1 - \alpha)$, which is the amount that a producer can extract from the consumers. Note that if the monopolist is perfectly discriminating, then the ratio is just 1, whereas if the market is competitive, the ratio is 0. Our notion of bargaining power therefore captures the extent to which producers and consumers have different incentives in the market.

4.2 Testable Implications

We provide three testable implications. The first two are on characteristics of projects that are suitable for different funding methods. The third is on ex post performance.

First, we discuss how various characteristics affect the likelihood that a project gets bank funding versus crowdfunding without resale. In Section 2 we analyze three characteristics: the competitiveness of the product market (α), the continuation probability of the project (δ), and the consumers' liquidity shock (λ). In Proposition 3, whether a project gets bank funding or crowdfunding (without resale) depends on the trade-off between the competitiveness (α) and the liquidity premium ($\frac{\delta\lambda}{1-\delta+\delta\lambda}$). The liquidity premium is high if the project has a long horizon, i.e. the project is likely to produce output in the far future, or if the consumers have a short horizon, i.e. their preference for the output is likely to change very quickly. Hence, a project is more likely to crowdfunding (without resale) than bank funding if competition outweighs liquidity premium, i.e.

- i The output of the project is sold in a competitive industry;
- ii The project has a short horizon;
- iii The consumers' preference for the output is persistent.

Second, we analyze the implications of the resale market and speculation. In Section 3 we show that speculation mitigates underinvestment but can cause overinvestment. A project is more likely to suffer overinvestment due to speculation if either bank funding or crowdfunding without resale was already close to achieving first best, i.e. either competition or liquidity premium was low in the first place, i.e.

- i The output is sold in a non-competitive industry;
- ii The project has a short horizon.

Lastly, we consider ex post (observed) performance. We have demonstrated conditions under which a broader range of projects are crowdfunded than under traditional methods. As the bank's decision is based on an anticipation of future cashflows, it implies that crowdfunded projects will on average appear to have worse performance. However, even though the financial performance is worse, it does not mean that the projects are socially inefficient. Hence, in a matched sample of bank funded and crowd funded projects with the same investment

- i. Crowdfunded projects will have lower cashflows and profitability measures than bank funded projects.
- ii. Crowdfunded projects with resale markets will have lower cashflows and profitability measures than crowdfunded projects without resale markets.

4.3 Policy Implications

We now discuss policy implications. First, we consider a lock-up period for the resale market. Second, we discuss potential implications of risk averse consumers.

Being a recent phenomenon, ICOs, which we model as crowdfunding with an active resale market, face less stringent regulation than its traditional counterpart, initial public offerings (IPOs). In particular, the investors of the IPOs are restricted from trading immediately after their stocks become publicly available. Intuitively, the lock-up period helps

insulate the funding decision from speculative premia in the secondary market. The lack of a similar regulation in ICOs can amplify the negative effect of speculation, ending up with funding inefficient projects.⁶ We think imposing a lock-up period can help prevent overinvestment. Given our testable implication from earlier section, this regulation would be especially important when speculation is likely to cause overinvestment, i.e. the industry is less competitive or the project has a short horizon.

In our model, all agents are risk neutral so there is no natural reason for consumers to hold portfolios. However, we note that crowdfunding does have one particular characteristic – by construction, there is an induced correlation between financial wealth and consumption satiety. To see this, assume the econometrician has access to the entire population of consumers. (In this case, we do not have to consider the properties of the sample.) First, it is easy to see that projects that are bank funded will not have any obvious effect on the correlation of consumption with portfolio performance. However with crowdfunding, this is not the case.

Proposition 7. *A project that is crowdfunded leads to covariance between the consumer’s portfolio performance and consumption. The sign of the covariance is the sign of $P_h - P_\ell > 0$.*

It is difficult to know how to quantify the welfare effects of the increase in variance of utility outcome that a correlation between consumption and wealth induces. As far as we know, this was not part of the discussion around the limits to investment mandated by the JOBS act. However, policy makers should be aware of this natural consequence of crowdfunding with resale.

5 Conclusion

We have presented a simple model of crowdfunding that emphasizes the role of consumer surplus. It is standard to take Fisher separation as given. However, with crowdfunding, this

⁶Howell, Niessner and Yermack (2018) discuss a case of ICO, where the lock-up period is voluntarily imposed.

distinction is no longer in place and so we should expect the properties of funded projects to differ from those that are funded by traditional methods.

We stress that crowdfunded projects are not different because firms and entrepreneurs now have access to cheaper capital and face a smaller regulatory hurdle, but because the criteria for “a good project” differ. Further, given that consumers typically have shorter horizons than traditional funders, whether the crowdfunding method allows them to resell their claims will affect the types of projects that they are willing to invest in, and may also explain part of the interest in ICOs.

Finally, we note that a long literature considers the benefit of patents to encourage innovation and investment. The role of a patent is to protect market share, and allow the innovating firm to extract rents or in other words to reduce the consumer surplus. Crowdfunding in as much as it acts as a pre-commitment by the consumers to buy the output of the project, can encourage projects that might not be financed by traditional financing channels: It is analogous to a patent issued by the consumer.

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A Proofs

Proof of Proposition 1. It directly follows from the main text. \square

Proof of Proposition 2. It directly follows from the main text. \square

Proof of Proposition 3. It directly follows from the main text. \square

Proof of Proposition 4. It directly follows from the main text. \square

Proof of Proposition 5. Denote by $q(h)$ and $q(l)$ the probabilities of staying in state h and l respectively from the perspective of the owner of claim in each state in equilibrium.

Then the equilibrium price P_h^* and P_l^* solve

$$P_h^* = \frac{\delta q(h)}{1 - \delta q(h)} (1 - \alpha)v + \frac{\delta(1 - q(h))}{1 - \delta q(h)} P_l^* \quad \text{and} \quad P_l^* = \frac{\delta(1 - q(l))}{1 - \delta q(l)} (v + P_h^*). \quad (32)$$

The owner of the claim in each state is the trader who is willing to pay the most for the claim in that state. To determine the owner of the claim in high state, notice that

$$\frac{\partial P_h^*}{\partial q(h)} = \frac{\delta}{(1 - \delta q(h))^2} ((1 - \alpha)v - (1 - \delta)P_l^*). \quad (33)$$

Thus, $\frac{\partial P_h^*}{\partial q(h)} > 0$ if and only if

$$P_l^* < \frac{(1 - \alpha)v}{1 - \delta}, \quad (34)$$

which is always the case because the RHS is the present value of receiving $(1 - \alpha)v$ in all states. And from (32), we have

$$\frac{\partial P_l^*}{\partial q(l)} < 0. \quad (35)$$

P_h^* increases in $q(h)$ holding P_l^* constant and P_l^* decreases in $q(l)$ holding P_h^* constant.

Now, consider the four exclusive and exhaustive cases: (i) $q'_h > q_h$ and $q'_l \geq q_l$; (ii) $q'_h \leq q_h$ and $q'_l < q_l$; (iii) $q'_h \leq q_h$ and $q'_l \geq q_l$; (iv) $q'_h > q_h$ and $q'_l < q_l$.

In case (i), the speculator holds the claim in high state and the consumers hold the claim in low state. In case (ii), the consumers hold the claim in high state and the speculator holds the claim in low state. Hence, the price is higher than their independent valuations, i.e. there is a speculative premium in cases (i) and (ii). In case (iii), the consumers hold the claim in both states. In case (iv), the speculator holds the claim in both states. Hence, there is no speculative premium in cases (iii) and (iv).

To find the equilibrium price, we substitute the owner's probability into Equation (32). In case (i), substituting $q(h) = q'_h$ and $q(l) = q_l$ into Equation (32) and solving for P_h^* and P_h^* yields Equation (27). In case (ii), substituting $q(h) = q_h$ and $q(l) = q'_l$ into Equation (32) and solving for P_h^* and P_h^* yields (26).

□

Proof of Proposition 6 From Equation (30), speculation causes overinvestment (i.e. $\tilde{V}^s > \tilde{V}$) if and only if

$$\alpha V_h < P_h^* - P_h. \quad (36)$$

Since $P_h = (1 - \alpha)V_h$, we can write above as

$$\frac{\alpha}{1 - \alpha} < \frac{P_h^*}{P_h} - 1. \quad (37)$$

Rearranging this, we have

$$\alpha < 1 - \frac{P_h}{P_h^*}. \quad (38)$$

Hence, speculation mitigates underinvestment if $\alpha \geq \alpha_{min}$ and causes overinvestment if $\alpha < \alpha_{min}$.

□

Proof of Proposition 7 Let $[\pi_h \pi_\ell]$ denote the unique stationary distribution, where

$$\pi_h = \frac{(1 - q_\ell)}{(1 - q_h) + (1 - q_\ell)} \quad (39)$$

$$\pi_\ell = \frac{(1 - q_h)}{(1 - q_h) + (1 - q_\ell)} \quad (40)$$

If the consumers crowd fund a project, then the covariance of consumption with financial wealth is

$$(\pi_h v P_h + (1 - \pi_h) 0 P_\ell) - \pi_h v (\pi_h P_h + (1 - \pi_h) P_\ell).$$

which is positive if and only if

$$P_h \geq P_\ell.$$

Using the expressions presented in Equations 18, we obtain

$$\begin{aligned} P_h &\geq P_\ell \\ q_h + \delta(1 - q_h - q_\ell) &\geq (1 - q_\ell) \end{aligned}$$

A sufficient condition is If $q_h + q_\ell > 1$. \square