Momentum Trading Through Reference Dependent Preferences

Sean Crockett*
Baruch College, Department of Economics and Finance
City University of New York (CUNY)

March 2010

Abstract

The endowment effect is a well-known behavioral regularity in which a person values a good more when he is endowed with it. In their generalization of prospect theory to consumption bundles with multiple attributes, Tversky and Kahneman [1991] imply the endowment effect as a consequence of loss aversion and diminishing sensitivity in gains. It has since frequently been presumed that this form of reference dependent preferences will inhibit trade. However, in this paper it is demonstrated that loss aversion and diminishing sensitivity in gains also imply a dynamic momentum trading effect that increases exchange, so the net effect of such preferences on trading volume is ambiguous. In fact, the momentum trading effect is shown to completely cancel out the endowment effect in an important class of examples.

Keywords: Loss aversion, reference dependence, prospect theory, endowment effect, status quo bias, momentum trading

JEL Classification Numbers: D11, D51

*Contact: sean.crockett@baruch.cuny.edu. I gratefully acknowledge the advice of Roberto Weber, John Duffy, Ryan Oprea, and Stephen Spear, and the financial support of the Tepper School of Business at Carnegie Mellon University and the Social and Information Sciences Laboratory (SISL) at Caltech. All mistakes are my own.

Introduction

The idea that a person's expected utility from owning a particular portfolio is conditional on his point of reference has gained much traction since it was introduced in the economics literature as *prospect theory* by Kahneman and Tversky [1979]. While prospect theory specifically characterizes expected utility over lotteries on one dimension (wealth) and remains a very active area of research, a parallel literature has developed concerning utility over deterministic multidimensional portfolios. Designated *reference dependence* by Tversky and Kahneman [1991], this literature had until recently been dominated by laboratory research into a consistent discrepancy between an individual's elicited willingness to accept (WTA) a good and his willingness to pay (WTP) for it (see Knetsch and Sinden [1984], Knetsch [1989], Kahneman, Knetsch and Thaler [1990], Loewenstein and Adler [1995], Bateman, Munro, Rhodes, Starmer and Sugden [1997], Myagkov and Plott [1997], and List [2004], or Kahneman [2003] for a short review; Plott and Zeiler [2005] examine inconsistencies with eliciting valuations in these studies).

An important conclusion of this literature is that individuals endowed with a good tend to report a higher WTA than the WTP reported by individuals who are not; that is, individuals are biased towards maintaining the status quo. To motivate this result, suppose members of a group of individuals are brought one-at-a-time into a booth and asked to choose between a coffee mug and a chocolate bar, and that half of them choose the mug and half choose the chocolate. Under standard neoclassical preferences we would assume that if each member of a new group drawn from the same distribution was given one of the goods and asked if he would prefer to trade it for the other, about half of the group would prefer to trade. However, List [2004] reports that 81% of his subjects keep an endowed chocolate bar and 77% keep an endowed mug[1]. In a related set-up where members of one group are endowed with a consumption good and members of another are endowed with fungible tokens, less trade occurs than is

[1] These were inexperienced subjects. List [2003; 2004] also finds that experienced traders appear to overcome this status quo bias.

predicted by neoclassical theory [Kahneman et al. 1990]. Buttressed by such results, Kahneman [2003] writes, “Loss-aversion [a formulation of reference dependence which implies the status quo bias] contributes to stickiness in markets, because loss-averse agents are much less prone to exchanges than final-states agents.”

However, in these experiments the opportunity to trade was one-shot, so what was observed was a static endowment effect. The goal of the present paper is to demonstrate that loss aversion (LA) and diminishing sensitivity in gains (DSG), behavioral axioms adopted by Tversky and Kahneman [1991], Munro and Sugden [2003], and Kőszegi and Rabin [2006] that imply this static endowment effect and which appear to be robust descriptions of behavior, also imply a dynamic momentum effect that actually increases the propensity to trade.

Imagine there are two individuals, Robinson and Crusoe, who exhibit LA and DSG. Suppose these individuals have identical preferences and value a chocolate bar and a coffee mug equally when they own neither, but in fact Robinson owns a chocolate bar and Crusoe owns a coffee mug. LA and DSG imply that Robinson prefers the chocolate bar, Crusoe prefers the mug, and they will not trade; this is the endowment effect. But now imagine that Robinson owns ten chocolate bars and Crusoe owns ten coffee mugs. The endowment effect will still inhibit exchange, but if convexity of preferences is satisfied we might reasonably expect they will trade a little. So suppose they exchange two coffee mugs for two chocolate bars, and for simplicity let this reallocation be Pareto optimal prior to the exchange taking place. It will be demonstrated in this paper that LA and DSG will cause Robinson to increase his relative preference for coffee mugs and Crusoe his relative preference for chocolate, and thus they will benefit from trading again, chocolate for mugs, respectively. This “momentum trading” is robust to myopic and perfectly anticipated reference dependence, and can be quite powerful. In fact, in an important class of examples momentum trading and the endowment effect are equal but opposing influences on trade, so that reference dependent individuals are not necessarily prone to less exchange as Kahneman suggests.

The intuition for reference dependent momentum trading partially extends to more than two goods. For example, if a reference dependent individual owns a portfolio of goods 1, 2, and 3 and then trades some of good 1 to obtain more of goods 2 and 3, under LA and DSG his preferences will adjust in such a way that he has a stronger preference for new net trades with the same sign as the one he just executed. However, the results in this paper are limited to the case of two goods because LA and DSG do not permit a characterization of preference adjustment in indirectly affected orthonts of net trade. That is to say, in the example above LA and DSG are silent on what happens to the relative preference for portfolios that involve trading good 2 for goods 1 and 3 after the first exchange.

A Model of Reference Dependent Behavior

There exist two perfectly divisible goods, 1 and 2, where a *portfolio* of these goods is an element of $\mathbb{R}^2_+$. Portfolios will be denoted a, r, s, x, y, and z. Preferences $\succeq_r$ depend on reference portfolio r and exist for each $r \in \mathbb{R}^2_+$. Let $I_r(y) \equiv \{z : z \sim_r y\}$, $R_r(y) \equiv \{z : z \succeq_r y\}$, and $P_r(y) \equiv \{z : z \succ_r y\}$ be the indifference and preferred-to sets. The following assumptions are reference dependent analogues to standard microeconomic theory.

A1 Completeness: *For all r: $\succeq_r$ is complete.*

A2 Transitivity: *For all r: $\succeq_r$ is transitive.*

A3 Strict monotonicity: *For all r, x, y: if $x > y$ then $x \succ_r y$.*

A4 Continuity for a given reference point: *For all r, x : $\{y | \, y \succeq_r x\}$ and $\{z | \, x \succeq_r z\}$ are closed.*

A5 Continuity for a change in reference point: *For all x, y: $\{r | x \succeq_r y\}$ is closed.*

Tversky and Kahneman [1991] develop three axioms describing how preferences adjust in response to a change in reference point: Loss aversion (LA), diminishing sensitivity in gains (DSG), and diminishing sensitivity in losses (DSL).[2] In this paper the first two are assumed, but DSL is replaced with non-diminishing sensitivity in losses (NDSL). Munro and Sugden [2003] point out that the WTP/WTA discrepancy and many other reference dependent regularities[3] can be explained entirely by LA and DSG (p. 412); in fact, multidimensional DSL explains no regularities reported in the literature. Masatlioglu and Uler [2008] test four theories of reference dependence, including a special case of LA and DSG, and found that 91% of subjects behaved consistently with this specification; unfortunately DSL was not tested in the paper because it is difficult to enforce losses in laboratory research.[4]

The assumption of NDSL greatly facilitates analysis, because its effect complements LA and DSG while the effect of DSL works in the opposite direction. NDSL also represents a far less substantial deviation from standard preferences, since DSL imposes non-convexity of preferences while convexity can be maintained under NDSL. Further, it is demonstrated in Appendix III that all of the results in this paper are robust to some DSL, provided its impact on preferences in the region of potential exchange is everywhere smaller than the combined influence of LA and DSG. Since the literature strongly supports LA and DSG while it is silent on DSL, NDSL appears to be a reasonable assumption, particularly given the robustness of the results to limited DSL. The reference dependence axiom in Munro and Sugden (which they label A7) implies LA, DSG, and increasing sensitivity in losses (ISL), and the results in the present paper apply to this model of reference dependence. The results also apply to Kőszegi and Rabin [2006] if DSL is replaced in their model by NDSL; demonstration of this fact is presented after Lemma 1.

[2]These assumptions are related to their namesakes in prospect theory. In this paper these terms will always imply reference dependence over multiple goods and *not* over lotteries.

[3]Including conservatism, the equivalent gain effect, the equivalent loss effect, the inner asymmetric dominance effect, the outer asymmetric dominance, the relative closeness effect, and the advantages/disadvantages effect.

[4]92% of the subjects also behaved consistently with a theory of reference dependence proposed by Masatlioglu and Ok [2006]. The other two theories tested did much worse.

Reference Dependence: *Let r, s, x, and y be elements of $\mathbb{R}^2_+$. Suppose $x_1 > y_1$, $y_2 > x_2$ and $r_2 = s_2$. Consider the change in reference point from r to s.*

A6 Loss Aversion: *If $x_1 \geq s_1 > r_1 = y_1$, then $x \sim_r y \Rightarrow x \succ_s y$.*

A7 Diminishing Sensitivity in Gains: *If $y_1 \geq r_1 > s_1$, then $x \sim_r y \Rightarrow y \succ_s x$.*

A8 Non-diminishing Sensitivity in Losses: *If $s_1 > r_1 \geq x_1$, then $x \sim_r y \Rightarrow x \succeq_s y$.*

The assumption of DSL would require $y \succ_s x$ in A8. To motivate A6, notice that from the perspective of reference point r, y is relatively advantageous to x in good 2 and x is relatively advantageous to y in good 1. Moving from r to s, the relative advantage of x in good 1 from the new reference point decreases by exactly the same quantity as the relative disadvantage of y increases in good 1, while the comparison in good 2 remains the same. Loss aversion implies that the effect of a disadvantage on preferences is greater than the effect of an advantage of identical size. DSG operates through a decreasing relative advantage in good 1 of x over y. For the sake of completeness, DSL (not assumed in this paper) acts on a decreasing relative disadvantage in good 1 of y over x.

So what are reference points? In this paper it is assumed that an individual's reference point is simply his current portfolio; thus his point of reference changes immediately with portfolio adjustment. Bateman, Kahneman, Munro, Starmer and Sugden [2005] specifically test this "current endowment hypothesis" and find reasonably strong support, even though the context was one where Kahneman, in particular, believed this hypothesis was likely to fail. The current endowment hypothesis is also consistent with the above-referenced experimental literature, since preferences in these experiments were elicited almost immediately after a change in subject endowments.[5]

[5] There is also evidence that subjects do not anticipate their own reference dependence, although myopia is neither assumed nor ruled out in this paper. Loewenstein and Adler [1995] report that subjects do not anticipate that their preferences change when new endowments are adopted. Related literature includes Loewenstein, O'Donoghue and Rabin [2003], Van Boven, Lowenstein and Dunning [2003], and Dhar, Huber and Kahn [2007]. Benartzi and Thaler [1995] use a myopic loss aversion model to explain the equity premium puzzle in one-dimensional stochastic setting.

Suppose an individual who obeys A1-8 and whose reference point is his current portfolio acquires some quantity of one good in exchange for some quantity of the other. That is, he begins with portfolio x and acquires portfolio y, such that either $x_1 > y_1$ and $x_2 < y_2$, or $x_1 < y_1$ and $x_2 > y_2$. Without loss of generality, the former is assumed to be the case. It is now shown that the indifference curve through any portfolio $z \in \mathbb{R}^2_+$ will pivot counter-clockwise given the change in reference point from x to y.

Lemma 1 Preference rotation under exchange: *Assume A1- A8, and let a, a', x, y, $z \in \mathbb{R}^2_+$, with $x_1 > y_1$ and $x_2 < y_2$. Suppose trade occurs from x to y. For any z such that $z_1 > y_1$ or $z_2 > x_2$, consider $a \in I_x(z)$ and $a' \in I_y(z)$ such that $a'_1 = a_1$. If $a_1 < z_1$, then $a'_2 < a_2$. If $a_1 > z_1$, then $a'_2 > a_2$. Similarly, if trade occurs from y to x, then if $a_1 < z_1$, $a'_2 > a_2$, and if $a_1 > z_1$, then $a'_2 < a_2$.*

Proof. See Appendix I. ■

Thus for any $z \in \mathbb{R}^2_+$ not dominated on both dimensions by both x and y (that is, it is not the case that both $z_1 \leq y_1$ and $z_2 \leq x_2$), if exchange of good 1 for good 2 takes place, the indifference curve through z will rotate counter-clockwise about z. If exchange of good 2 for good 1 takes place, the indifference curve through z will rotate clockwise. See Figure 1. Under ISL the indifference curve rotation occurs in the dominated region, as well, while under constant sensitivity in losses (CSL) no rotation takes place in this region. Lemma 2 will rule out the possibility that trade ever occurs in the dominated region.

Let $MRS\,(a|b)$ be the marginal rate of substitution at portfolio a given reference point b. Lemma 1 is obviously equivalent to the condition that $MRS\,(z|y) < MRS\,(z|x)$ for permissible z. It is now shown that this condition is satisfied in Kőszegi and Rabin [2006] if their model is altered by assuming NDSL rather than DSL. Let $u\,(a|b) = \sum_{j=1}^{2} m_j\,(a_j) + \mu\,(m_j\,(a_j) - m_j\,(b_j))$ be the utility of portfolio a given reference point b. The functions m_j and μ are continuous, strictly monotonic, and twice-differentiable. Kőszegi and Rabin assume $\mu''(z) \leq 0$ for $z > 0$ (weak DSG) and $\mu''(z) \geq 0$ for $z < 0$

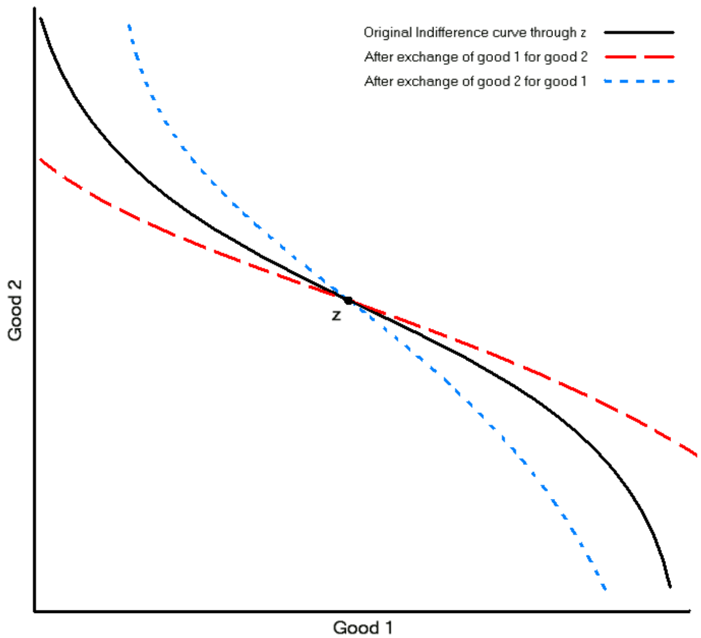

Figure 1: Indifference Curve Rotation Under Exchange

(weak DSL). But consider for the moment DSG and NDSL as in the present paper. Given x, y, and z as in Lemma 1 we have

$$MRS(z|x) = \frac{m_1(z_1)' [1 + \mu'(v_1)]}{m_2(z_2)' [1 + \mu'(v_2)]},$$

where $v_j = m_j(z_j) - m_j(x_j)$ for $j = 1, 2$. By strict monotonicity, it must be the case that

$$MRS(z|y) = \frac{m_1(z_1)' [1 + \mu'(v_1 + \varepsilon_1)]}{m_2(z_2)' [1 + \mu'(v_2 - \varepsilon_2)]},$$

where $\varepsilon_j > 0$ for $j = 1, 2$. NDSL and DSG imply $\mu'(v_1 + \varepsilon_1) \leq \mu'(v_1)$ and $\mu'(v_2 - \varepsilon_2) \geq \mu'(v_2)$, respectively. At least one of these inequalities will be strict unless z is dominated on both dimensions by both x and y. Thus the change in reference point from x to y decreases the marginal rate of substitution at z for all allowable z in Lemma 1. It is worth emphasizing again that there is no empirical evidence of DSL in a deterministic multi-attribute setting while there is ample support for LA and DSG. If one is willing to assume NDSL (or the weaker condition that LA and DSG everywhere dominate DSL, as discussed in Appendix III), Kőszegi and

Rabin implies momentum trading as does Tversky and Kahneman and Munro and Sugden. Analysis of non-smooth models of reference dependence (where indifference curves are kinked at the current endowment) is beyond the scope of the present paper.

Sagi [2006] recognizes that for $x_1 > y_1$ and $x_2 < y_2$ where $y \succeq_x x$, under A1-A8 there exists z such that $z_1 < y_1$ and $z_2 > y_2$ and $y \succ_x z$ but $z \succ_y y$. While he does not specifically provide a characterization of indifference curve rotation under A1-A8, he undoubtedly anticipates such a result. But rather than explore the consequences of indifference curve rotation through reference dependent preferences as in this paper, he objects to the possibility of such preference reversals and rules them out explicitly with his Axiom 1. He provides several justifications for this objection, both normative and behavioral, where the latter is supported by several manifestations of individual regret. Sagi conveys a broad perspective in summarizing his position, stating that "from a modeling perspective, it seems useful to develop an understanding of the types of models that do and do not satisfy 'no regret'," (p. 287) thus placing his Axiom 1 as a benchmark rather than a necessary condition in the study of reference-dependent preferences. Whether individuals actually produce the preferences reversals implied by A1-A8 is an important empirical question that awaits resolution. In this paper I take up Segi's challenge and further the understanding of a large class of models that expressly do not satisfy 'no regret'.

Reference Dependent Exchange

A bit more microeconomic structure is required before considering exchange among agents. It is assumed that preferences relative to any reference point are convex, and that individuals do not engage in utility-diminishing transactions.

A9 (Strict) Convexity of preferences: *Suppose* $r, x, y, z \in \mathbb{R}^2_+$, $x \neq z$, *and* $x, z \in R_r(y)$. *Then for* $\alpha \in (0, 1)$, $\alpha x + (1 - \alpha)z \in P_r(y)$.

A10 (Weak) Utility improvement: *For $x, y \in \mathbb{R}^2_+$, if x is the current portfolio then y will be voluntarily adopted only if $y \succeq_x x$.*

Let $x^i_t \in \mathbb{R}^2_+$ denote the portfolio of individual i at time t, where $t \in \{1, 2, \ldots\}$ and $i \in \{1, 2, \ldots, M\}$ with M finite. In a convenient abuse of notation, M will also denote the set of all agents when the context is clear. When necessary, $x^i_{j,t}$ will denote the quantity of a particular good, $j = 1, 2$, owned by i at time t. Subscripts and superscripts will continue to be suppressed for notational convenience when possible.

There are $M \geq 2$ individuals in a pure exchange economy. An *allocation* $\mathbf{x}$ is defined as an M-tuple of portfolios $\left[x^1, x^2, \ldots, x^M\right]$. There is a fixed, finite quantity of goods available in the economy, $q = (q_1, q_2)^T \in \mathbb{R}^2_{++}$. Denote by Ψ the set of feasible allocations, or

$$\Psi = \left\{ \mathbf{x} \in \mathbb{R}^{2M}_+ : \sum_{i=1}^{M} x^i_j \leq q_j,\, j = 1, 2 \right\}.$$

By A3 and A10 feasibility will always be satisfied with equality. An allocation $\mathbf{x}$ which is Pareto optimal relative to reference allocation $\mathbf{r}$ is denoted $\mathbf{x} \in ps(\mathbf{r})$; if $\mathbf{x}$ is also individually rational relative to $\mathbf{r}$ it may be denoted $\mathbf{x} \in cs(\mathbf{r})$ (i.e., $\mathbf{x}$ is in the contract set relative to $\mathbf{r}$). The special case when $\mathbf{x}$ is Pareto optimal relative itself is called reflexive Pareto optimality. Formally,

Definition 1 *A feasible allocation* $\mathbf{x}$ *is a* reflexive Pareto optimum *if there does not exist* $\mathbf{y} \in \Psi$ *such that* $y^i \succeq_{x^i} x^i$ *for each* $i \in 1, 2, \ldots, N$ *and* $y^i \succ_{x^i} x^i$ *for some* i.

Munro and Sugden propose two assumptions on exchange which, along with with A1-A10, guarantee convergence to a reflexive Pareto optimum.[6] One is an innocuous non-triviality condition on exchange. Intuitively, realized gains from trade are not permitted to vanish asymptotically unless available gains from trade do so as well. Specifically, for $\mathbf{x}, \mathbf{z} \in \Psi$ let $\Phi^i(\mathbf{x}, \mathbf{z}) = \{\mathbf{y} \in \Psi | y^i \succeq_{x^i} x^i \ \wedge \ y^i < z^i\}$, with $L\left(\Phi^i[\mathbf{x}, \mathbf{z}]\right)$ the Lesbesgue measure of this set. Thus, when $\mathbf{z}$ is Pareto-improving from $\mathbf{x}$,

[6] For this result they assume weaker behavioral restrictions than A1-A10, which they label C1-C7. It is a simple exercise to show their convergence result is unaffected by replacing C1-C7 with A1-A10 in this paper.

$L\left(\Phi^i\left[\mathbf{x},\mathbf{z}\right]\right)$ is a measure of the gains from trade for agent i in moving from $\mathbf{x}$ to $\mathbf{z}$.

A11 Limit non-trivial improvement: *If* $\sum_{i=1}^{M} L\left(\Phi^i\left[\mathbf{x}_t,\mathbf{x}_{t+1}\right]\right) \to 0$ *as* $t \to \infty$, *then* $\max_{z\in\Psi} \sum_{i=1}^{M} L\left(\Phi^i\left[\mathbf{x}_t,\mathbf{z}\right]\right) \to 0$ *as* $t \to \infty$.

The other assumption is limit acyclicity of portfolios, which rules out the possibility that an agent prefers to return to an open neighborhood of x^i. It is difficult to motivate acyclicity without implicitly assuming that a reference dependent individual is sufficiently forward-looking to not engage in a sequence of utility-improving trades that will leave him where he started, but such anticipatory behavior has not been assumed (or ruled out) in this paper. However, the restriction is crucial to rule out money pumps and guarantee the stability of exchange. Fortunately when there are only two types of goods, A1-A11 guarantee a version of limit acyclicity sufficient to obtain convergence to a reflexive Pareto optimum.

Lemma 2 Discrete limit acyclity of portfolios: *Assume A1-A11 for all* $i \in M$. *Let* $\langle x_t^i \rangle_{t=1}^T$ *be a sequence of feasible portfolios. Suppose* $d\left(x_t^i, x_{t+1}^i\right) > 0$ *for all* $t \in [1, T-1]$, *where* $d(\cdot)$ *is the Euclidean metric. Then there exists* $\varepsilon > 0$ *such that if* $d\left(x_1^i, x_T^i\right) < \varepsilon$, *then* $x_{t+1}^i \not\succeq_{x_t^i} x_t^i \ \forall t \in [1, T-1]$.

Proof. See Appendix II. ■

This version of limit acyclicity is slightly stronger than C7** in Munro and Sugden. C7** (conforming to present notation) assumes the following: For the sequence $\langle x_t^i \rangle_{t=1}^T$, suppose $z \in \mathbb{R}_+^2$ and $d(z, x_s^i) > 0$ for some $s \in [2, T-1]$. Then there exists $\varepsilon' > 0$ such that if $d\left(x_1^i, z\right) < \varepsilon'$ and $d\left(x_T^i, z\right) < \varepsilon'$, then $x_{t+1}^i \not\succeq_{x_t^i} x_t^i \ \forall t \in [1, T-1]$. Lemma 2 is stronger than C7** in two ways. First, exchange is required to be discrete, rather than the weaker requirement that the distance between the first and some other portfolio in the sequence be positive. Otherwise, Lemma 2 is the special case of C7** when $z = x_1^i$. It is easily verified that the convergence proof in Munro and Sugden goes through under these two conditions.

The assumption that exchange is discrete plays an important role in the proof of Lemma 2. If exchange is continuous it is possible for an individual to return to a previous portfolio. Imagine that some agent engages in continuous exchange exactly orthogonal to his current utility gradient at all points in time. Such exchange will trace out a long run indifference curve (see Munro and Sugden), whose local properties are at all points equivalent to the current indifference curve. An individual could trade along his long run indifference curve for some distance and return to his initial endowment without violating A10. However, any discrete exchange in the current weakly preferred-to set must necessarily lie on a new long run indifference curve that lies strictly above the old, so that the Lesbesgue measure of the feasible region between the new and previous long run indifference curves is strictly positive.[7]

Momentum trading with two agents

Suppose there are two agents in the economy, and a Pareto-improving trade from $\mathbf{r}$ to $\mathbf{x}$ has occurred as in Figure 2, where $r_1^1 > x_1^1$ and $r_2^1 < x_2^1$. The inner (light) lens-shaped region represents the individually rational set at $\mathbf{x}$ when the reference point is $\mathbf{r}$, while the dark and light lens-shaped regions together represent the individually rational set at $\mathbf{x}$ after trade has occurred. The adjusted individually rational set contains the original by Lemma 1, so more potential trades which satisfy A10 are available after the shift in preferences. Also note that further exchange must necessarily occur in the same direction as before; that is, agent 1 would give good 1 to agent 2 in exchange for good 2. In fact, it is now shown that the entire contract

[7] To maintain limit acyclicty under continuous exchange, A11 can be strengthened as follows: For $i \in M$, if $d\left(x_t^i, x_{t+k}^i\right) > 0$ for $k > 0$, then $\int_{m=0}^{k} L\left(\Phi^i\left[\mathbf{x}_{t+m}, \mathbf{x}_{t+m+1}\right]\right) dm > 0$. This condition guarantees that if an individual's portfolio is eventually adjusted by a discrete amount, then the measure of his gains from trade is strictly positive. The proof of Lemma 2 is then easily adjusted to apply to this individual. Intuitively, this assumption keeps each agent a strictly positive distance above his initial long run indifference curve if his net exchange of goods has been strictly positive.

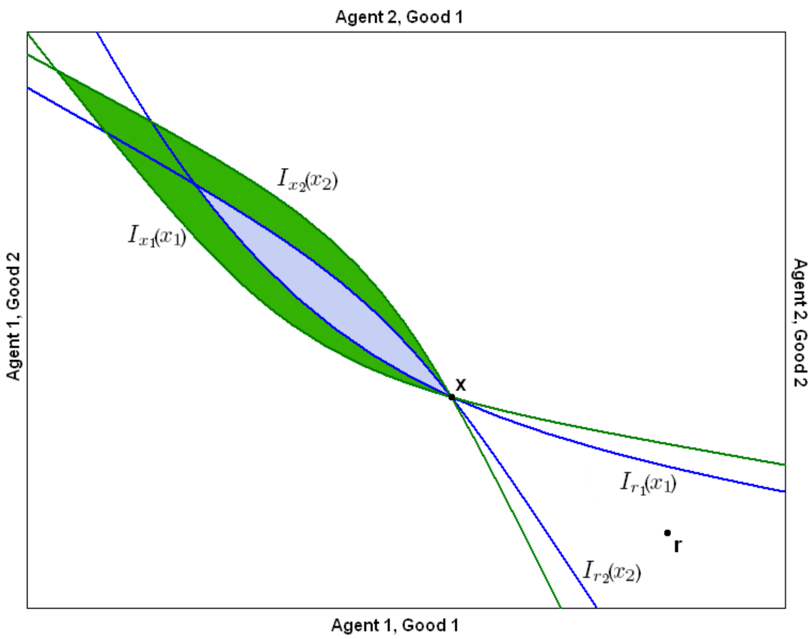

Figure 2: Shift in the Individually Rational Set Due to Reference Dependence

set relative to **r** shifts up and to the left after the reallocation to **x**, and any reflexive Pareto optima reached under A1-A11 is separated from **r** by $ps(\mathbf{r})$.

Proposition 1 Momentum Trading (Two Agents): *Suppose there are two agents and assume A1-A11. Let* $\omega \in \Psi$ *be a non-optimal initial endowment. Let* $\mathbf{x} \in cs(\omega)$*, and without loss of generality suppose* $x_1^1 < \omega_1^1$ *and* $x_2^1 > \omega_2^1$*. If* $\mathbf{y} \in cs(\mathbf{x})$ *then* $y_1^1 < x_1^1$ *and* $y_2^1 > x_2^1$*. Further, for any reflexive Pareto optimum* $\mathbf{z}$ *attainable from* ω*, there exists an allocation* $\mathbf{a} \in ps(\omega)$ *such that* $a_1^1 > z_1^1$ *and* $a_2^1 < z_2^1$*.*

Proof. Under the standing assumptions on preferences the Pareto set relative to any feasible reference allocation is a 1-dimensional manifold from origin to origin in the Edgeworth box. Beginning with agent 1's origin and tracing a path to agent 2's origin, by strict monotonicity the Pareto set is also monotonically increasing in both goods for agent 1. Since the contract set relative to any feasible allocation and reference point is non-empty, $ps(\omega)$ partitions the set of non-Pareto optimal allocations into two sets, $se(\omega)$ and $nw(\omega)$ (*southeast* and *northwest*). For $\mathbf{a} \in se(\omega)$, $MRS^1(a^1|\omega^1) <$

$MRS^2(a^2|\omega^2)$. For $\mathbf{b} \in nw(\omega)$, $MRS^1(b^1|\omega^1) > MRS^2(a^2|\omega^2)$. By assumption $MRS^1(y^1|x^1) = MRS^2(y^2|x^2)$. By Lemma 1, $MRS^1(y^1|\omega^1) < MRS^1(y^1|x^1)$ and $MRS^2(y^2|\omega^2) > MRS^2(y^2|x^2)$. Thus $y \in nw(\omega)$, proving the first part of the proposition.

It is trivial given marginal rates of substitution at ω that any initial exchange must take place in the northwest quadrant of ω. Further note that at any attainable allocation $\mathbf{b}$ in this quadrant, it must be the case that $b^i \succ_{b^i} \omega^i$ for $i \in \{1, 2\}$ by Lemma 2. This fact directly rules out exchange ever taking place in the southwest or northeast quadrants of ω. Since both agents respect A9 (convexity), allocations in the southeast quadrant are ruled out, as well. Thus for any reflexive Pareto optimum $\mathbf{z}$ attainable through voluntary exchange, $z_1^1 > \omega_1^1$ and $z_2^1 > \omega_2^1$. Suppose $\mathbf{z} \in se(\omega)$. By assumption, $MRS^1(z^1|\omega^1) < MRS^2(z^2|\omega^2)$. But by Lemma 1, $MRS^1(z^1|z^1) < MRS^1(z^1|\omega^1)$ and $MRS^2(z^2|z^2) > MRS^2(z^2|\omega^2)$. Thus $MRS^1(z^1|z^1) < MRS^2(z^2|z^2)$, contradicting the assumption that $\mathbf{z}$ is a reflexive Pareto optimum. ■

Therefore, in a two agent economy where the initial endowment ω is not Pareto optimal the final allocation adopted represents a greater volume of exchange than would be expected if attention were restricted to the initial Pareto set. For a parametric example of this momentum trading effect, consider the following reference dependent CES function from Munro and Sugden:

$$u(x, r) = Q(r)\left[\sum\nolimits_j \gamma_j r_j^{\rho-\beta} x_j^{\beta}\right]^{1/\beta},$$

where $\sum_j \gamma_j = 1$ and $1 > \rho \geq \beta > -\infty$ (agent superscripts have been suppressed). Let $Q(r) = 1$, $\rho = 0.75$, $\beta = 0.25$, and $\gamma_1 = \gamma_2 = 0.5$, and normalize the aggregate quantity of each good to one. Figure 3 illustrates how trade 'pushes' the current Pareto set in the same direction as net exchange. For example, a trade from $\mathbf{a}$ to $\mathbf{b}$ would shift the current Pareto set up and to the left, from $ps(\mathbf{a})$ to $ps(\mathbf{b})$. A trade from $\mathbf{a}$ to $\mathbf{d}$ would shift the current Pareto set from $ps(\mathbf{a})$ to $ps(\mathbf{d})$, and so forth.

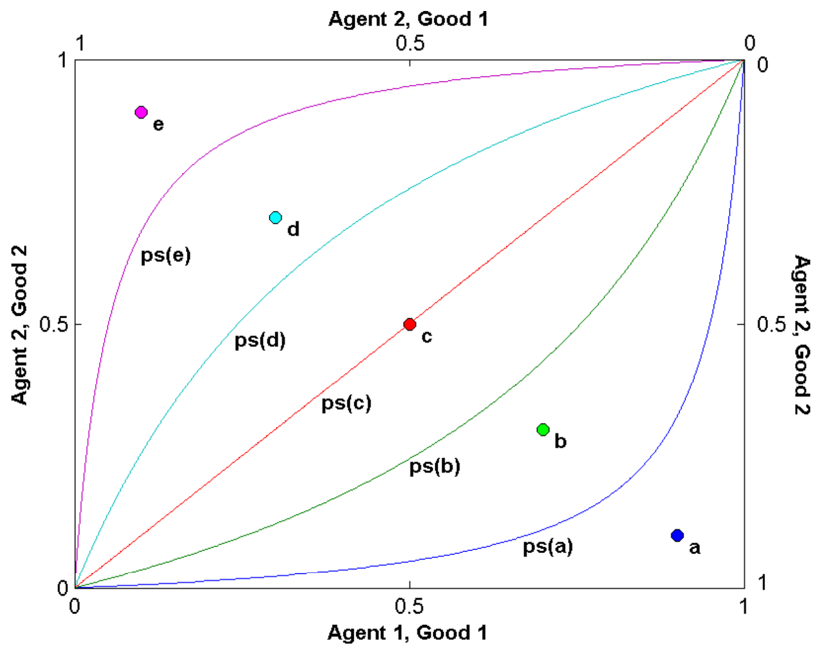

Figure 3: Contract Curves, Reference Dependent CES Function

In Figure 3, $ps(\mathbf{c})$ actually coincides with the entire reflexive Pareto set, a point that will be verified momentarily. So suppose the economy begins at initial endowment $\mathbf{a}$. If preferences are fixed at $\mathbf{a}$, it would be reasonable to expect exchange to eventually converge to some allocation in $ps(\mathbf{a})$. However, if preferences are instead reference dependent, we would expect that exchange will converge to an allocation in the reflexive Pareto set $ps(\mathbf{c})$. Clearly $ps(\mathbf{c})$ is further from $\mathbf{a}$ than $ps(\mathbf{a})$ in any reasonable sense, and thus reference dependence has generated a momentum trading effect.

Of course, this example has so far ignored the static endowment effect. Because at allocation $\mathbf{a}$ each agent i possesses mostly good i, his preferences at $\mathbf{a}$ are inclined towards that good, inhibiting the potential for exchange; this is why $ps(\mathbf{a})$ is so close to $\mathbf{a}$. So reference dependence pulls the Pareto set closer to the endowment (the static endowment effect), but through trade it also pushes it away (the momentum trading effect). Is it possible to determine if one effect dominates the other? For identical individuals with reference dependent CES preferences, the two effects actually cancel each other out. To see why, first note agent 1's marginal rate of substitution (the agent superscript has again been suppressed):

$$\frac{\gamma_1 r_1^{\rho-\beta} x_1^{\beta-1}}{\gamma_2 r_2^{\rho-\beta} x_2^{\beta-1}}$$

Under the current endowment hypothesis, $r_j = x_j$ for $j = 1, 2$, so the expression simplifies to $\gamma_1 x_1^{\rho-1}/\gamma_2 x_2^{\rho-1}$. For an allocation to be a reflexive Pareto optimum, it must be the case that the marginal rates of substitution of the two agents are equal at the current reference point; thus $\gamma_1 x_1^{\rho-1}/\gamma_2 x_2^{\rho-1} = \gamma_1 (1 - x_1)^{\rho-1} / \gamma_2 (1 - x_2)^{\rho-1}$, an expression that simplifies to $x_1 = x_2$ (the first equality is obtained by substituting the aggregate resource constraint for agent 2's portfolio). This is the reason why $ps(\mathbf{c})$ in Figure 3 represents the entire reflexive Pareto set.

But what if each individual shares an identical reference point which does not adjust with exchange; i.e., what if we have identical reference independent individuals?[8] The Pareto set relative to this common reference portfolio is, surprisingly, also characterized by the condition $x_1 = x_2$, *regardless of the reference point adopted* (when equating the marginal rates of substitution the common reference points cancel out). **Thus the reference independent Pareto set coincides with the reflexive Pareto set when agents share identical CES preferences.**

Returning to Figure 3, when **a** is the initial endowment the endowment effect causes the current Pareto set to shift from $ps(\mathbf{c})$ to $ps(\mathbf{a})$, provided that the individuals shared the same reference point prior to the endowment being distributed. Then through exchange the momentum trading effect moves the final Pareto set back to $ps(\mathbf{c})$. So we have a two agent economy where reference dependent individuals are not "much less prone to exchanges than final states agents" as claimed by Kahneman; they can be expected to generate the same volume of exchange.

[8] It would perhaps most natural to assume the individuals share the origin as a common initial reference point (this would certainly be the case in most experiments, where the entire endowment is assigned by the experimenter), although under CES preferences we may only get arbitrarily close to the origin.

Momentum trading with many agents

Let ω be the initial endowment and suppose $\mathbf{x} \in ps(\omega)$ is strictly interior. Further suppose that in moving from ω to $\mathbf{x}$ each agent must give up some quantity of one good in exchange for the other; i.e., there is full participation and no free lunch in moving to $\mathbf{x}$. Let B be the set of agents such that for all $i \in B$, $x_1^i - \omega_1^i > 0$ ("buyers"). Let S be the set of agents such that for all $j \in S$, $x_1^j - \omega_1^j < 0$ ("sellers"). By assumption, $B \bigcup S = M$. It is now shown that there exists an open set of reflexive Pareto optima attainable by Pareto-improving trade from $\mathbf{x}$ such that for any allocation in this set, on net buyers remain buyers and sellers remain sellers.

Proposition 2 Momentum Trading (Existence): *Assume A1-A11 for all $k \in M$. Let $\omega \in \Psi$ be a non-optimal initial endowment. Let $\mathbf{x} \in ps(\omega)$ be a strictly interior feasible Pareto optimum relative to ω such that $\left(x_1^k - \omega_1^k\right) * \left(x_2^k - \omega_2^k\right) < 0$ for all $k \in M$. Then there exists an open set of reflexive Pareto optima Z such that for each $\mathbf{z} \in Z$, $sgn \sum_{k \in A} \left(z^k - x^k\right) = sgn \sum_{k \in A} \left(x^k - \omega^k\right)$ for $A \in \{B, S\}$ (here sgn is the signum function).*

Proof. Let $\mathbf{x}_{s(t)}$ be an allocation at time t and stage s, where $\mathbf{x}_{0(0)} = \omega$ and $\mathbf{x}_{0(1)} = \mathbf{x}$. For each stage s, a sequence $\langle \mathbf{x}_{s(t)} \rangle_{t=0}^{\infty}$ of Pareto-improving trades is constructed such that $\underset{t \to \infty}{lim}\ \mathbf{x}_{s(t)} = \mathbf{x}_{s+1(0)}$ is a reflexive Pareto optimum restricted to a subset of agents, and $sgn \sum_{k \in A} \left(x_{s+1(0)}^k - x^k\right) = sgn \sum_{k \in A} \left(x^k - \omega^k\right)$ for $A \in \{B, S\}$. In each successive stage this subset will strictly contain the previous one, and since M is finite there exists a stage where the limiting allocation is a reflexive Pareto optimum.

Consider a reallocation from $\mathbf{x}_{0(0)}$ to $\mathbf{x}_{0(1)}$. $MRS_i\left(x_{0(1)}^i | x_{0(0)}^i\right) = MRS_j\left(x_{0(1)}^j | x_{0(0)}^j\right)$ for all $i, j \in M$ by assumption. By Lemma 1, $MRS_i\left(x_{0(1)}^i | x_{0(1)}^i\right) > MRS_j\left(x_{0(1)}^j | x_{0(1)}^j\right)$ for all $i \in B$, $j \in S$. Let $\underline{p}_{s(t)} = \underset{j \in S}{max}\ MRS^j\left(x_{s(t)}^j | x_{s(t)}^j\right)$ and $\bar{p}_{s(t)} = \underset{i \in B}{min}\ MRS^i\left(x_{s(t)}^i | x_{s(t)}^i\right)$. Letting good 2 be the numeraire, at each time t of stage s a price $p_{s(t)}$ will be chosen such that $p_{s(t)} \in \left(\underline{p}_{s(t)}, \bar{p}_{s(t)}\right)$. Let the *long-run demand* for $k \in M$ be the set of locally optimal portfolios on the budget line defined by

$x^k_{s(t)}$ and price $p_{s(t)}$ (if long-run indifference sets are strictly convex this portfolio is unique). Let $y^k_{0(1)}$ be the element of long-run demand where $sgn\left(y^k_{0(1)} - x^k_{0(1)}\right) = sgn\left(x^k_{0(1)} - x^k_{0(0)}\right)$ and net trade is minimized; such a portfolio exists trivially by Lemma 1 and the relationship between $p_{0(1)}$ and k's marginal rate of substitution. If $\sum_{k \in M} y^k_{0(1)} = q$, $\mathbf{y}_{0(1)}$ is a reflexive Pareto optimum and the proposition holds. If not, without loss of generality assume good 1 is in excess demand (by Walras' Law we need only consider good 1).

For all $j \in S$, let $x^j_{0(2)} = y^j_{0(1)}$. For interior j, $MRS^j\left(x^j_{0(2)}|x^j_{0(2)}\right) = p_{0(1)}$. For j possessing only good 2, $MRS^j\left(x^j_{0(2)}|x^j_{0(2)}\right) < p_{0(1)}$. Let $\alpha_{s(t)} < 1$ be the ratio of aggregate supply to demand of good 1 at stage s and time t. Then if $x^i_{0(2)} = x^i_{0(1)} + \alpha_{0(1)}\left[y^i_{0(1)} - x^i_{0(1)}\right]$ for all $i \in B$, $\mathbf{x}_{0(2)}$ represents a feasible reallocation. By Lemma 1, $MRS^i\left(x^i_{0(2)}|x^i_{0(2)}\right) > p_{0(1)}$ for all $i \in B$. If all $j \in S$ have corner portfolios the stage is complete. Otherwise, by assumption $p_{0(2)} \in \left(p_{0(1)}, \bar{p}_{0(2)}\right)$. Choose $p_{0(2)}$ so good 1 remains in excess demand; such a price exists since the aggregate excess demand of good 1 is strictly positive as $p_{0(2)}$ approaches $p_{0(1)}$.

Construct the sequence $\langle \mathbf{x}_{0(t)} \rangle$ following the procedure outlined above, appropriately restricting the choice of prices so that good 1 is in excess demand at every reallocation. One of two things occurs. At some finite time T, all $j \in S$ only possess good 2, in which case $\mathbf{x}_{1(0)} = \mathbf{x}_{0(T)}$. Or at $\mathbf{x}_{1(0)} = \lim_{t \to \infty} \mathbf{x}_{0(t)}$, some non-empty subset of agents $B_1 \subseteq B$ has a marginal rate of substitution equal to the limiting price in stage 0. If the former case is true, let $B_1 = \underset{i \in B}{argmin}\left[MRS^i\left(x^i_{1(0)}|x^i_{1(0)}\right)\right]$. If $B_1 = B$, a reflexive Pareto optimum has been reached. If not, the sellers in stage 1 are now $S \bigcup B_1$, and $p_{1(1)}$ is chosen such that there is excess demand at $\mathbf{y}_{1(1)}$. Stage 1 proceeds as stage 0, culminating in $B_2 \supset B_1$, a non-empty subset of buyers through stages 0 and 1 who become sellers in stage 2. In each stage a strictly positive number of buyers become sellers, and since there are a finite number of (original) buyers the process must converge to a reflexive Pareto optimum.

Note that all $j \in S$ remain sellers throughout. All $i \in B$ remained buyers in the first stage. Some buyers potentially became sellers in subsequent stages, but since this group formed a weak subset of active sellers, while all buyers remained $i \in B$, then $sgn \sum_{i \in B} \left(x^i_{s(0)} - x^i \right) = sgn \sum_{i \in B} \left(x^i - \omega^i \right)$. ■

Thus under A1-A11, there always exists an open set of attainable reflexive Pareto optima where momentum trading would be realized. However, more structure on preferences is necessary to guarantee momentum trading with more than two agents.

Proposition 3 Momentum Trading (Sufficient Condition): *Assume A1-A11 and further suppose that preferences are homothetic relative to any reference point for all $k \in M$. Let $\omega \in \Psi$ be a non-optimal initial endowment. Let $\mathbf{x} \in ps(\omega)$ be a feasible Pareto optimum relative to ω such that $\left(x^k_1 - \omega^k_1 \right) * \left(x^k_2 - \omega^k_2 \right) < 0$ for all $k \in M$. Then for all $\mathbf{z} \in ps(\mathbf{x})$ where $\left(z^k_1 - x^k_1 \right) * \left(z^k_2 - x^k_2 \right) < 0$ for all $k \in M$, $sgn \sum_{k \in A} \left(z^k - x^k \right) = sgn \sum_{k \in A} \left(x^k - \omega^k \right)$ for $A \in \{B, S\}$.*

Proof. Partition $i \in B$ into subsets B_1 and B_2, where $z^i_1 > x^i_1$ for all $i \in B_1$, and $z^i_1 < x^i_1$ for all $i \in B_2$. That is, B_1 is the set of good 1 buyers from ω to $\mathbf{x}$ who remain buyers from $\mathbf{x}$ to $\mathbf{z}$, and B_2 is the set of buyers who become sellers. Similarly, partition $j \in S$ into subsets S_1 and S_2 where $z^j_1 < z^j_1$ for all $j \in S_1$, and $z^j_1 > z^j_1$ for all $j \in S_2$ (S_1 is the set of sellers who remain sellers, S_2 is the set of sellers who become buyers). Let $p = MRS^k \left(x^k | \omega^k \right)$ for all $k \in M$.

For all $i \in B_2$, $MRS^i \left(x^i | x^i \right) > p$ by Lemma 1. By homotheticity, $MRS^i \left(z^i | x^i \right) > MRS^i \left(x^i | x^i \right)$, so $MRS^i \left(z^i | x^i \right) > p$ for all $i \in B_2$. Similarly, for all $j \in S_2$, $MRS^j \left(x^j | x^j \right) < p$ by Lemma 1. By homotheticity, $MRS^j \left(z^j | x^j \right) < MRS^j \left(x^j | x^j \right)$, so $MRS^j \left(z^j | x^j \right) < p$ for all $j \in S_2$. Thus $\mathbf{z} \notin ps(\mathbf{x})$ unless B_2 or S_2 (or both) are empty. Without loss of generality, suppose $S_2 = \emptyset$. The proposition trivially holds for S. Since $\sum_{i \in B_1} \left(z^i - x^i \right) = \sum_{i \in B_2} \left(z^i - x^i \right) + \sum_{j \in S_1} \left(z^j - x^j \right)$, the proposition holds for B as well. ■

Corollary 1 *Assume A1-A11 and further suppose that long-run preferences are strictly convex and homothetic relative to any reference point for all $k \in M$. Let $\omega \in \Psi$ be a non-optimal initial endowment. Let $\mathbf{x} \in ps(\omega)$ be a feasible*

Pareto optimum relative to ω *such that* $\left(x_1^k - \omega_1^k\right) * \left(x_2^k - \omega_2^k\right) < 0$ *for all* $k \in M$*. Then for all reflexive Pareto optima* $\mathbf{z}$ *where* $\left(z_1^k - x_1^k\right) * \left(z_2^k - x_2^k\right) < 0$ *for each* $k \in M$*,* $sgn \sum_{k \in A} \left(z^k - x^k\right) = sgn \sum_{k \in A} \left(x^k - \omega^k\right)$ *for* $A \in \{B, S\}$*.*

Proof. Obvious from the proof of Proposition 3. ■

It is worth noting that Munro and Sugden show long-run preferences are strictly convex and homothetic for the reference dependent CES function. Therefore, for this class of preferences we know by Corollary 1 that momentum trading *must necessarily occur.*

Conclusion

This paper delivers an unexpected result. In addition to implying the familiar endowment effect, loss aversion and diminishing sensitivity in gains also imply a competing momentum effect which can be large enough to erase the bias against exchange associated with the endowment effect. This momentum effect is derived from assumptions with strong support in the literature, and does not conflict with any of the observed reference dependent empirical regularities. It is important to emphasize that myopia was neither assumed nor ruled out to obtain the results. Provided that agents do not trade outside of their current individually rational sets and eventually exhaust all gains from trade, any degree of anticipatory rationality is permitted.

Why has momentum trading not been observed in the experimental economics literature? One reason is that goods are insufficiently divisible in most of these experiments, but more importantly preferences have always been elicited before and after one change in subject endowments, rather than during several changes in sequence. Interesting paths for future laboratory research include eliciting preferences over sequences of endowments, and testing the diminishing versus increasing sensitivity in losses hypotheses.

It is tempting to relate the momentum trading effect studied in this paper to the growing literature on momentum trading in finance. For example, Barber and Odean [2000] document that individual investors tend to lose money by over-trading in equity markets. However, it is not clear the mechanisms that generate these two different momentum trading results are at all the same; for example, over-confidence and herding are often expected to play a large role in the finance literature on momentum trading.

Much closer to the momentum trading story in this paper, Dhar et al. [2007] study the 'shopping momentum effect.' Consumers who choose to purchase an item in a department store are shown to be more likely to purchase a second item, after controlling for desirable aggregation of purchases to offset time and travel costs. "Shopping momentum arises from the idea that shopping has an inertial quality ... which once crossed makes further purchases more likely." (p. 3) Potentially benefitting from such shopping momentum, retailers use loss leaders to attract customers to a store in the expectation that sales of other goods will increase [Mulhern and Padgett 1995]. If one were to treat department store purchases as one type of good and cash as a composite second good, this observed shopping momentum effect is consistent with the momentum effect characterized in this paper. In any event, momentum exchange in a multi-attribute deterministic setting has been empirically observed and may potentially imply real economic consequences. The characterization of momentum trading in the present paper may usefully inform the exploration of this and potentially other such behavior.

Appendix I - Proof of Lemma 1

First note that if Lemma 1 is established for a change in reference point from x to y, then the consequences of a change from y to x are trivial, since the two sets $I_x(z)$ and $I_y(z)$ will already have been sufficiently characterized. It will be useful to introduce the portfolio $s = (y_1, x_2)$.

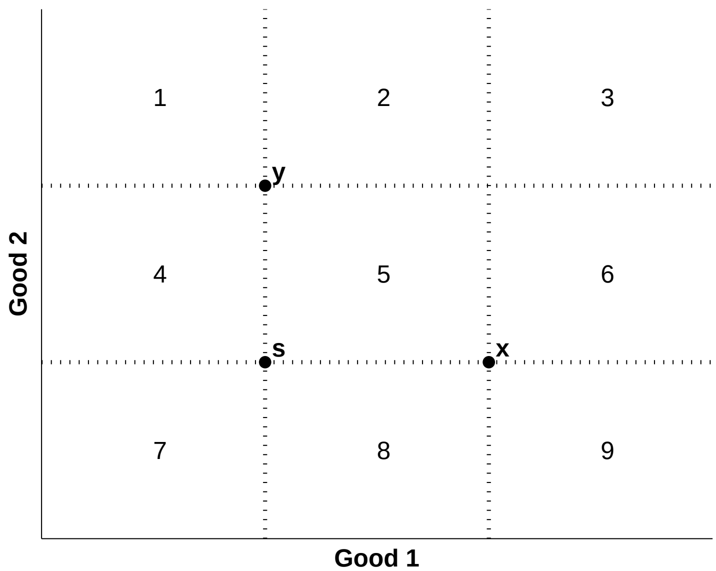

Figure 4: Distinct Regions of Reference Dependence

The strategy for the proof is as follows. In **Step A** it will be shown that for a change in reference point from x to s, for any z such that $z_1 > y_1$ the indifference curve through z will rotate counter-clockwise; this corresponds to z in regions $2, 3, 5, 6, 8$, and 9 in Figure 4. It will also be shown that under ISL the proposition holds for a change in reference point from x to s for any z such that $z_1 \leq y_1$ (regions $1, 4$, and 7), while under CSL the preference relation between any two portfolios in these regions is unaffected. Now fix the indifference map from reference point s.

In **Step B** it will be shown that for the new indifference map defined in **Step A**, a change in reference point from s to y will rotate the indifference curve through z counter-clockwise for any z such that $z_2 > x_2$; this corresponds to z in regions

$1, 2, 3, 4, 5$, and 6 in Figure 4. Under ISL this rotation also occurs in regions 7, 8, and 9, while under CSL the preference relation between any two portfolios in these regions is unaffected. It will then have been demonstrated that the proposition holds for z in any region but 7 in Figure 4, finishing the proof. If ISL is assumed, the proposition holds for z in region 7, as well. Under very mild rationality restrictions no sequence of trades will ever take the individual into region 7, so the fact that Lemma 1 does not hold in this region under CSL is of little consequence.

Step A: Change in reference point from x to s

There are three cases to consider, $z_1 \geq x_1$, $z_1 \in (y_1, x_1)$, and $z_1 \leq y_1$.

Case A1: $z_1 \geq x_1$**.** This corresponds to regions $3, 6$, and 9 in Figure 4.

Subcase A1a: $a_1 < z_1$**.**

- Suppose $a_1 \geq x_1$. A7 implies $a \succ_s z$[9].
- Suppose $a_1 \in (y_1, x_1)$. A6 implies $a \succ_{(a_1,x_2)} z$. By A3, A4 and IVT, $\exists \alpha > 0 : z \sim_{(a_1,x_2)} (a_1, a_2 - \alpha)$. By A7, $(a_1, a_2 - \alpha) \succ_s z$.
- Suppose $a_1 \leq y_1$. A6 implies $a \succ_s z$.

Thus, for all $a_1 < z_1$, by A3, A4 and IVT, $\exists \varepsilon > 0 : z \sim_s (a_1, a_2 - \varepsilon)$.

Subcase A1b: $a_1 > z_1$**.** Since $a_1 > z_1 \geq x_1$, A7 implies $z \succ_s a$. By A3, A4 and IVT, $\exists \delta > 0 : z \sim_s (a_1, a_2 + \delta)$.

Case A2: $z_1 \in (y_1, x_1)$**.** This corresponds to regions $2, 5$, and 8 in Figure 4.

Subcase A2a: $a_1 < z_1$**.** A8 implies $a \succeq_{(z_1,x_2)} z$. By A3, A4 and the intermediate value theorem (IVT), $\exists \alpha \geq 0 : z \sim_{(z_1,x_2)} (a_1, a_2 - \alpha)$.

[9]Actually, A7 implies that a change in reference point from s to x strictly favors z relative to a. Since reference dependence is assumed to be fixed rather than path dependent, then the change in reference point from x to s must strictly favor a relative to z

- Suppose $a_1 \in (y_1, z_1)$. A6 implies $(a_1, a_2 - \alpha) \succ_{(a_1,x_2)} z$. By A3, A4 and IVT, $\exists \beta > \alpha : z \sim_{(a_1,x_2)} (a_1, a_2 - \beta)$. A7 implies $(a_1, a_2 - \beta) \succ_s z$.
- Suppose $a_1 \leq y_1$. A6 implies $(a_1, a_2 - \alpha) \succ_s z$.

Thus, for all $a_1 < z_1$, by A3, A4 and IVT, $\exists \varepsilon > 0 : z \sim_s (a_1, a_2 - \varepsilon)$.

Subcase A2b: $a_1 > z_1$**.**

- Suppose $a_1 \geq x_1$. A6 implies $z \succ_{(z_1,x_2)} a$. By A3, A4 and IVT, $\exists \alpha > 0 :$ $z \sim_{(z_1,x_2)} (a_1, a_2 + \alpha)$. By A7, $z \succ_s (a_1, a_2 + \alpha)$.
- Suppose $a_1 \in (z_1, x_1)$. A8 implies $z \succeq_{(a_1,x_2)} a$. By A3, A4 and IVT, $\exists \beta \geq 0 : z \sim_{(a_1,x_2)} (a_1, a_2 + \beta)$. A6 implies $z \succ_{(z_1,x_2)} (a_1, a_2 + \beta)$. By A3, A4 and IVT, $\exists \gamma > \beta : z \sim_{(z_1,x_2)} (a_1, a_2 + \gamma)$. A7 implies $z \succ_s (a_1, a_2 + \gamma)$.

Thus, for all $a_1 > z_1$, by A3, A4 and IVT, $\exists \delta > 0 : z \sim_s (a_1, a_2 + \delta)$.

Case A3: $z_1 \leq y_1$**.** This corresponds to regions $1, 4$, and 7 in Figure 4.

Subcase A3a: $a_1 < z_1$**.** A8 implies $a \succeq_s z$. By A3, A4 and IVT, $\exists \varepsilon \geq 0 : z \sim_s (a_1, a_2 - \varepsilon)$. Note $\varepsilon > 0$ under ISL, and $\varepsilon = 0$ under CSL.

Subcase A3b: $a_1 > z_1$**.**

- Suppose $a_1 \leq y_1$. A8 implies $z \succeq_s a$. By A3, A4 and IVT, $\exists \delta \geq 0 : z \sim_s (a_1, a_2 + \delta)$. Note $\delta > 0$ under ISL, and $\delta = 0$ under CSL.
- Suppose $a_1 \in (y_1, x_1)$. A8 implies $z \succeq_{(a_1,x_2)} a$. By A3, A4 and IVT, $\exists \alpha \geq 0 : z \sim_{(a_1,x_2)} (a_1, a_2 + \alpha)$. A6 implies $z \succ_s (a_1, a_2 + \alpha)$. By A3, A4 and IVT, $\exists \delta > \alpha \geq 0 : z \sim_s (a_1, a_2 + \delta)$.
- Suppose $a_1 \geq x_1$. A6 implies $z \succ_s a$. By A3, A4 and IVT, $\exists \delta > 0 : z \sim_s (a_1, a_2 + \delta)$.

To summarize, take any $z \in \mathbb{R}^2_+$, and suppose $z \sim_x a$. It was demonstrated in **Step A** that for $a_1 < z_1$, there exists $\varepsilon \geq 0$ such that $z \sim_s (a_1, a_2 - \varepsilon)$, and for $a_1 > z_1$, there exists $\delta \geq 0$ such that $z \sim_s (a_1, a_2 + \delta)$. Further, if $z_1 \leq y_1$, $a_1 \leq y_1$, and CSL is assumed, then $\varepsilon = \delta = 0$. If at least one of these conditions is not met, then $\varepsilon > 0$ and $\delta > 0$.

Step B: Change in reference point from s to y

Case B1: $z_2 \geq y_2$. This corresponds to regions $1, 2$, and 3 in Figure 4.

Subcase B1a: $a_1 < z_1$. By A3, $a_2 - \varepsilon > z_2 \geq y_2$. Therefore, A7 implies $(a_1, a_2 - \varepsilon) \succ_y z$. Since $\varepsilon \geq 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$.

Subcase B1b: $a_1 > z_1$.

- Suppose $a_2 + \delta \leq x_2$. A6 implies $z \succ_y (a_1, a_2 + \delta)$.
- Suppose $a_2 + \delta \in (x_2, y_2)$. A7 implies $z \succ_{(y_1, a_2+\delta)} (a_1, a_2 + \delta)$. By A3, A4 and IVT, $\exists \alpha > 0 : z' = (z_1, z_2 - \alpha) \sim_{(y_1, a_2+\delta)} (a_1, a_2 + \delta)$.
  - If $z'_2 \geq y_2$, A6 implies $z' \succ_y (a_1, a_2 + \delta)$, and thus by A3, $z \succ_y (a_1, a_2 + \delta)$.
  - If $z'_2 < y_2$, A6 implies $z' \succ_{(y_1, z'_2)} (a_1, a_2 + \delta)$. By A3, A4 and IVT, $\exists \beta > \alpha : z'' = (z_1, z_2 - \beta) \sim_{(y_1, z'_2)} (a_1, a_2 + \delta)$. By A8, $z'' \succeq_y (a_1, a_2 + \delta)$. Thus by A3, $z \succ_y (a_1, a_2 + \delta)$.
- Suppose $a_2 + \delta \geq y_2$. A7 implies $z \succ_y (a_1, a_2 + \delta)$.

Since $\delta \geq 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 > a_2$.

Case B2: $z_2 \in (x_2, y_2)$. This corresponds to regions $4, 5$, and 6 in Figure 4.

Subcase B2a: $a_1 < z_1$. A7 implies $(a_1, a_2 - \varepsilon) \succ_{(y_1, z_2)} z$. By A3, A4 and IVT, $\exists \alpha > 0 : (a_1, a_2 - \varepsilon) \sim_{(y_1, z_2)} z' = (z_1, z_2 + \alpha)$.

- If $a_2 - \varepsilon \geq y_2$, then A6 implies $(a_1, a_2 - \varepsilon) \succ_y z'$, and by A3, $(a_1, a_2 - \varepsilon) \succ_y z$.

- If $a_2 - \varepsilon < y_2$, then A6 implies $(a_1, a_2 - \varepsilon) \succ_{(y_1, a_2-\varepsilon)} z'$. By A3, A4 and IVT, $\exists \beta > \alpha : (a_1, a_2 - \varepsilon) \sim_{(y_1, a_2-\varepsilon)} z'' = (z_1, z_2 + \beta)$. A8 implies $(a_1, a_2 - \varepsilon) \succeq_y z''$. Thus by A3, $(a_1, a_2 - \varepsilon) \succ_y z$.

Since $\varepsilon \geq 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$.

Subcase B2b: $a_1 > z_1$**.**

- Suppose $a_2 + \delta \leq x_2$. A6 implies $z \succ_{(y_1, z_2)} (a_1, a_2 + \delta)$. By A3, A4 and IVT, $\exists \alpha > 0 : z' = (z_1, z_2 - \alpha) \sim_{(y_1, z_2)} (a_1, a_2 + \delta)$. By A8, $z' \succeq_y (a_1, a_2 + \delta)$. Then A3 implies $z \succ_y (a_1, a_2 + \delta)$.
- Suppose $a_2 + \delta \in (x_2, z_2)$. A7 implies $z \succ_{(y_1, a_2+\delta)} (a_1, a_2 + \delta)$. By A3, A4 and IVT, $\exists \beta > 0 : (a_1, a_2 + \delta) \sim_{(y_1, a_2+\delta)} z'' = (z_1, z_2 - \beta)$. By A6, $z'' \succ_{(y_1, z''_2)} (a_1, a_2 + \delta)$. By A3, A4 and IVT, $\exists \gamma > \beta : (z_1, z_2 + \gamma) \sim_{(y_1, z''_2)} (a_1, a_2 + \delta)$. A8 implies $(z_1, z_2 + \gamma) \succeq_y (a_1, a_2 + \delta)$. Thus by A3, $z \succ_y (a_1, a_2 + \delta)$.

Since $\delta > 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 > a_2$.

Case B3: $z_2 \leq x_2$ **and** $z_1 > y_1$**.** This corresponds to regions 8 and 9 in Figure 4. Since $z_1 > y_1$, it was established in **Step A** that $\varepsilon > 0$ and $\delta > 0$.

Subcase B3a: $a_1 < z_1$**.**

- Suppose $a_2 - \varepsilon \geq y_2$. By A6, $(a_1, a_2 - \varepsilon) \succ_y z$.
- Suppose $a_2 - \varepsilon \in (x_2, y_2)$. By A6, $(a_1, a_2 - \varepsilon) \succ_{(y_1, a_2-\varepsilon)} z$. By A3, A4, and IVT, $\exists \alpha > 0$ such that $(a_1, a_2 - \varepsilon) \sim_{(y_1, a_2-\varepsilon)} z' = (z_1, z_2 + \alpha)$. Note by A3 $z'_2 < a_2 - \varepsilon$. By A8, $(a_1, a_2 - \varepsilon) \succeq_y z'$. By A3, $(a_1, a_2 - \varepsilon) \succ_y z$
- Suppose $a_2 - \varepsilon \leq x_2$. By A8, $(a_1, a_2 - \varepsilon) \succeq_y z$.

Since $\varepsilon > 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$.

Subcase B3b: $a_1 > z_1$**.** By A8, $z \succeq_y (a_1, a_2 + \delta)$. Since $\delta > 0$, by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 > a_2$.

Case B4: $z_2 \leq x_2$ **and** $z_1 \leq y_1$**.** This corresponds to region 7 in Figure 4.

Subcase B4a: $a_1 < z_1$**.** Note from **Step A** that $\varepsilon = 0$ in this circumstance.

- Suppose $a_2 \geq y_2$. By A6, $a \succ_y z$. By A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$.
- Suppose $a_2 \in (x_2, y_2)$. By A6, $a \succ_{(y_1,a_2)} z$. By A3, A4, and IVT, $\exists \alpha > 0$ such that $a \sim_{(y_1,a_2)} z' = (z_1, z_2 + \alpha)$. Note by A3 $z'_2 < a_2$. By A8, $a \succeq_y z'$, so by A3, $a \succ_y z$. By A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$.
- Suppose $a_2 \leq x_2$. By A8, $a \succeq_y z$. Under ISL, $a \succ_y z$, so by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 < a_2$. Under CSL, the individual is indifferent between z and a.[10].

Subcase B4a: $a_1 > z_1$**.**

- Suppose $a_1 > y_1$. By A8, $z \succeq_y (a_1, a_2 + \delta)$. From **Step A** $\delta > 0$, so by A3, $z \succeq_y a$. By A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 > a_2$.
- Suppose $a_1 \leq y_1$. From **Step A** $\delta = 0$. Under ISL $z \succ_y a$, so by A3, A4, and IVT, $\exists a' \in I_y(z)$ such that $a'_1 = a_1$ and $a'_2 > a_2$. Under CSL, the individual is indifferent between z and a (subject to the caveat noted in the previous footnote). ■

[10]In fact to maintain A1-5, for some a it is necessary for $a \succ_y z$. This characterization is not relevant to the proposition, so it is not presented in more detail.

Appendix II - Proof of Lemma 2

Agent superscripts have been suppressed for notational convenience. The contrapositive of the proposition will be proven; that is, if $x_{t+1} \succeq_{x_t} x_t$ for all $t \in [1, 2, \ldots, T-1]$, then there exists $\varepsilon > 0$ such that $d\,(x_{t+1}, x_1) > \varepsilon$. Assume trade occurs in every period, so that $x_{t+1} \neq x_t$ for $t \in [1, T-1]$. This is without loss of generality, and avoids introducing notation for the maximal subsequence of $\langle x_t \rangle_{t=1}^{T}$ such that $x_{t+1} \neq x_t$. Further suppose $x_{1,2} < x_{1,1}$ and $x_{2,2} > x_{2,1}$. When the individual instead trades good 2 for good 1 in the first period the argument is symmetric.

First consider the case where $x_{1,t+1} < x_{1,t}$ and $x_{2,t+1} > x_{2,t}$ for $t \in [1, T-1]$, so that each successive exchange involves giving up good 1 to acquire good 2. By A10, $x_{t+1} \succeq_{x_t} x_t$ for all t. By Lemma 1, $x_{t+1} \succ_{x_{t+1+m}} x_t$ for $m \geq 0$. Then since $(x_T \succ_{x_T} x_{T-1})\,, (x_{T-1} \succ_{x_T} x_{T-2})\,, (x_{T-2} \succ_{x_T} x_{T-3})\,, \ldots, (x_2 \succ_{x_T} x_1)$, by A2 $x_T \succ_{x_T} x_1$. Since $d\,(x_1, x_T) > 0$ by assumption, then by A3, A4, and IVT, there exists $\alpha > 0$ such that $x_T \sim_{x_T} x' = x_1 + (0, \alpha)$. Let $b_\varepsilon\,(x_1)$ be the ε-ball in $\mathbb{R}^2$ centered on x_1 such that $\varepsilon = \frac{1}{2}\alpha$. This ball is convex so by A9 $x_T \notin b\,(\varepsilon)$.

Now consider exchange as above through period $k \in [2, T-1]$, but in period $k+1$ the agent acquires good 1 for good 2, that is $x_{1,k+1} > x_{1,k}$ and $x_{2,k+1} < x_{2,k}$. As demonstrated above, there exists $\varepsilon > 0$ such that $I_{x_k}\,(x_k)$ lies strictly outside $b_\varepsilon\,(x_1)$. By A3 x_{k+1} cannot contain less of both goods than x_1, so there are three cases to consider:

Case 1: $x_{1,k} > x_{1,1} \wedge x_{2,k} \leq x_{2,1}$**.** By A3, A4, and IVT, there exists $\alpha > 0$ such that $x_1 \sim_{x_k} (x_{1,k+1}, x_{2,k+1} - \alpha)$. By Lemma 1, $(x_{1,k+1}, x_{2,k+1} - \alpha) \succ_{x_1} x_1$. Thus by A3, $x_{k+1} \succ_{x_1} x_1$. But then it is without generality to prove the proposition for the subsequence $\langle x_1, x_{k+1}, x_{k+2}, \ldots, x_T \rangle$. If $x_{1,k+t+1} > x_{1,k+t}$ and $x_{2,k+t+1} < x_{2,k+t}$ for $t \in [1, T-k-1]$, the proposition has already been shown to hold. Otherwise, Case 1, 2, or 3 applies anew.

Case 2: $x_{1,k} \leq x_{1,1} \wedge x_{2,k} > x_{2,1}$**.** Let $\underline{m}$ be the minimum $m \in [1, k-1]$ such that $x_{k+1} \succeq_{x_m} x_m$. That $\underline{m}$ exists is trivial, since $x_{k+1} \succ_{x_{k-1}} x_{k-1}$ by Lemma 1.

Claim: It must be the case that $x_{1,k+1} < x_{1,\underline{m}}$. If $\underline{m} = 1$, this is true by assumption. So suppose $\underline{m} > 1$ and $x_{1,k+1} \geq x_{1,\underline{m}}$. By A10, $x_{\underline{m}} \succeq_{x_{\underline{m}-1}} x_{\underline{m}-1}$. By Lemma 1, $x_{k+1} \succ_{x_{\underline{m}-1}} x_{\underline{m}}$. Then by A2, $x_{k+1} \succ_{x_{\underline{m}-1}} x_{\underline{m}-1}$, violating the assumption that $\underline{m}$ is the minimum $m \in [1, k-1]$ such that $x_{k+1} \succeq_{x_m} x_m$. Thus $x_{1,k+1} < x_{1,\underline{m}}$, and $x_{2,k+1} > x_{2,\underline{m}}$ by A3. But then it is without generality to prove the proposition for the subsequence $\langle x_1, x_{\underline{m}}, x_{k+1}, x_{k+2}, \ldots, x_T \rangle$. If $x_{1,k+t+1} < x_{1,k+t}$ and $x_{2,k+t+1} > x_{2,k+t}$ for $t \in [1, T-k-1]$, the proposition has already been shown to hold. Otherwise, Case 1, 2, or 3 applies anew.

Case 3: $x_{1,k} > x_{1,1} \wedge x_{2,k} > x_{2,1}$**.** It is without loss of generality to consider the subsequence $\langle x_\tau \rangle_{\tau=k+1}^{T}$, for which all possibilities have already been considered. Since it is not possible for x_T to return to a neighborhood of x_{k+1}, by A3 neither can x_T return to a neighborhood of x_1.

It has been demonstrated that a subsequence of $\langle x_t \rangle_{t=1}^{T}$ which begins and ends with $t = 1$ and $t = T$, respectively, can always be constructed such that each element of this subsequence is preferred to the last from the last, and that this subsequence is either monotonically increasing in good 1 and decreasing in good 2, or monotonically increasing in good 2 and decreasing in good 1. It has already been demonstrated that in such a sequence x_T cannot return to some neighborhood of x_1. ■

Appendix III - Introducing Diminishing Sensitivity in Losses

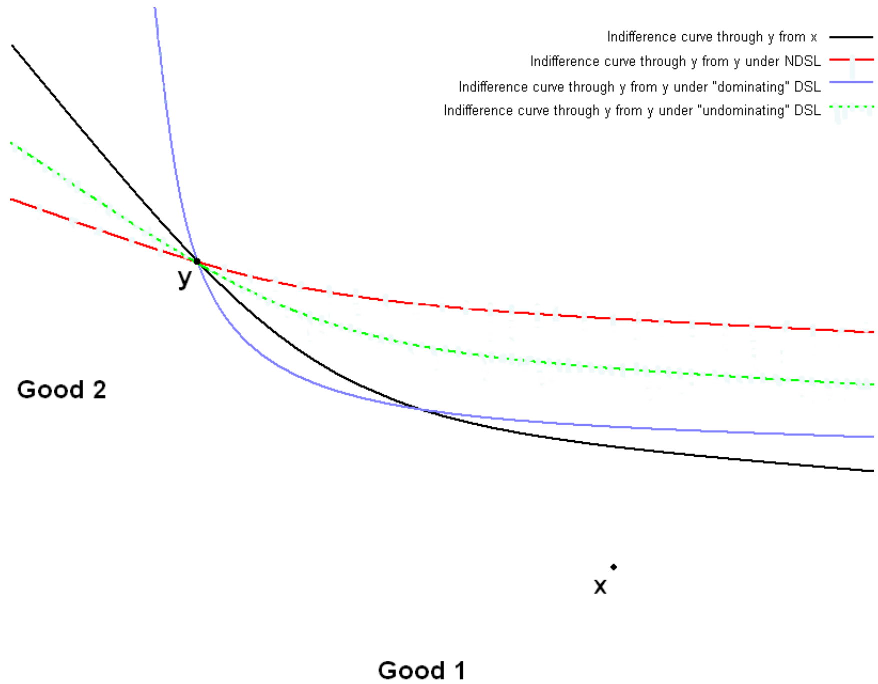

Figure 5: Ambiguous Indifference Curve Rotation under ISL

If NDSL is replaced by DSL, the rotation of indifference curves characterized in Lemma 1 no longer necessarily applies. To demonstrate, consider how the indifference curve through y (particularly important in the context of exchange) adjusts under DSL as trade takes place from x to y. Again suppose $x_1 > y_1$, $y_2 > x_2$, and $y \succ_x x$, and now assume $z \in I_x(y)$ such that $z_2 > y_2$. As in the proof of Lemma 1, the change in reference point from x to y is broken into two parts, from x to $s = (y_1, x_2)$, and then from s to y. DSL implies that $y \succ_s z$, so A3, A4 and the intermediate value theorem imply $(z_1 + \alpha, z_2) \sim_s y$ for some $\alpha > 0$. However, the move from s to y favors $(z_1 + \alpha, z_2)$ relative to y by DSG (A7), shifting the portion of $I_y(y)$ above y back in the direction of $I_x(y)$ (that is, back relative to $I_s(y)$). In Figure 5 the section of the light solid curve above y represents a circumstance where the effect of DSL dominates DSG, and the section of the small-dashed curve above y represents a circumstance where the effect of DSL is dominated by DSG. The flattest curve (large-dashed) maintains the assumption of NDSL.

On the other hand, DSL does not impact the indifference curve through y to the right of x given a change in reference point from x to y, so we have the result from Lemma 1 that $I_y(y)$ is located above $I_x(y)$ in this region. The indifference curve through y in the region between x and y after the change in reference point is ambiguous due to the competing influences of DSL and A6-A7, but to preserve continuity and monotonicity it must be the case that the various indifference curves in this region look similar to Figure 5. Apparently when DSL is a dominant influence on preferences, the effect of exchange is to make goods more complementary to each other, which is itself a testable hypothesis.

Lemma 1 can be maintained by replacing A8 (NDSL) with DSL provided that the impact of LA and/or DSG everywhere dominate the impact of DSL. From the proof of Lemma 1 one may readily verify that when $z_1 > y_1$ or $z_2 > x_2$, DSL influences how the indifference curve through z adjusts only in conjunction with LA and/or DSG. When $z_1 \leq y_1$ and $z_2 \leq x_2$ (region 7 in Figure 4), DSL exerts exclusive influence on any point on the indifference curve through z in the same region. In order for convexity and continuity to be maintained, it is necessary for DSL to dominate LA and DSG in some portion of regions 4 and 8 in Figure 4. However, if we only require that convexity and continuity obtain outside of region 7, we can have LA and DSG everywhere dominating DSL at the price of a preference discontinuity and/or non-convexity between regions 4 and 7 and regions 4 and 8. Since Lemma 2 guarantees trade will never occur in region 7 relative to any previously adopted allocation, this relaxation of A4 and/or A9 comes at little cost, and with it comes the advantage of the robustness of Propositions 1, 2, and 3 to the assumption of "dominated" DSL; i.e., DSL whose influence outside of region 7 is always dominated by LA and/or DSG.

References

Barber, B. and T. Odean (2000). "Trading is harzardous to your wealth: The common stock investment performance of individual investors." *Journal of Finance*, **55**(2), 773–806.

Bateman, I., D. Kahneman, A. Munro, C. Starmer and R. Sugden (2005). "Testing competing models of loss aversion: An adversarial collaboration." *Journal of Public Economics*, **89**, 1561–1580.

Bateman, I., A. Munro, B. Rhodes, C. Starmer and R. Sugden (1997). "A test of the theory of reference-dependent preferences." *Quarterly Journal of Economics*, pp. 479–505.

Benartzi, S. and R. Thaler (1995). "Myopic loss aversion and the equity premium puzzle." *Quarterly Journal of Economics*, **109**(1), 73–92.

Dhar, R., J. Huber and U. Kahn (2007). "The shopping momentum effect." *Journal of Marketing Research*, **44**, 370–78.

Kahneman, D. (2003). "A psychological perspective on economics." *The American Economic Review*, **93**(2), 162–168.

Kahneman, D., J. Knetsch and R. Thaler (1990). "Experimental tests of the endowment effect and the Coase theorem." *Journal of Political Economy*, **98**, 1325–48.

Kahneman, D. and A. Tversky (1979). "Prospect theory: An analysis of decision under risk." *Econometrica*, **47**, 263–91.

Kőszegi, B. and M. Rabin (2006). "A model of reference-dependent preferences." *Quarterly Journal of Economics*, **121**(4), 1133–1165.

Knetsch, J. (1989). "The endowment effect and evidence of nonreversible indifference curves." *The American Economic Review*, **79**, 1277–1284.

Knetsch, J. and J. Sinden (1984). "Willingness to pay and compensation demanded: Experimental evidence of an unexpected disparity in measures of value." *Quarterly Journal of Economics*, **99**(3), 507–521.

List, J. (2003). "Does market experience eliminate anomolies?" *Quarterly Journal of Economics*, **118**(1), 41–71.

List, J. (2004). "Neoclassical theory versus prospect theory: Evidence from the marketplace." *Econometrica*, **72**(2), 615–625.

Loewenstein, G. and D. Adler (1995). "A bias in the prediction of tastes." *The Economic Journal*, **105**, 929–937.

Loewenstein, G., T. O'Donoghue and M. Rabin (2003). "Projection bias in predicting future utility." *Quarterly Journal of Economics*, **118**(4), 1209–1248.

Masatlioglu, Y. and E. A. Ok (2006). "Reference-dependent procedural decision making." Working paper.

Masatlioglu, Y. and N. Uler (2008). "Contrasting reference dependent choice models." Working paper.

Mulhern, F. and D. Padgett (1995). "The relationship between retail price promotions and regular price purchases." *Journal of Marketing*, **59**, 83–90.

Munro, A. and R. Sugden (2003). "On the theory of reference-dependent preferences." *Journal of Economic Behavior and Organization*, **50**(4), 407–428.

Myagkov, M. and C. Plott (1997). "Exchange economies and loss exposure: Experiments exploring prospect theory and competitive equilibria in market environments." *The American Economic Review*, **87**(5), 801–828.

Plott, C. and K. Zeiler (2005). "The willingness to pay - willingness to accept gap, the 'endowment effect,' subject misconceptions, and experimental procedures for eliciting valuations." *The American Economic Review*, **95**(3), 530–545.

Sagi, J. (2006). “Anchored preference relations.” *Journal of Economic Theory*, **130**, 283–295.

Tversky, A. and D. Kahneman (1991). “Loss aversion in riskless choice: A reference-dependent model.” *Quarterly Journal of Economics*, **106**(4), 1039–61.

Van Boven, L., G. Lowenstein and D. Dunning (2003). “Mispredicting the endowment effect: Underestimation of owners’ selling prices by buyer’s agents.” *Journal of Economic Behavior and Organization*, **51**, 351–365.