EFFECTS OF INFORMATION IN TRAFFIC NETWORKS

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Introduction

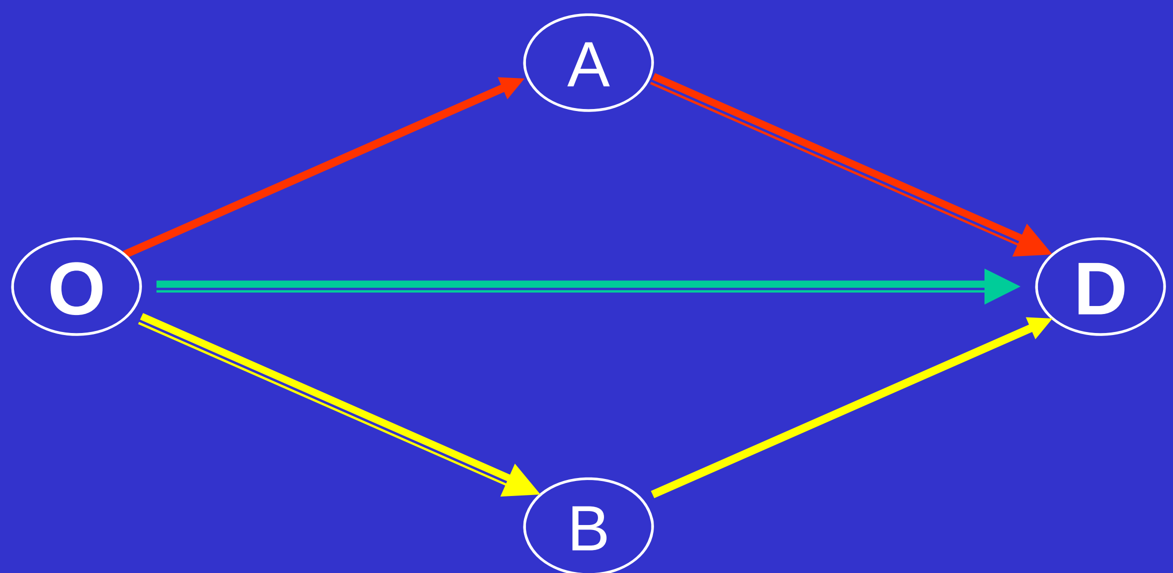
- Networks form the infrastructure for the functioning of modern societies and economies.

- Well-known physical networks in which nodes correspond to locations in space and links to appropriate connections with associated flows include transportation and communication networks.

Transportation networks have evolved over the centuries through advances in science and technology and come in many forms: road, rail, air, or waterway with a variety of associated modes of travel.

Communication networks enable the transmission of voice, data, information, and videos, and today involve telephones, computers, as well as satellites and microwaves.

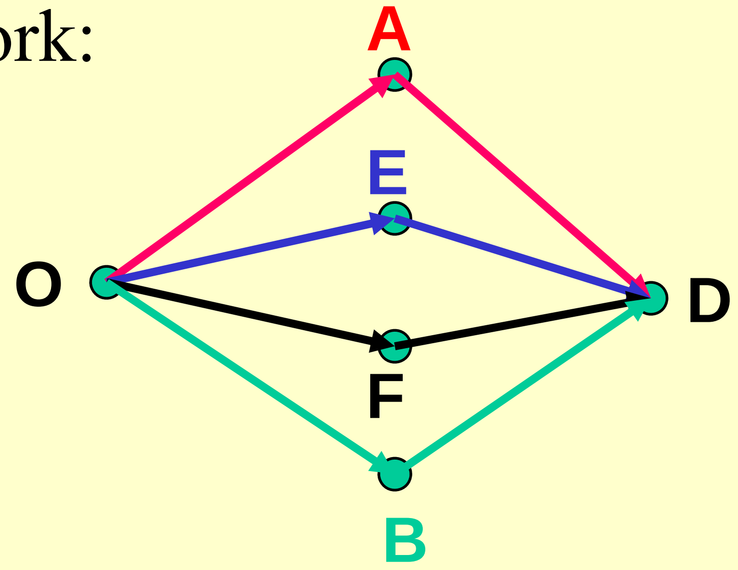
In modeling the flow in the network, it is commonly assumed that in choosing a path from a given source (origin) to a given destination, selfish agents (players, users) wish to minimize their own cost.



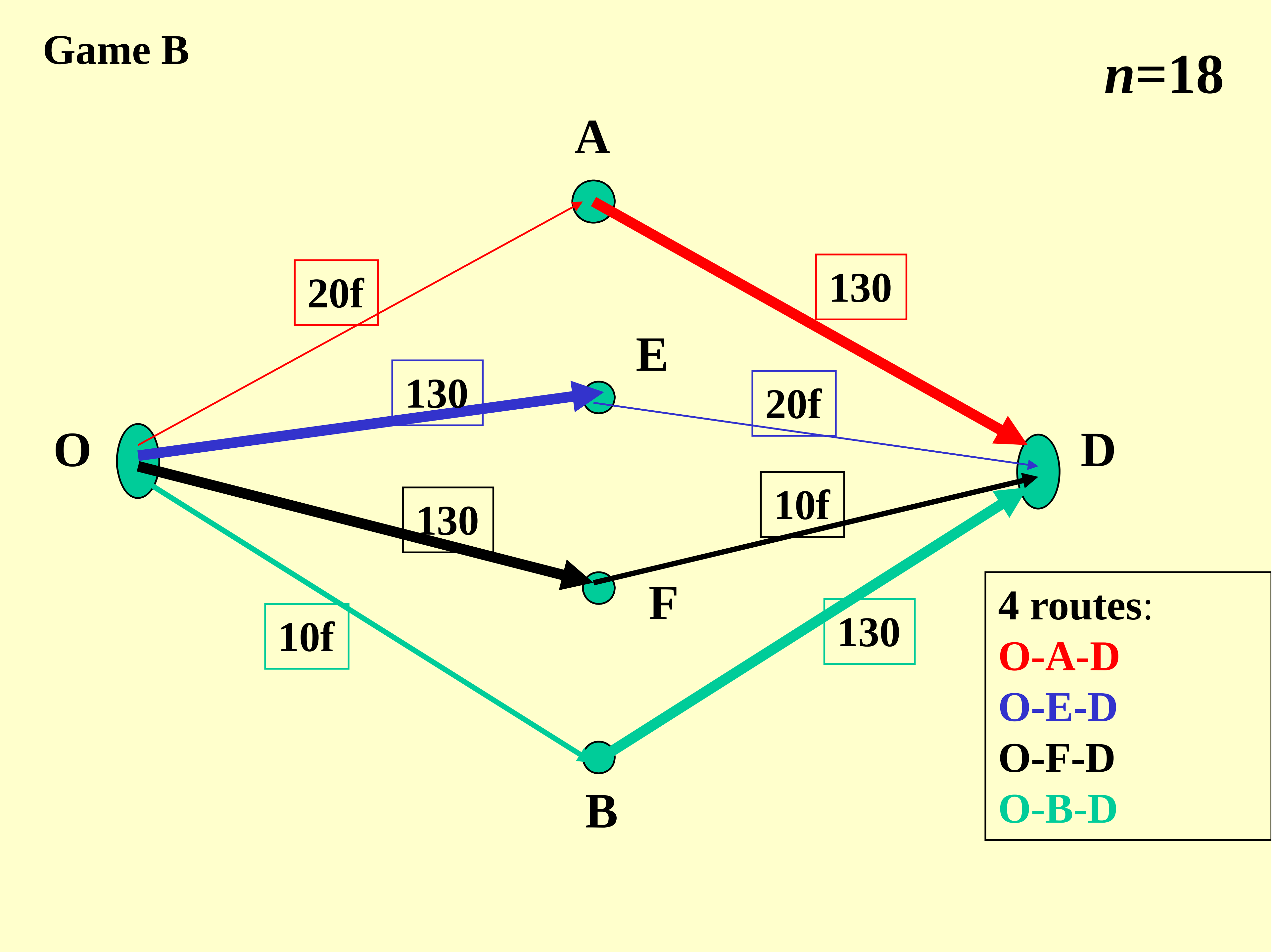
Consider the traffic network on the next page.
It has 4 one-way routes from a common origin (on the left) denoted by O to a common destination (on the right) denoted by D.

There are 4 routes in the network:

O-A-D
O-E-D
O-F-D
O-B-D



There are ***n*=18** commuters. Each has to choose a route from the origin O to the destination D.
Routes are chosen independently.


Game B
n=18
A
E
O
D
F
B
20f
130
130
20f
130
10f
10f
130
4 routes:
O-A-D
O-E-D
O-F-D
O-B-D

Travel is never cost-free. There are costs associated with choosing each route. They reflect time to travel, road conditions, congestion on the road, or mode of transportation (e.g., car, train).

In general, the cost of traveling a road segment x-y is given by

$$cost(x\text{-}y) = a + bf_{x\text{-}y}$$

where a and b are non-negative constants, the same for each commuter, and $f_{x\text{-}y}$ denotes the number of commuters traversing the road segment x-y.

Example Suppose that the cost of some road segment x-y is $20f_{x-y}$. Then,

- If you are the only commuter taking this road, then your cost is 20.

- If you are one of 4 commuters taking this road, then your cost is 20×4=80.

- If you are one of 18 commuters taking this road, then your cost is 20×18=360.

The cost of a route (that may include more than a single segment) is the sum of costs of its separate segments.

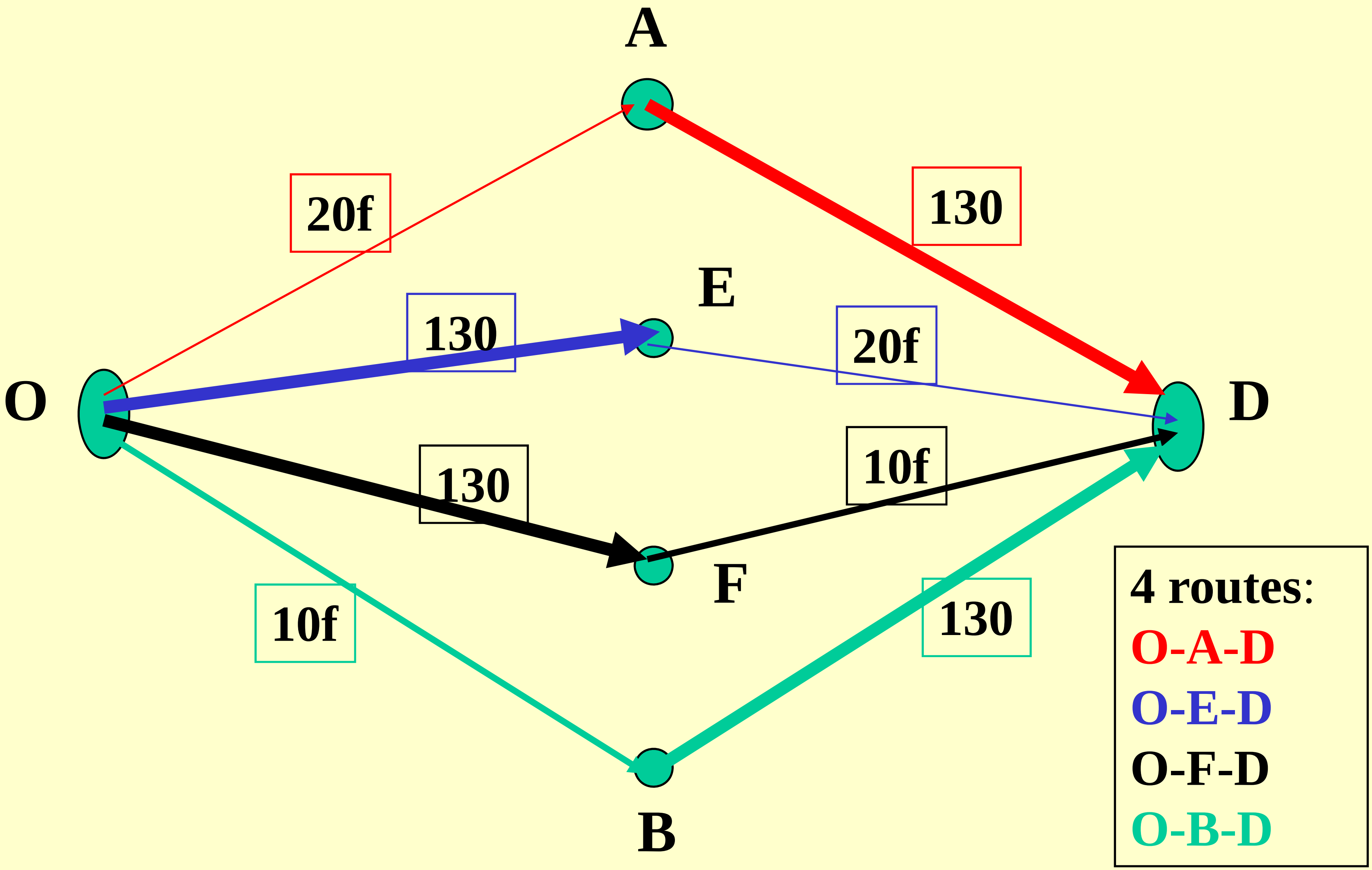
In the traffic network with the 4 routes there are three types of road segments.

- Two segments are heavily subject to congestion:
  $c(x\text{-}y)=20f_{x\text{-}y}$
- Two segments are moderately subject to congestion:
  $c(x\text{-}y)=10f_{x\text{-}y}$
- Four routes are not subject to congestion: $c(x\text{-}y)=190$.

Refer to this choice problem as Game B (for Basic).

Game B

n=18



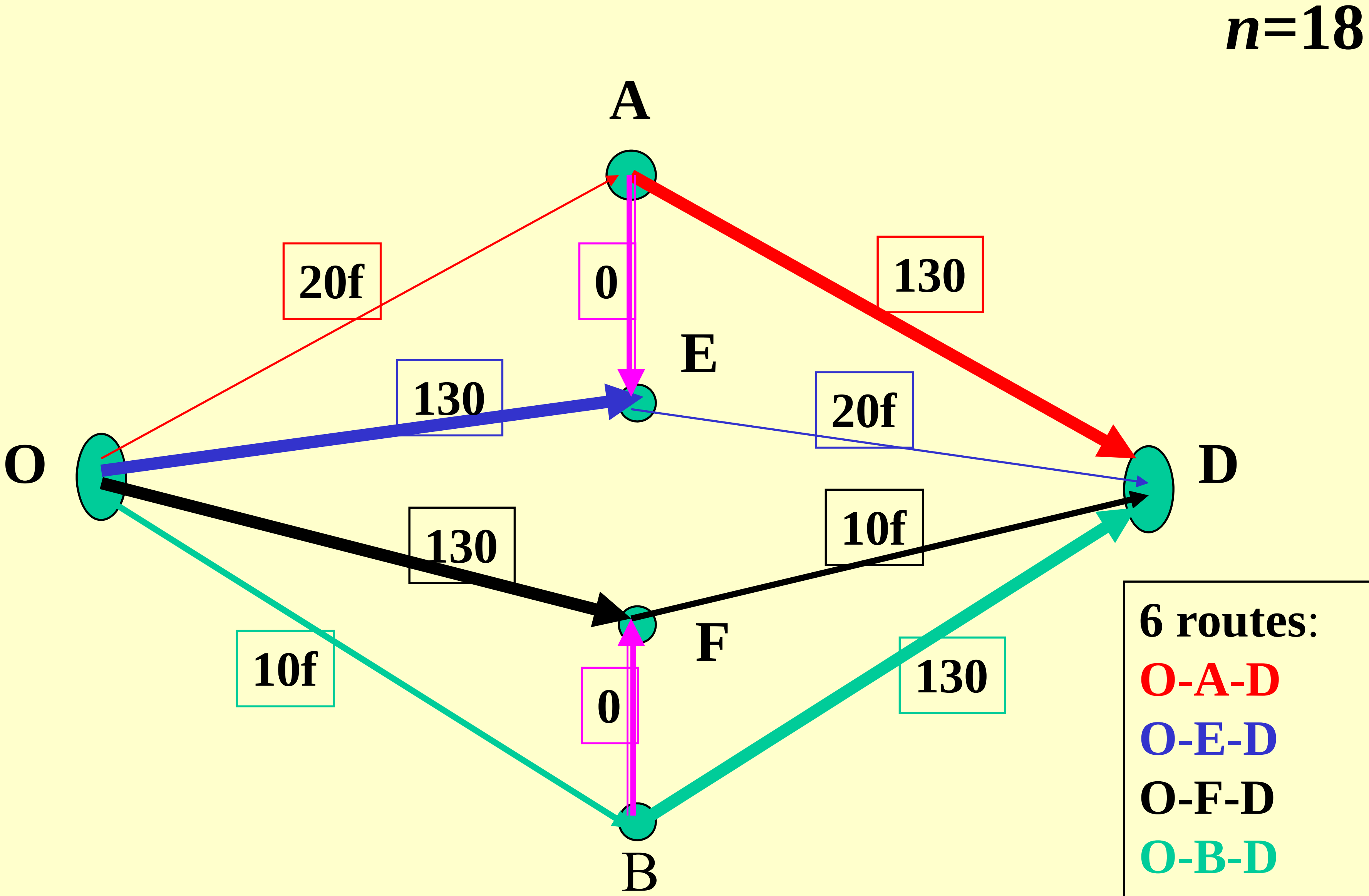
Please choose one of the 4 routes that minimizes cost

A central transportation authority has decided to improve the congested traffic network by adding two one-way new road segments (“bridges”). These two segments are short and wide with a 0 cost of traversing each of them.

In this new route choice problem, which we call Game A (for Augmented), there are now 6 possible routes:

- O-A-D
- O-E-D
- O-F-D
- O-B-D
- O-A-E-D
- O-B-F-D

Game A



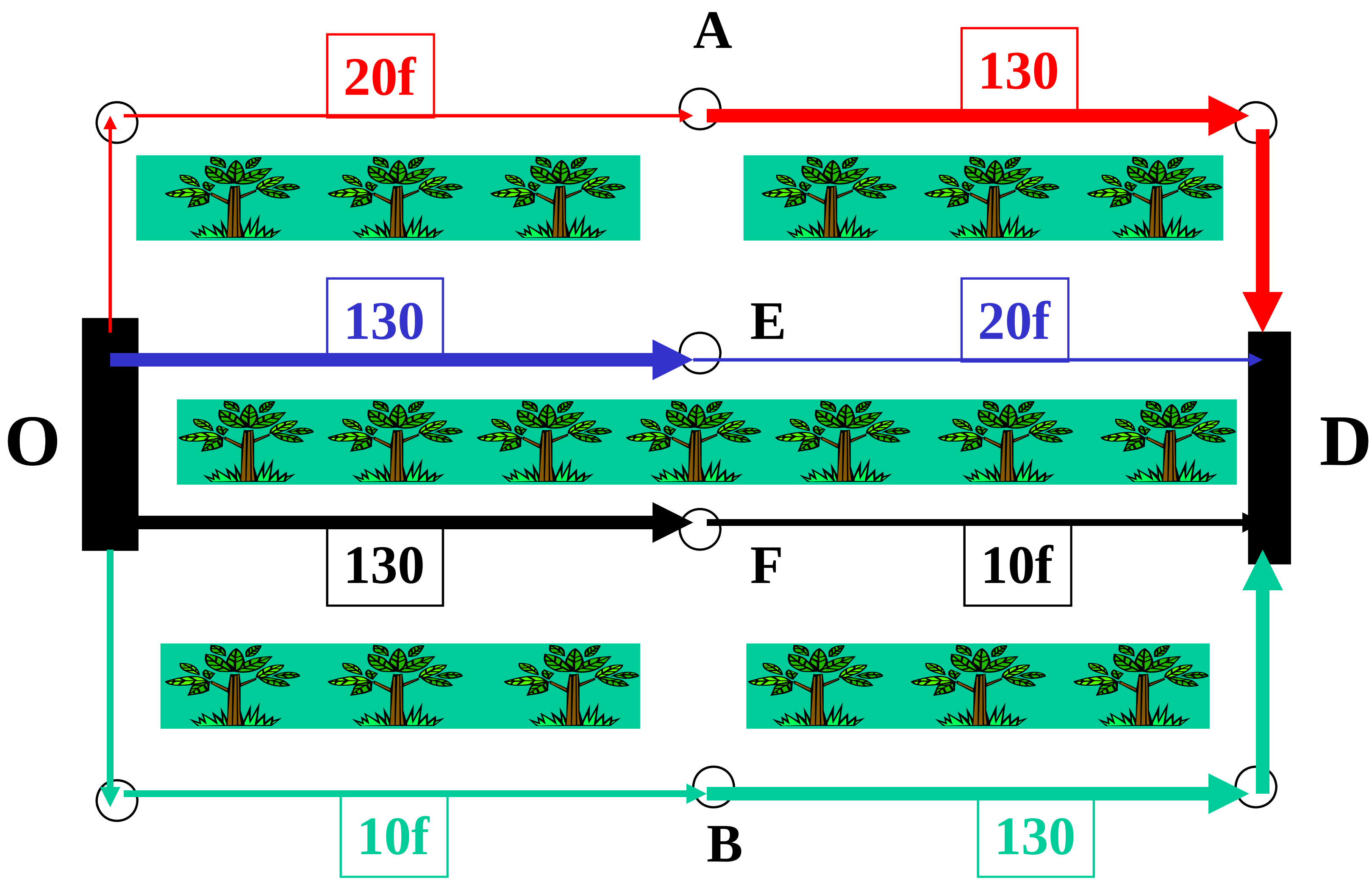
6 routes:
- O-A-D
- O-E-D
- O-F-D
- O-B-D
- O-A-E-D
- O-B-F-D

Please choose one of the 6 routes that minimizes travel cost

The two next slides exhibit the same Game B and Game A but they use a different, more common and familiar, topology.

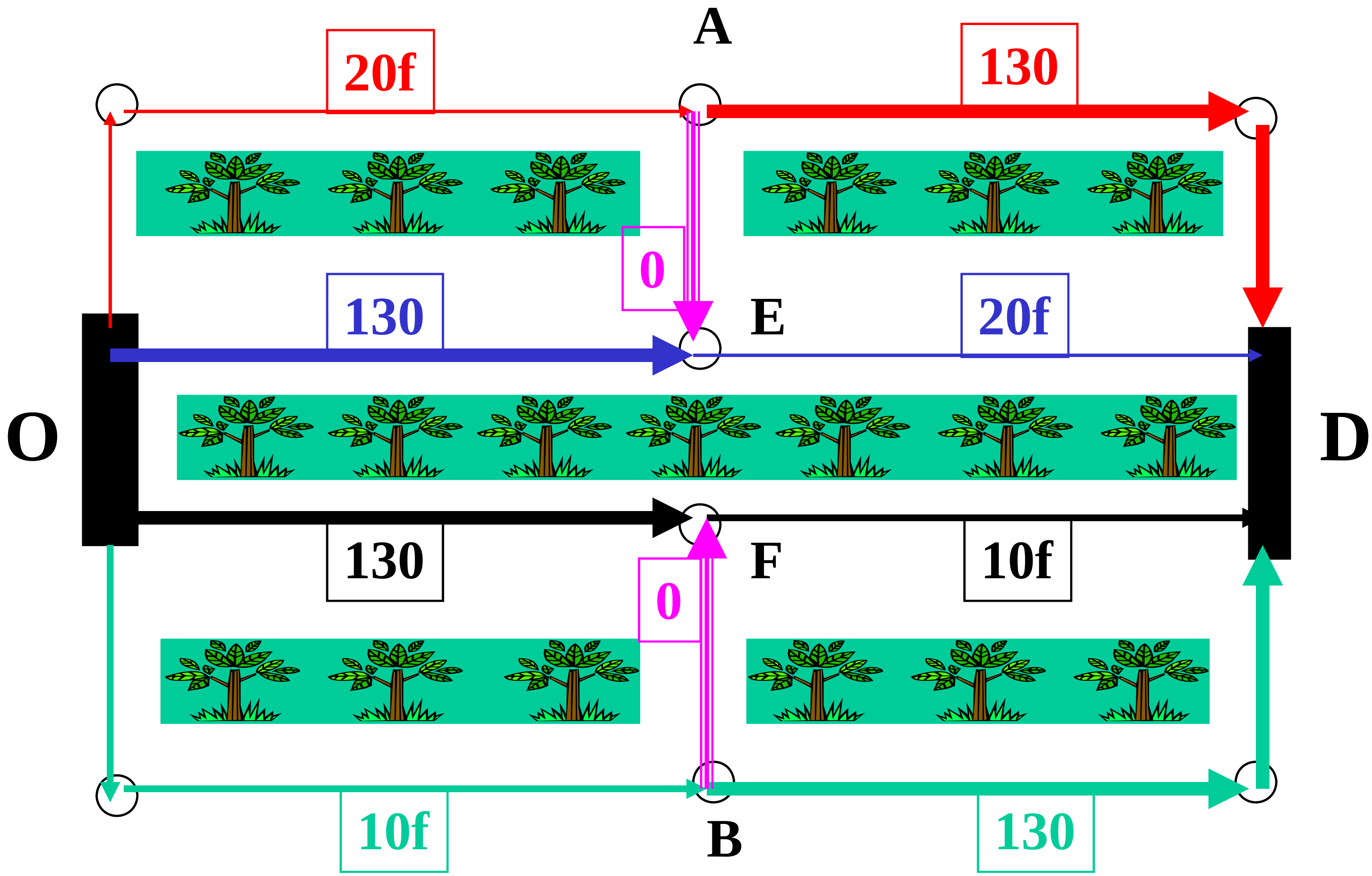
Game B

n=18



Choose one of **4** routes: (**O-A-D**), (**O-E-D**), (**O-F-D**), (**O-B-D**)

Game A n=18



Choose one of **6** routes: (**O-A-D**), (**O-E-D**), (**O-F-D**), (**O-B-D**)
(**O-A-E-D**), (**O-B-F-D**)

Normative Approach

How should the 18 commuters choose their routes?

What sort of theory should we be looking for?

To answer this question, we invoke the game-theoretical equilibrium solution. This is the major solution concept for non-cooperative games.

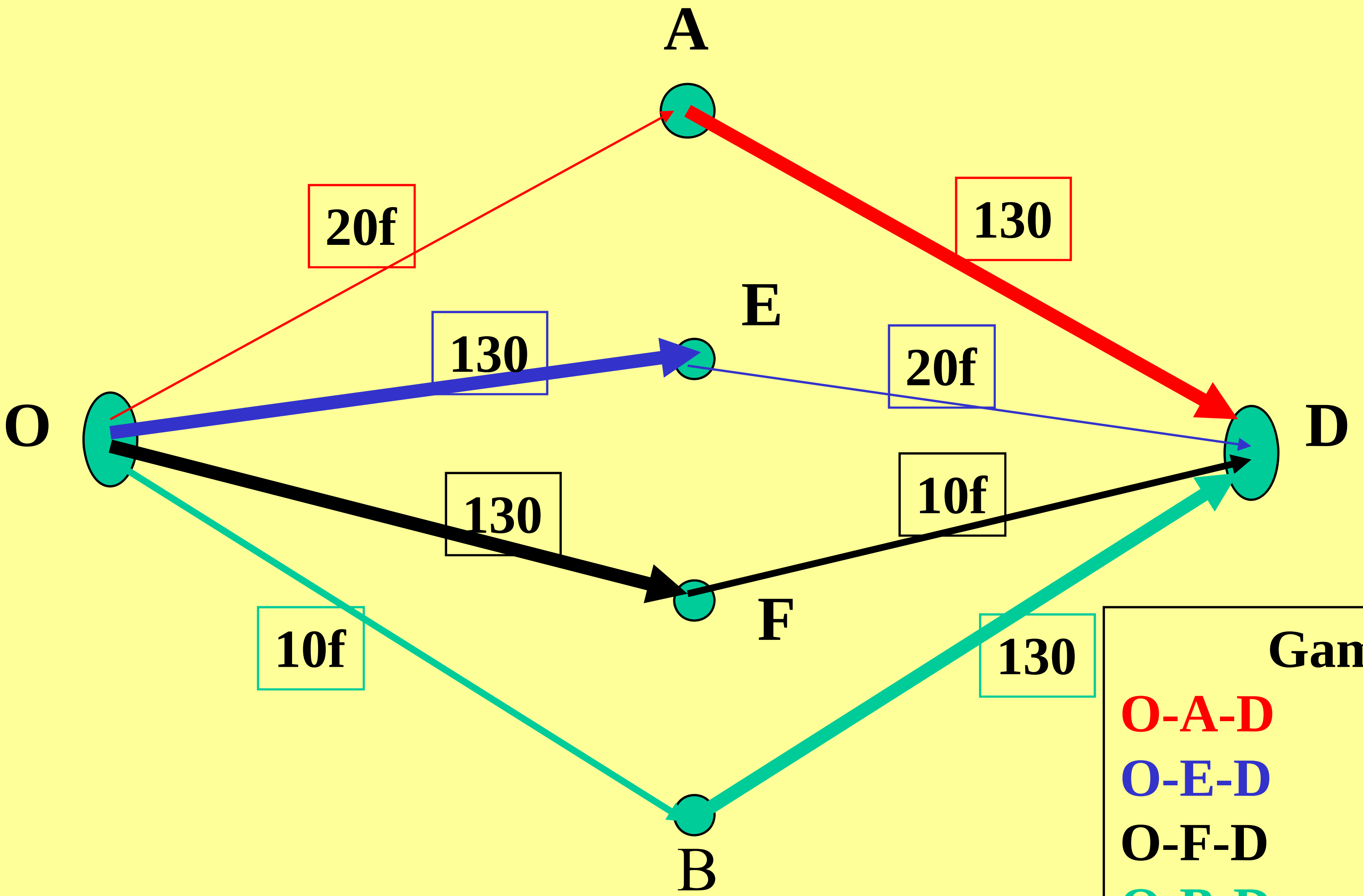
The solution specifies a distribution (strategy profile) of the commuters over all the routes such that no single commuter wishes to deviate from this profile unilaterally.

In other words, each individual choice of route is a best response to the choices of all the others. As such, the solution is self-enforcing.

Game B

Equilibrium Solution

***n*=18**

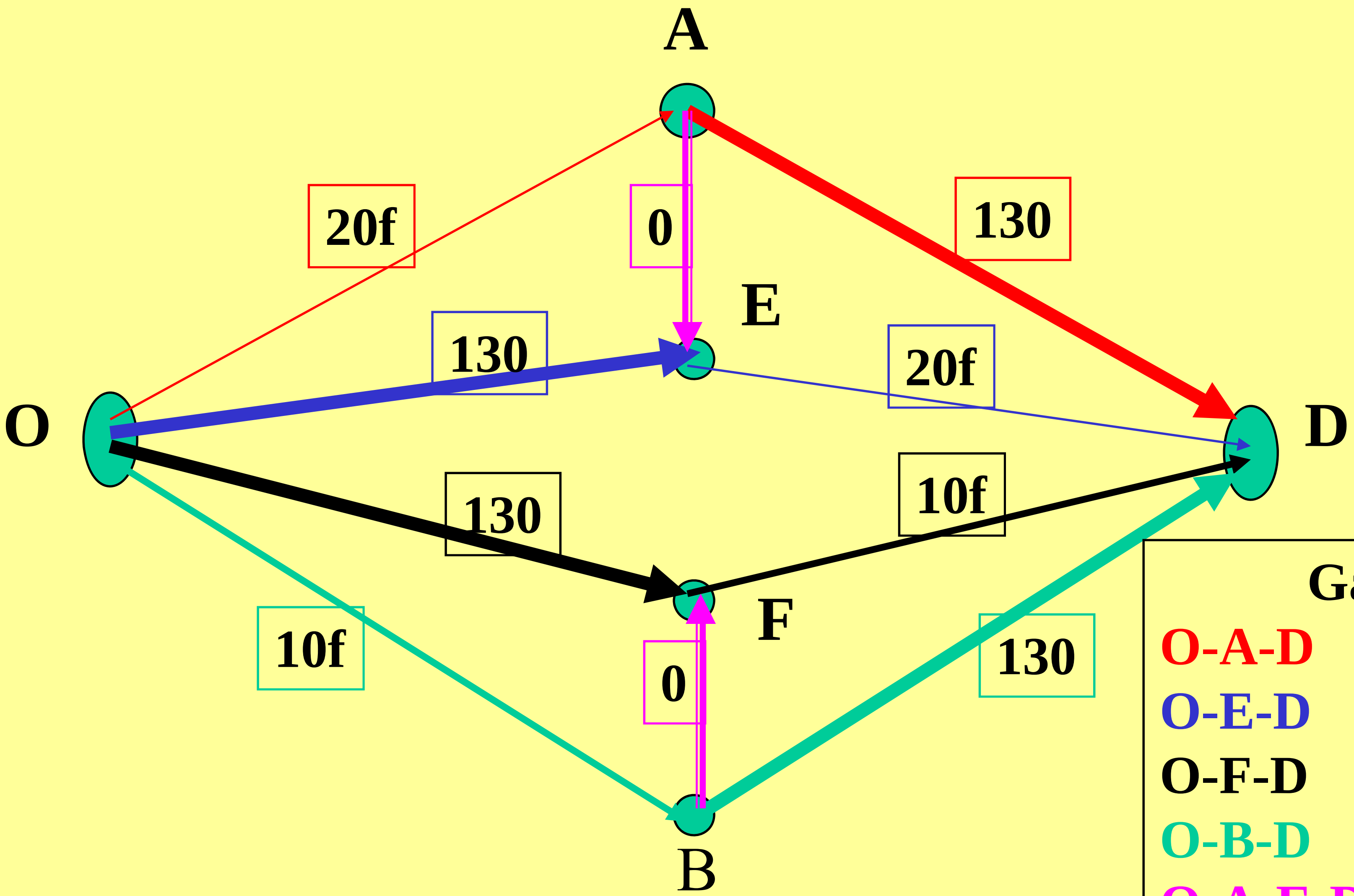


Game B	
O-A-D	3
O-E-D	3
O-F-D	6
O-B-D	6

Game A

Equilibrium Solution

***n*=18**



Game A	
O-A-D	0
O-E-D	0
O-F-D	0
O-B-D	0
O-A-E-D	6
O-B-F-D	12

Properties of the Equilibrium Solution

- ♦ The equilibrium solution specifies how many commuters should choose each route.
- ♦ It does not tell individual commuters which route to choose.
- ♦ It is a static concept.
- ♦ It is silent with respect to the process by which commuters choose a specific equilibrium (e.g., by introspection, intuition, learning).

In fact, the solution seems to be of little or no interest because there may be multiple equilibria. In Game B, there are 18!/(3!3!6!6!) = 343,062,720 pure-strategy equilibria.

These millions of equilibria are not Pareto rankable; one is not preferred over the other. In fact, under each of these equilibria the travel cost of each commuter is 190.

The same problem of multiple equilibria also plagues Game A. In Game A there are

$$18!/(6!12!) = 37{,}128$$

pure-strategy equilibria. Although this number is smaller than the one for Game B, it is still very large.

The travel cost under each of these equilibria is 240!

So what is the predictive power of this solution?

This example illustrates an important finding:

Paradoxically, when one or more links are ***added*** *to a network and each user independently seeks her best possible route from O to D, at the new equilibrium the cost of travel for each user* ***may increase***.

In fact, in moving from Game B to Game A (with ***n***=18) individual travel cost increases by 26.3% from 190 to 240.

The Braess Paradox

The situation illustrated in Games B and A is a manifestation of the **Braess Paradox** **(1968)**.

The Braess Paradox (BP) has attracted considerable attention and instigated much ***theoretical*** research in transportation science, engineering, telecommunication, and computer science.

However, the BP has **NOT** been studied ***experimentally***.

Is it nothing more than an interesting curiosity resulting from equilibrium analysis of the two networks?

OR

Is it a result of behavioral and practical importance?

Inefficient equilibria: The Prisoner's Dilemma game, the Centipede game, the Braess Paradox

Empirical Evidence

- There is some evidence that the BP might actually have occurred during road construction in the city of **Stuttgart**. Murchland writes (1970, p. 394):

“Braess’s paradox is also described by Knödel (1969). Knödel remarks that the example may seem contrived, but a recent experience in Stuttgart shows that it can occur in reality. Major road investments in the city centre, in the vicinity of Schlossplatz, failed to yield the benefits expected. They were only obtained when a cross street, the lower part of Königstrasse, was subsequently withdrawn from traffic use.”

- The **New York Times** reported counterintuitive consequences of road closures. See the report: “What if they closed 42nd street and nobody noticed?” (Dec. 25, 1990):

- **Fisk and Pallottino** (1981) tested for manifestations of the BP in a localized section of road network in Winnipeg. They report positive results.

- Evidence that adding capacity in the **British Telecom** network resulted in worse performance.

Empirical evidence is very meager and mostly anecdotal.

The BP is ideally suited to be studied in the ***laboratory***.

How Severe Can the BP Be?

- The BP is as likely to appear as not (Steinberg & Zangwill, 1983).
- Detecting the BP even in its worst possible manifestation is algorithmically difficult (Roughgarden, 2002)
- If the cost function is affine, then the flow at the Nash equilibrium has total cost of at most **4/3** times that of the optimal flow (Roughgarden & Tardos, 2002)
- With continuous, non-negative, and non-decreasing cost functions, the effect of the BP can be considerably higher (Roughgarden, 2005).

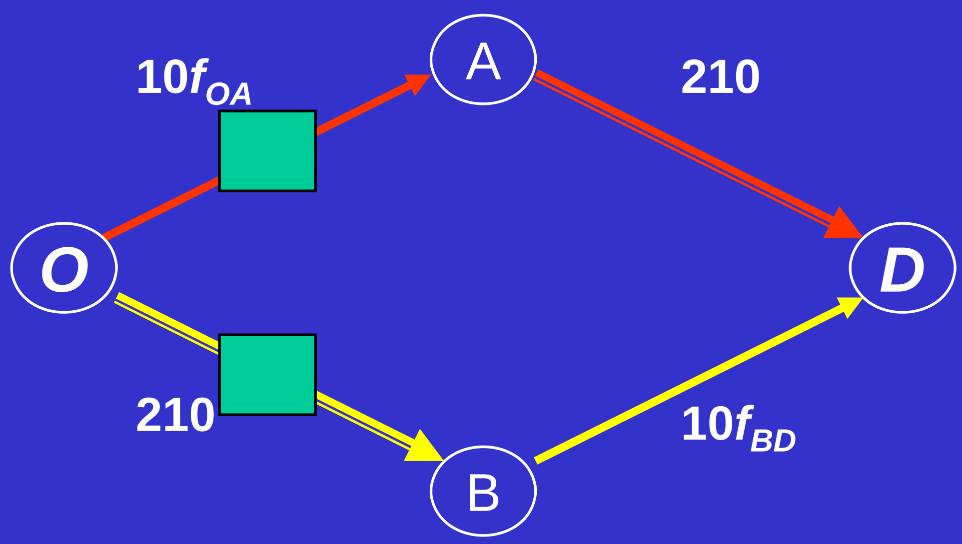
Before answering this question whether the BP has any practical or behavioral importance with the 4-route network game that was presented before, I wish to present a summary of experimental results from a much simpler game with only two routes in the Basic game and an added link in the Augmented game.

Game 1A:

$n = 18$

Two routes: (***O-A-D***) and (***O-B-D***)

- What are the equilibrium solutions of this game?
- What are the associated costs of travel?



Game 1A: $n = 18$.

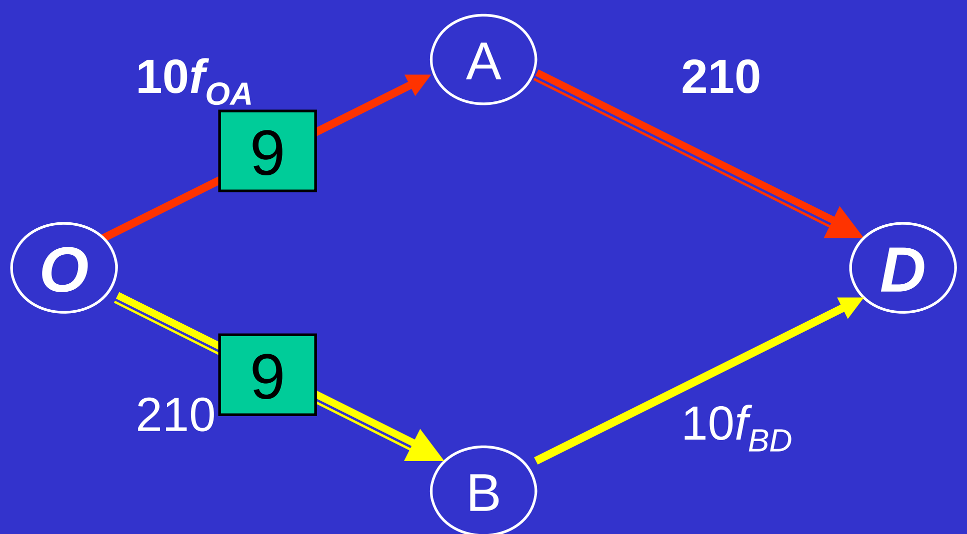
In (pure-strategy) equilibrium:

9 players choose (***O-A-D***)

9 players choose (***O-B-D***).

There are 18!/(9!9!)=48,620 such equilibria.

Total cost of travel for each player is 10×9+210=300



There is also a symmetric mixed-strategy equilibrium where

route (*O*-*A*-*D*) is chosen with probability 0.5
route (*O*-*B*-*D*) is chosen with probability 0.5

The expected payoff associated with the mixed strategy equilibrium is 305

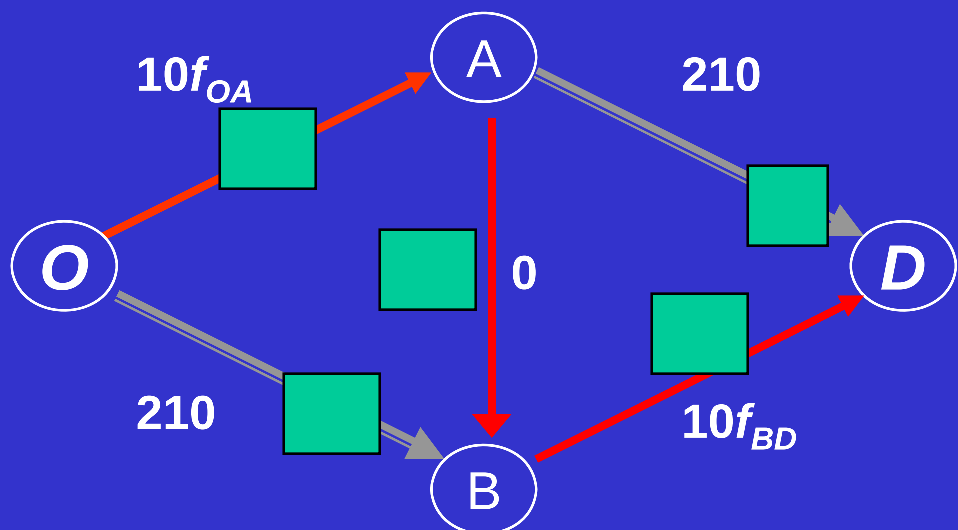
Note: This expected payoff exceeds the one under pure-strategy play.

Game 1B: Augment the network in Fig. 1A by **adding** a **cost-free** link from vertex *A* to vertex *B.*

As a result, we have three routes:

(*O-A-D*), (*O-B-D*), (*O-A-B-D*).

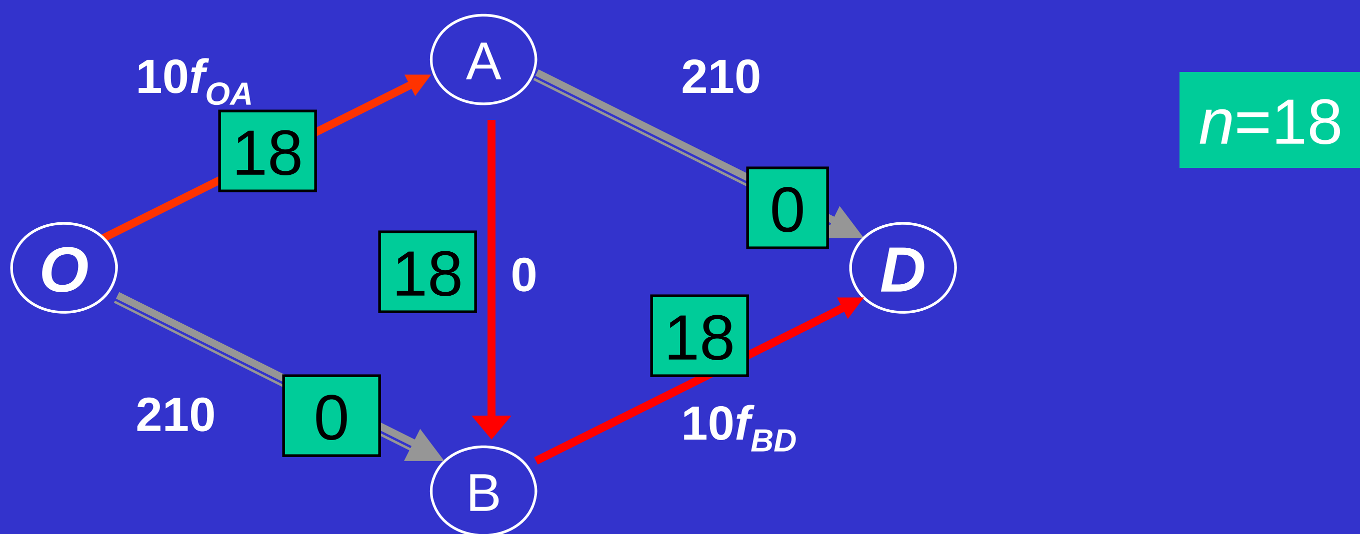
- ♦ What are the equilibria of this game?
- ♦ What are the associated costs of travel?



n=18

Game 1B: Augment the network in Fig. 1A by **adding** a **cost-free** link from vertex *A* to vertex *B*.

In equilibrium, all ***n***=18 users choose (***O-A-B-D***).
The cost for each user is **(10×18)+0+(10×18)=360**



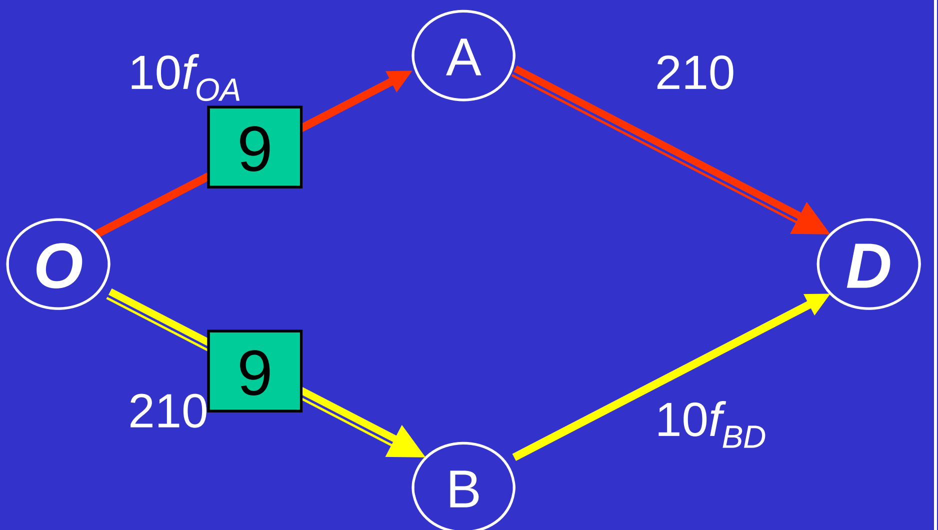
Experimental Evidence

Experiment 1: Two routes, one cost-free link, symmetric players, **108** subjects

All the experiments are computer-controlled with payoff contingent on performance.

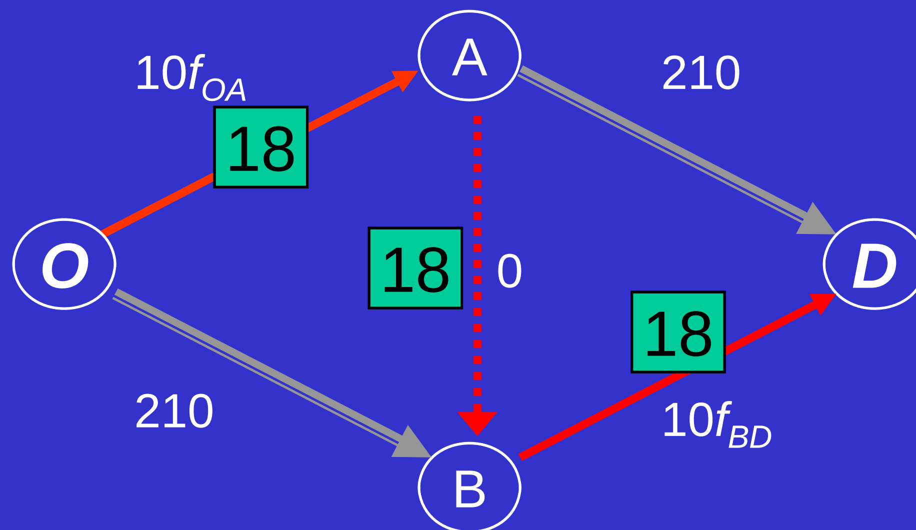
Experiment 1 –Games 1A and 1B

Game 1A



Eq. Cost = 10×9+210=300

Game 1B

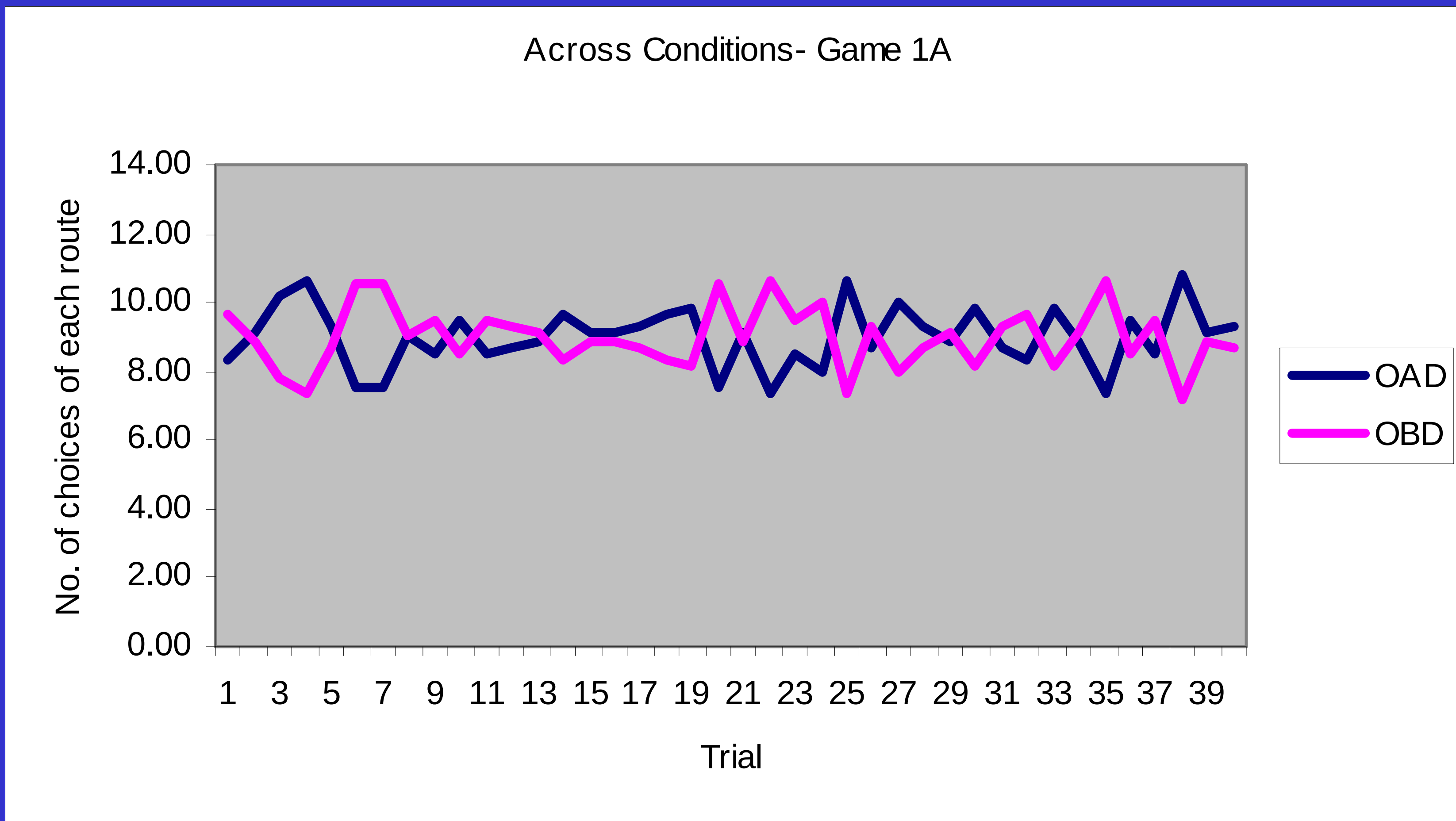


Eq. Cost = 10×18+10×18=360

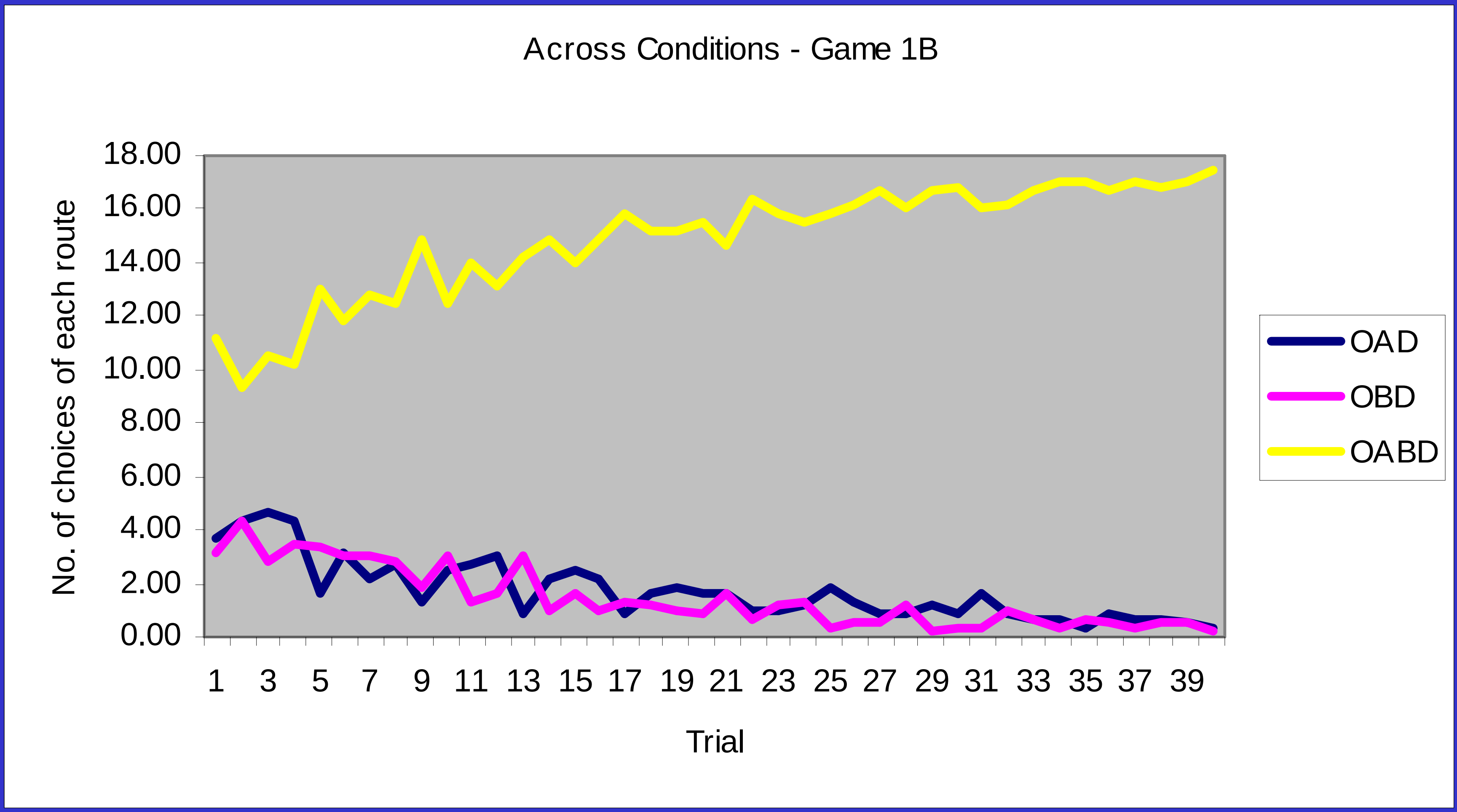
Experimental Design

- **Two experimental conditions:**
  Condition ADD (Game 1A, then Game 1B)
  Condition DELETE (Game 1B, then Game 1A)
- **Within-subject design**
- **40 iterations of each game**
- **3 sessions of *n*=18 in each condition**
- **Same endowment of 400 for each game**
- **Equilibrium payoff is**
  400-300=100 for Game 1A
  400-360= 40 for Game 1B

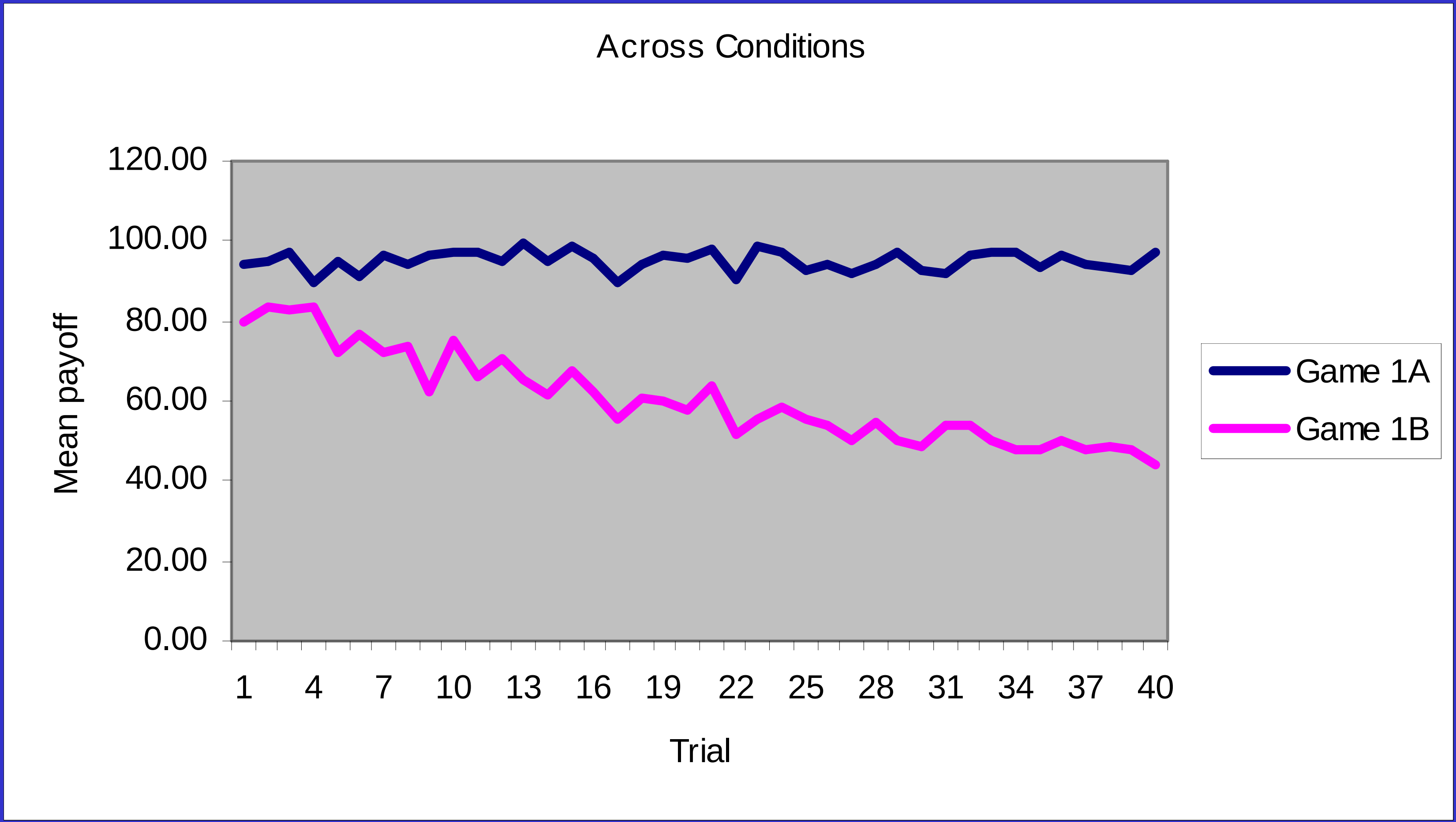
Across Both Conditions: Game 1A
Mean Number of Choices of Each Route by Trial



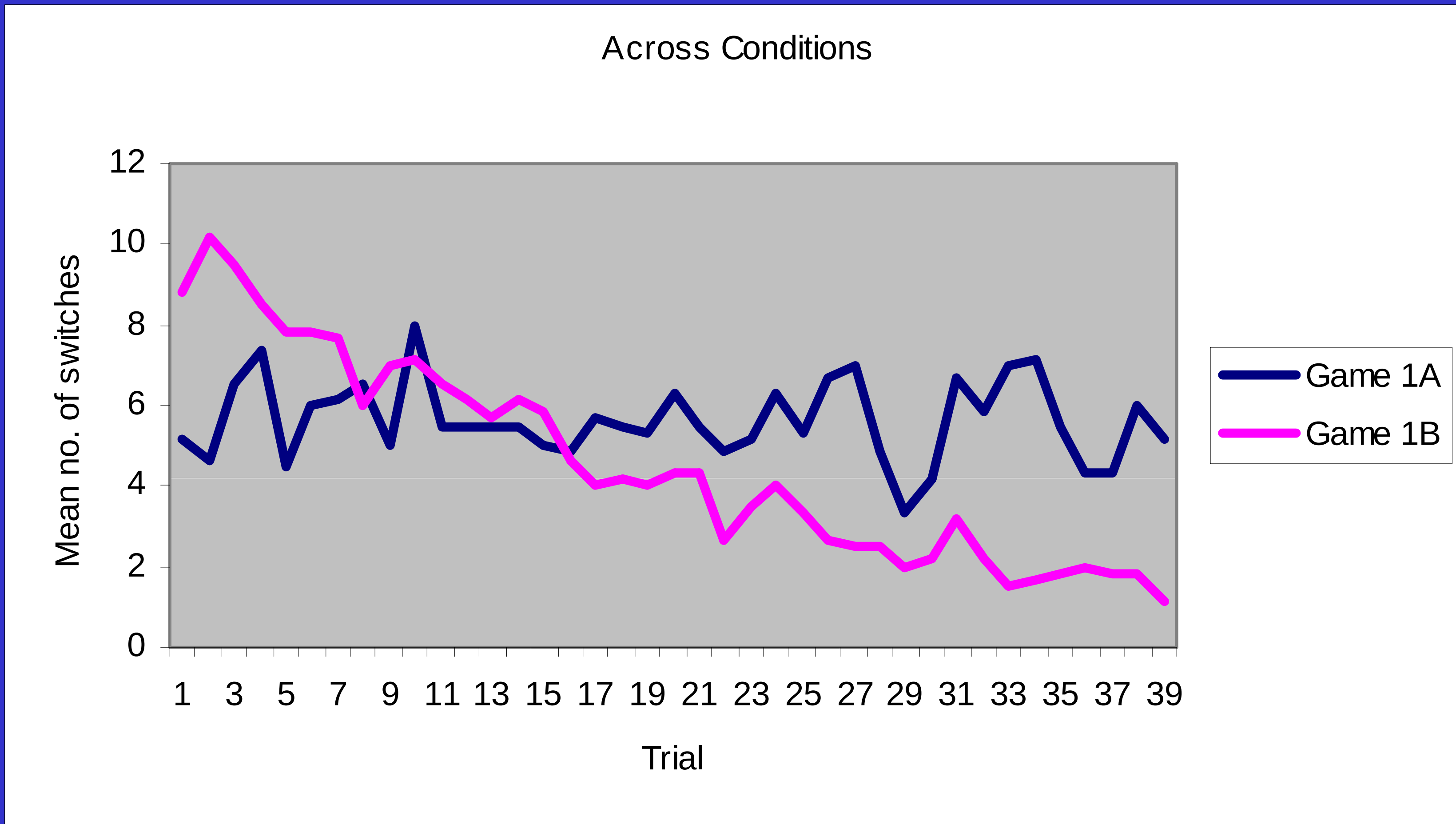
Across Both Conditions : Game 1B
Mean Number of Choices of Each Route by Trial



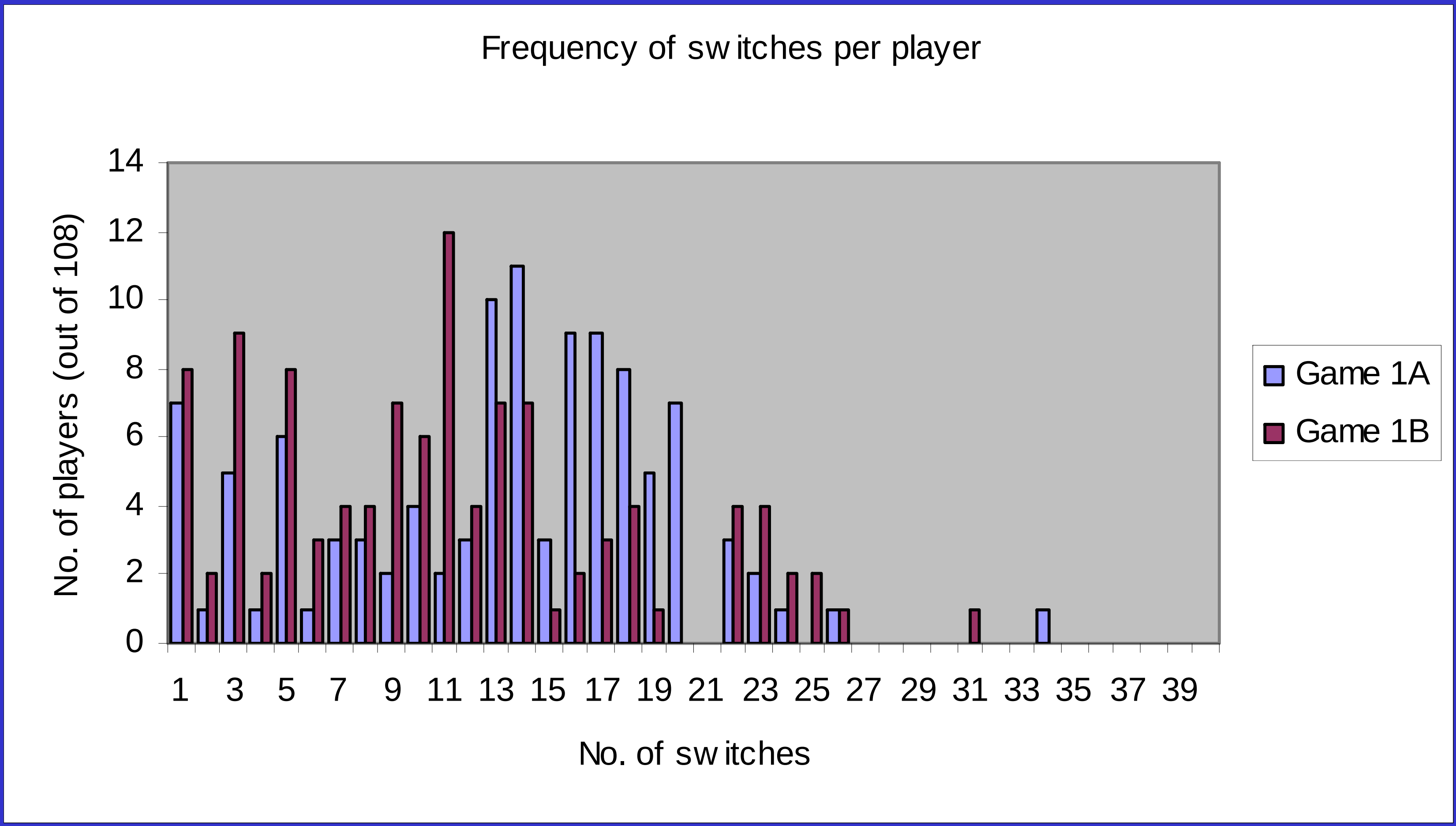
Across Conditions: Mean Payoff by Trial and Game



Across Conditions:
Mean Number of Route Switches between Trials



Across Conditions:
Frequency Distributions of Individual Number of Switches



Mean Frequency of Route Choice, Trials 21-40

Condition	Game 1A		Game 1B		
	O-A-D	*O-B-D*	*O-A-D*	*O-B-D*	*O-A-B-D*
ADD	8.95	9.05	0.85	0.57	16.58
DELETE	9.12	8.88	1.03	0.72	16.25
ACROSS	9.03	8.97	0.94	0.64	16.42
P.S. Eq.	9.00	9.00	0	0	18.00

Mean Payoff by Game, Condition, and Trials

Condition	Game 1A			Game 1B		
	1-40	21-40	40	1-40	21-40	40
ADD	92.63	94.37	98.15	58.55	50.74	42.59
DELETE	92.92	95.52	96.67	59.75	53.12	45.19
ACROSS	92.77	94.95	97.41	59.15	51.93	43.89
P.S. Eq.	100	100	100	40	40	40
M.S. Eq.	95	95	95			

Conclusions of the First Experiment on the Two-route Network

- We have presented and illustrated the *Braess Paradox*: a directed network where (theoretically) adding a single link to an existing network may cause all network users to be ***worse off***.

- The normative model is based on ***equilibrium*** analysis of the basic and augmented games.

- The BP has inefficient equilibria.

Conclusions (con't)

- Using a within-subject design and inflating the effect of the BP, we have experimentally tested the model by eliciting independent route choices of financially motivated subjects in a simple network that is susceptible to the BP.

- The results show clearly that the BP is not merely an interesting curiosity, but a significant result of potential practical importance.

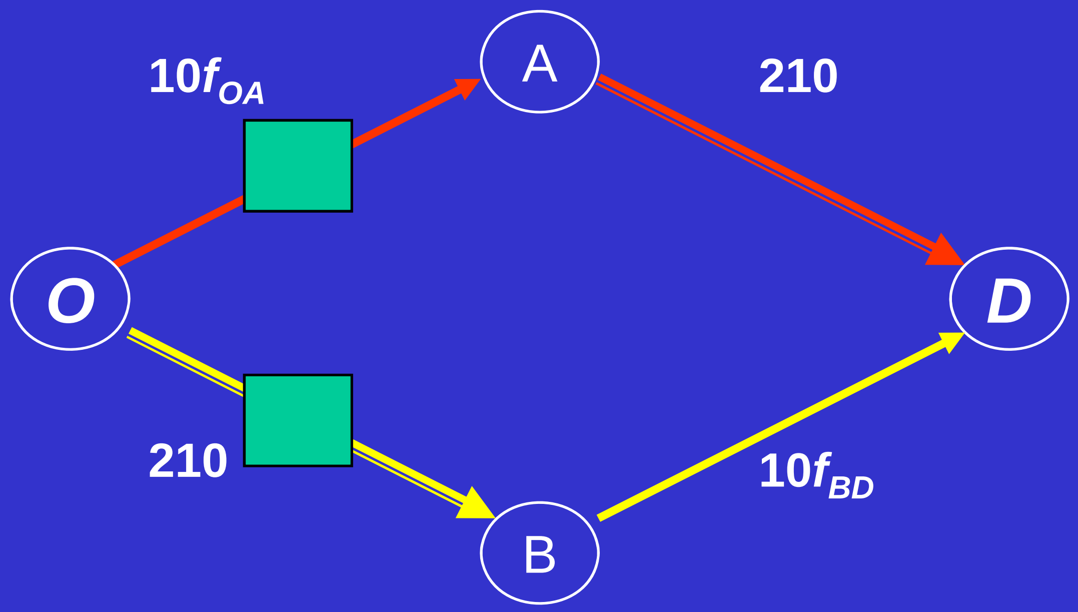
Limitations

There are two major limitations of our first two-route network experiment.

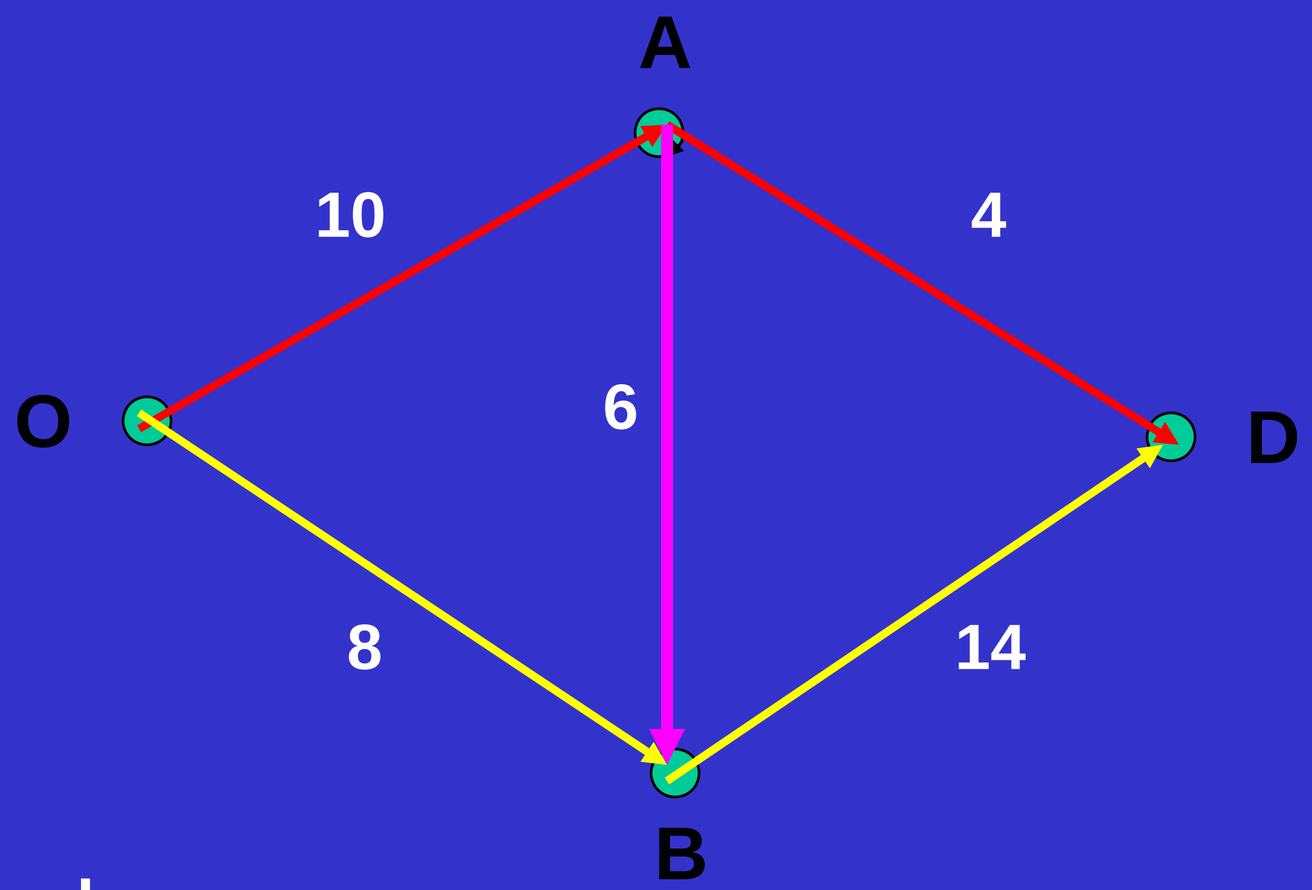
First, the network that we have studied is too simple. It has only two routes and the costs are symmetric. Our results may not generalize to richer networks with more routes and no symmetry in the cost structure.

Second, providing complete information at the end of each round on the route choices of all the commuters is not realistic. In traffic networks in the wild, commuters are only informed of the cost associated with the route they have actually chosen and the congestion on this route.

Note that the effects of information in our simple two-route network cannot be assessed because each commuter may deduce the route choices of all the commuters.



Although less obvious, each commuter may also deduce the route choices of all the commuters in the augmented game.



Example

Major Purposes of This Study

Therefore, we have decided to study a richer network in order to answer the following two research questions.

1. Do the results that support the BP in the simple network generalize to richer networks?

2. Is there support for the BP when commuters are only informed about the congestion and travel cost on the route they actually choose?

3. Hypotheses: No to both questions.

Experimental Design

- ♦ **Two experimental conditions**

  Complete Information

  Incomplete Information

- ♦ **Within-subject design in each condition**

  Play Game B for 60 rounds

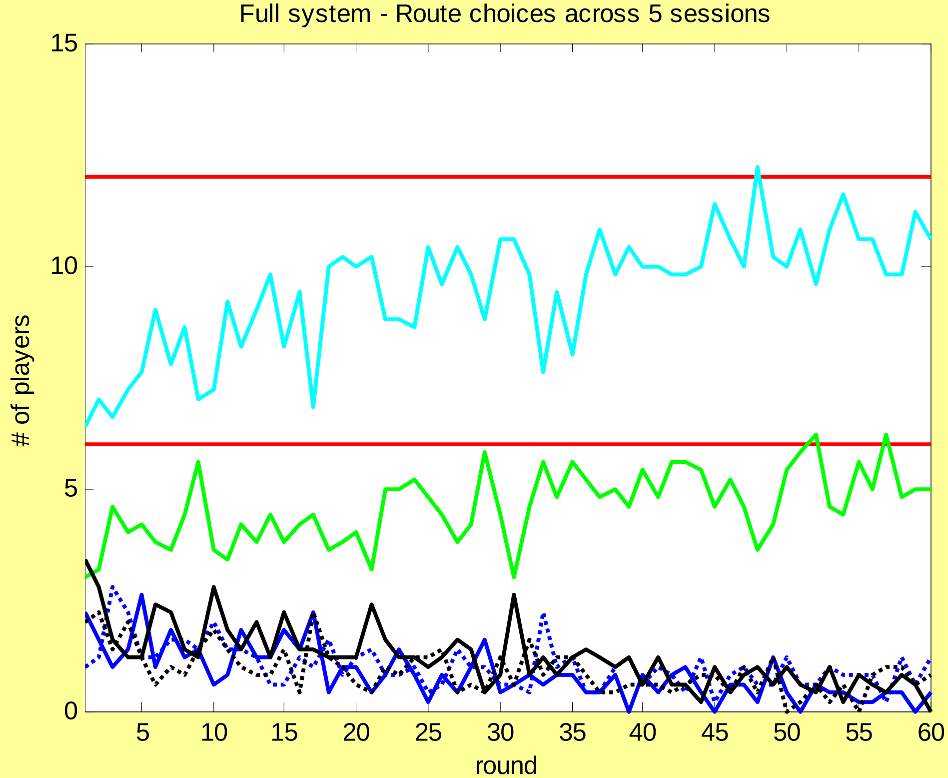
  Play Game A for 60 rounds

- ♦ **5 sessions in each condition**
- ♦ **18 subjects in each session**
- ♦ **Same endowment of 290 for each game**

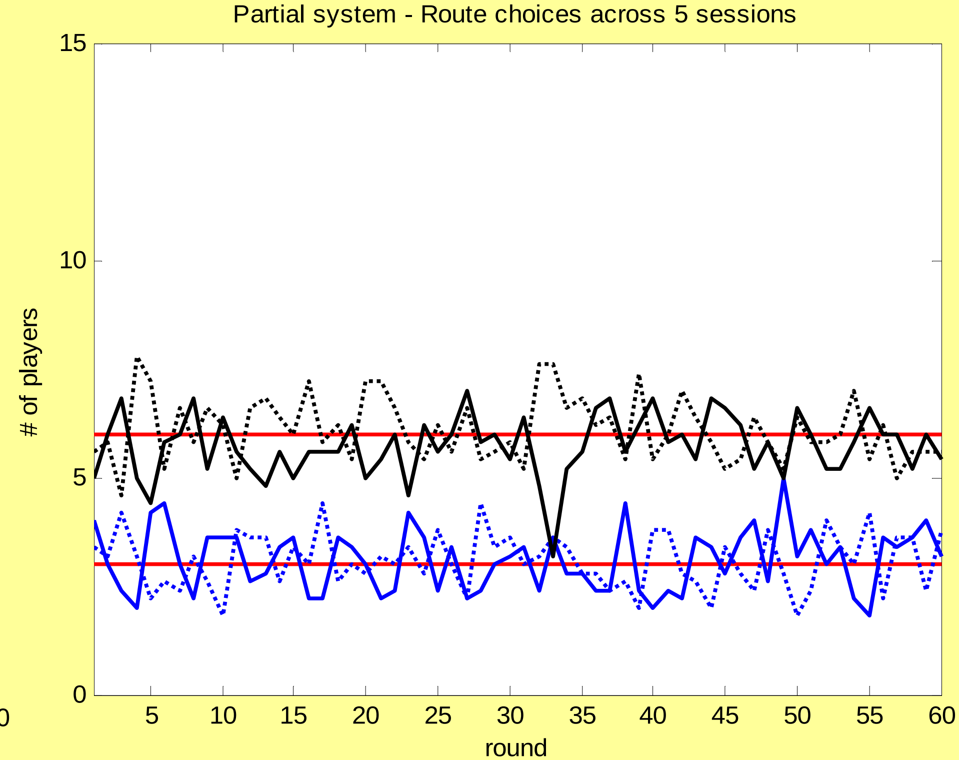
  The resulting equilibrium payoff is

  290-190=100 for Game B

  290-240= 50 for Game A



Game A

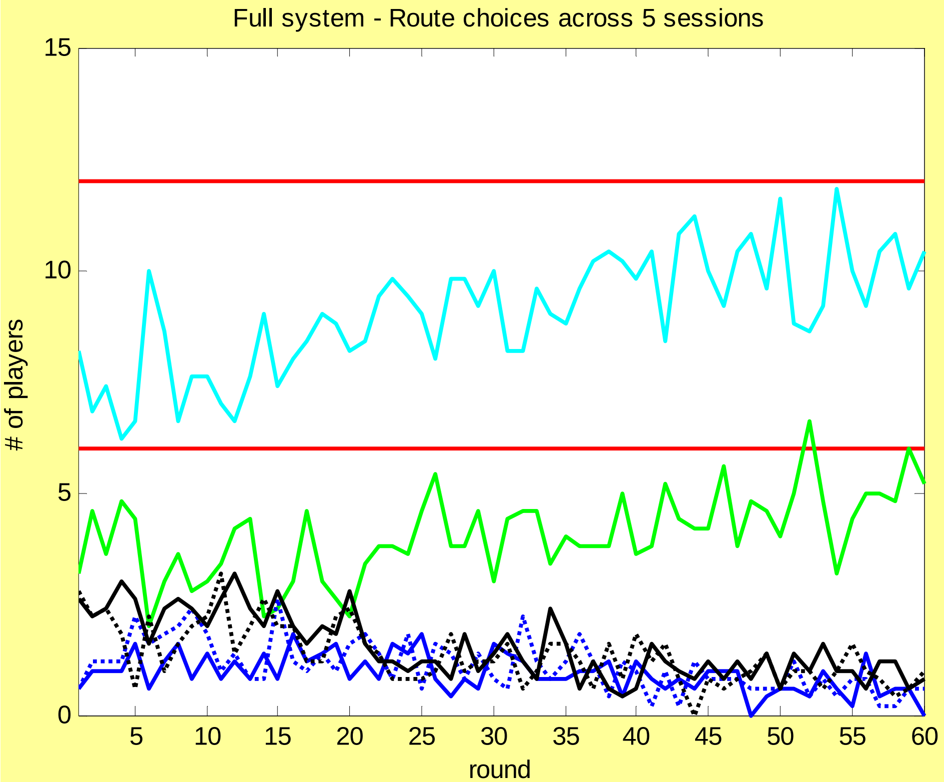


Game B

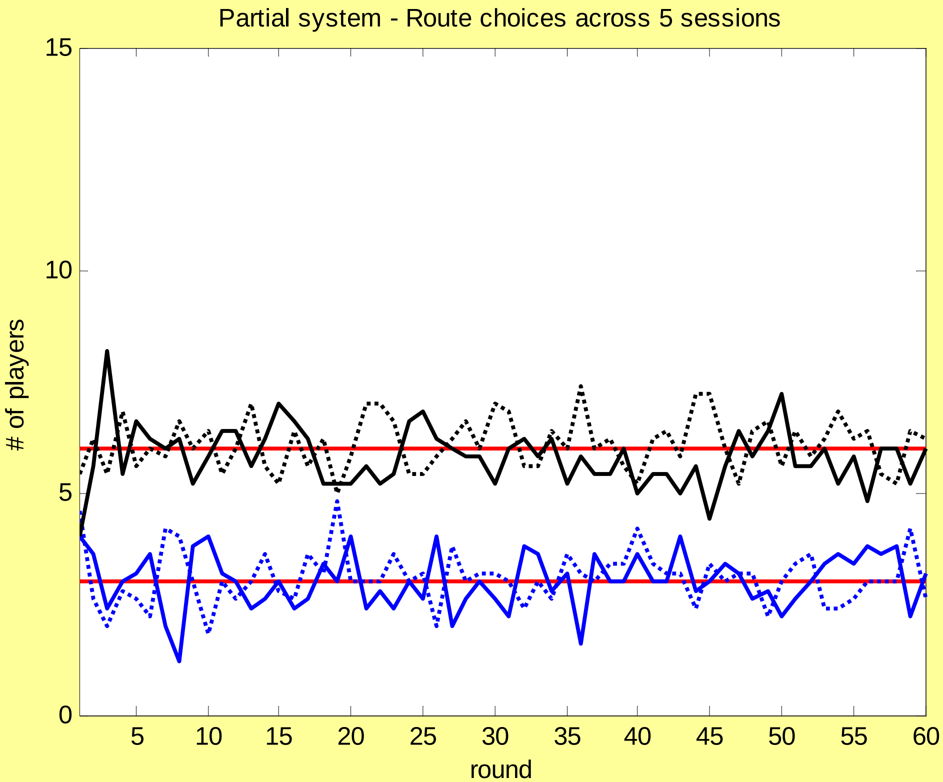
Equilibrium decisions are denoted by red lines:

Game A top line:	**12**
Game A bottom line:	**6**
Game B top line:	**6**
Game B bottom line:	**3**

Mean number of route choice in the Full Information condition by game type (A or B) and route.



Game A



Game B

Equilibrium decisions are denoted by red lines:

Game A top line:	**12**
Game A bottom line:	**6**
Game B top line	**6**
Game B bottom line:	**3**

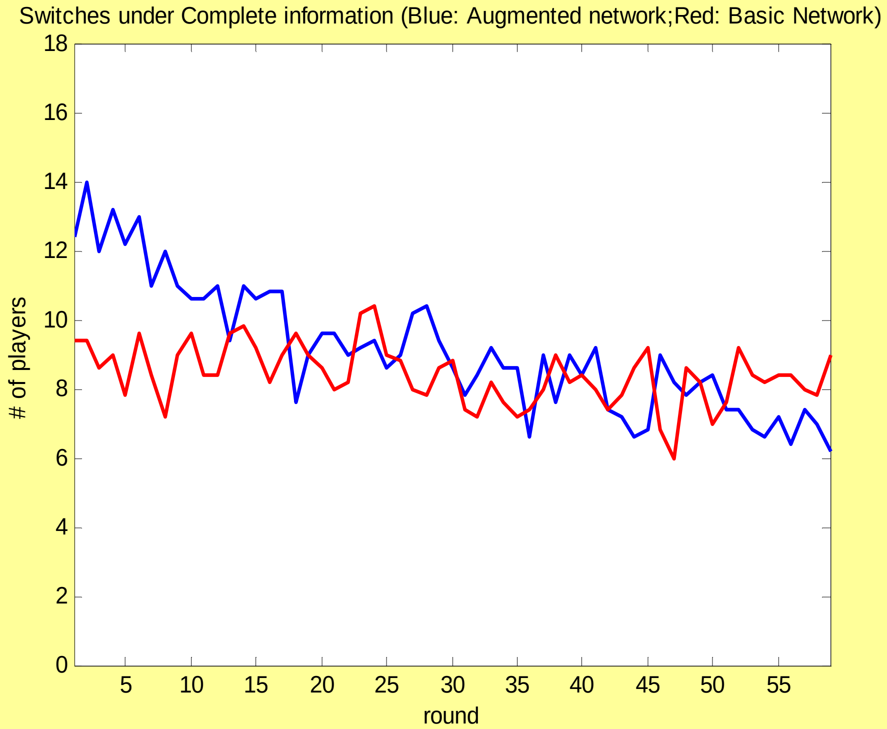
Mean number of route choice in the Incomplete Information condition by game type (A or B) and route.

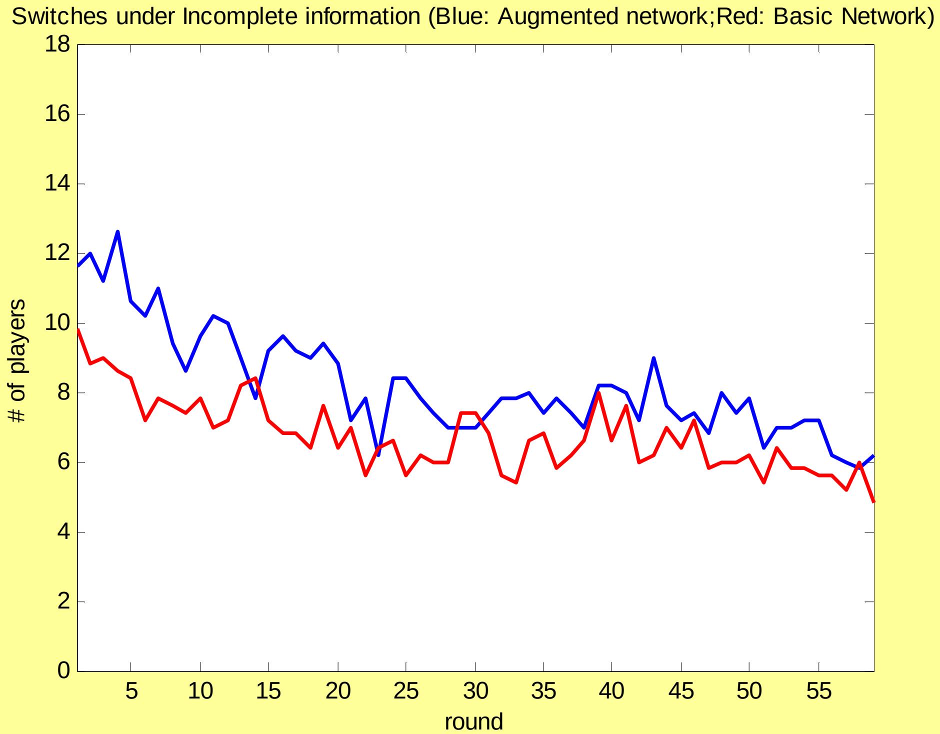
Mean results may not represent the results for individual sessions.

Therefore, I have ten slides at the end of this presentation that exhibit the route choices for each session separately.

First, we display the mean route choices for Condition Complete Information, and next the mean route choices for Condition Incomplete Information.

Note how similar the sessions are to one another.

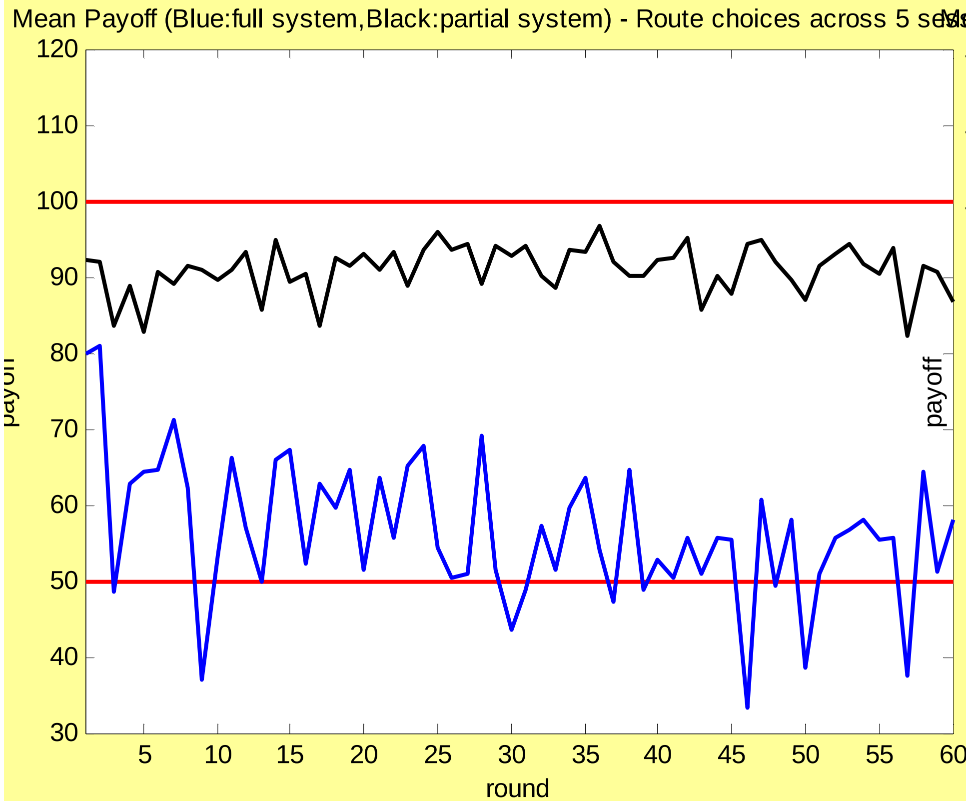




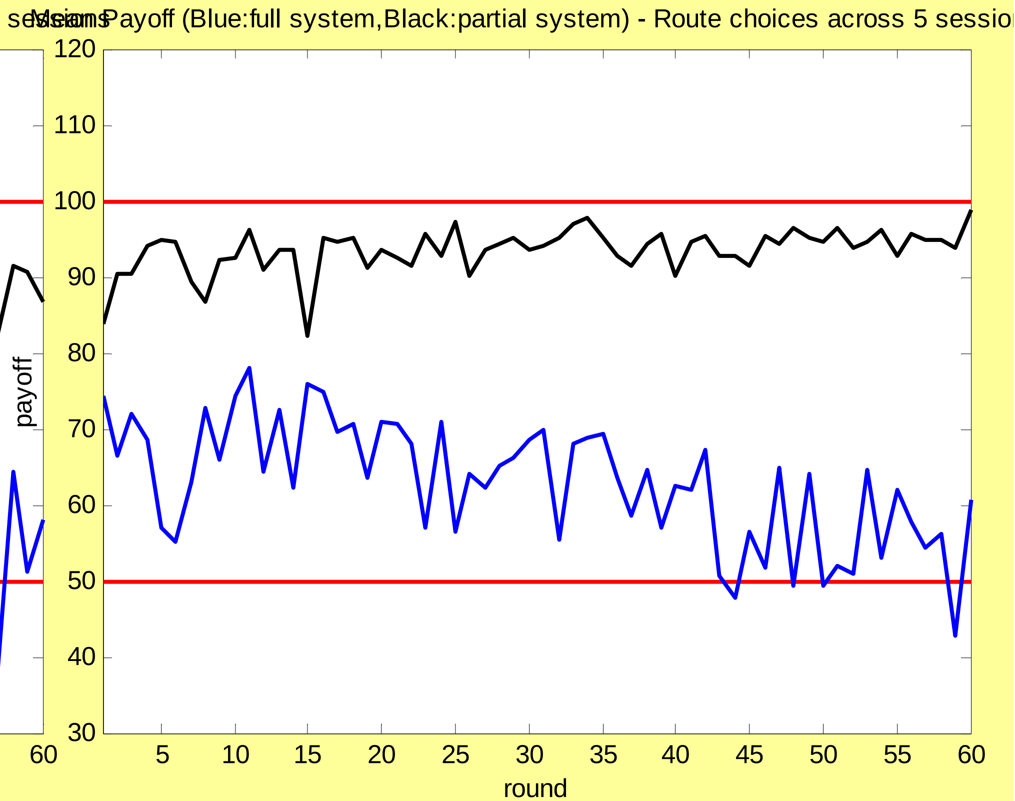
Complete Information **Incomplete Information**

Mean Number of switches between routes by game type and condition.

Blue line: Game A (Augmented)
Red Line: Game B (Basic)



Complete information



Incomplete information

Horizontal lines show the equilibrium prediction.

Top line (100): Game B
Bottom line (50): Game A.

<u>Mean payoff</u> across sessions by condition, game type (A or B) and round of play.

General Conclusions

- We observe systematic and replicable patterns of aggregate behavior in fairly rich networks with decentralized decisions.

- Moreover, with experience in traversing the network these aggregate patterns of behavior converge to the equilibrium solution.

- More surprisingly, converge to equilibrium behavior supports the Braess Paradox: subjects converge to equilibrium when the network is expanded and, consequently, their payoff is cut by half.

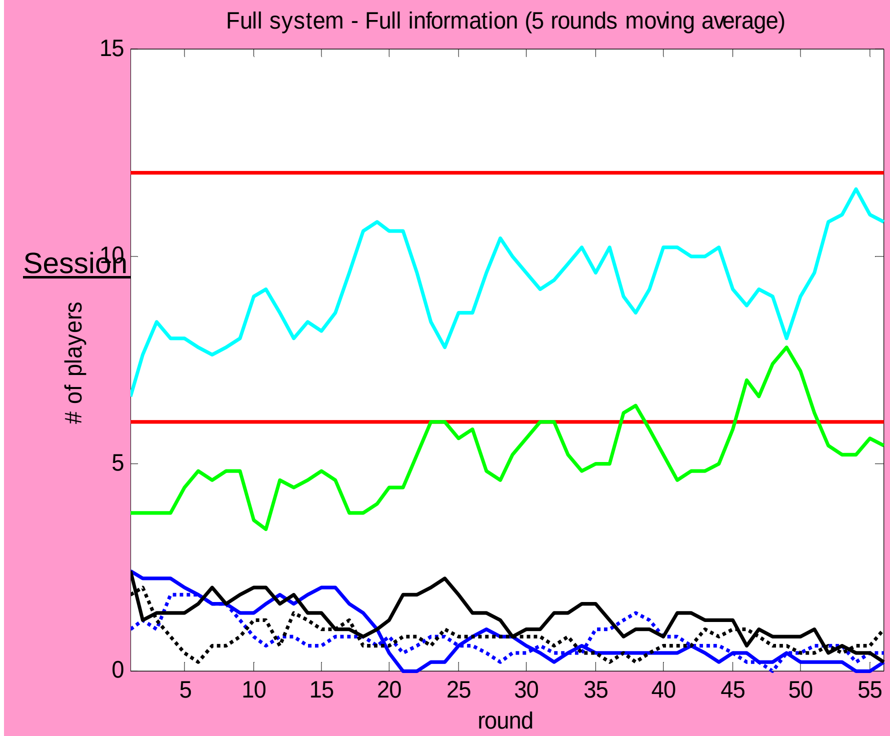
- All of this happens when the same subjects encounter the Basic network and the Augmented network.

- Finally, we observe no differences between the two information conditions in 1) route choice, 2) frequency of switches, and 3) rate of learning.

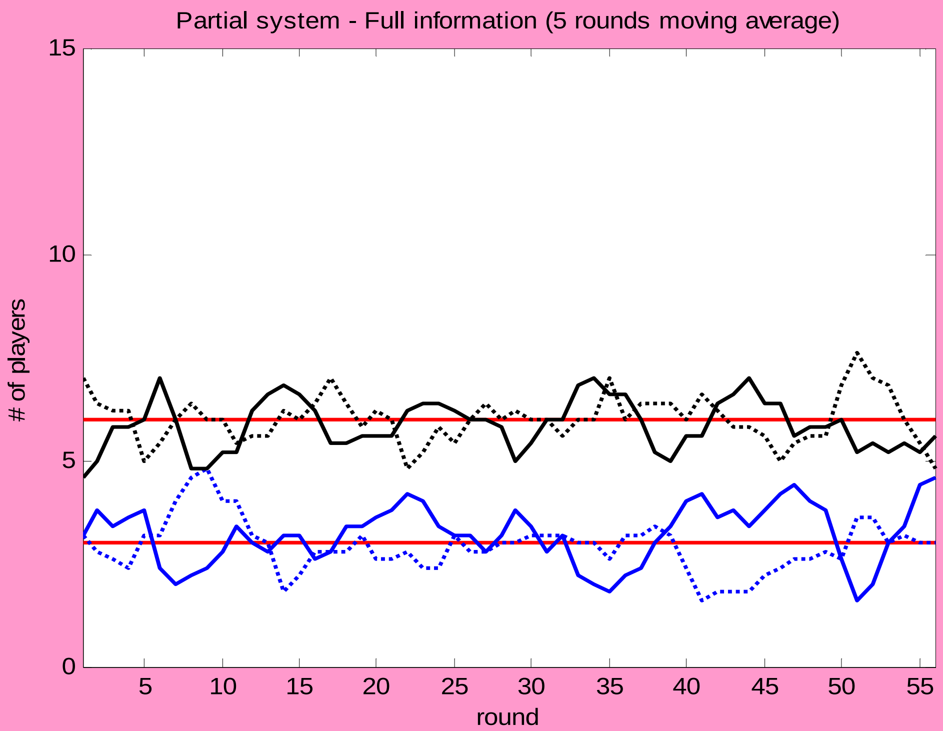
The Challenge

The challenge is to explain these results.

The efficient distribution of limited resources by decentralized individual decisions, which most likely is task dependent, is still an open question in many network systems.



Game A



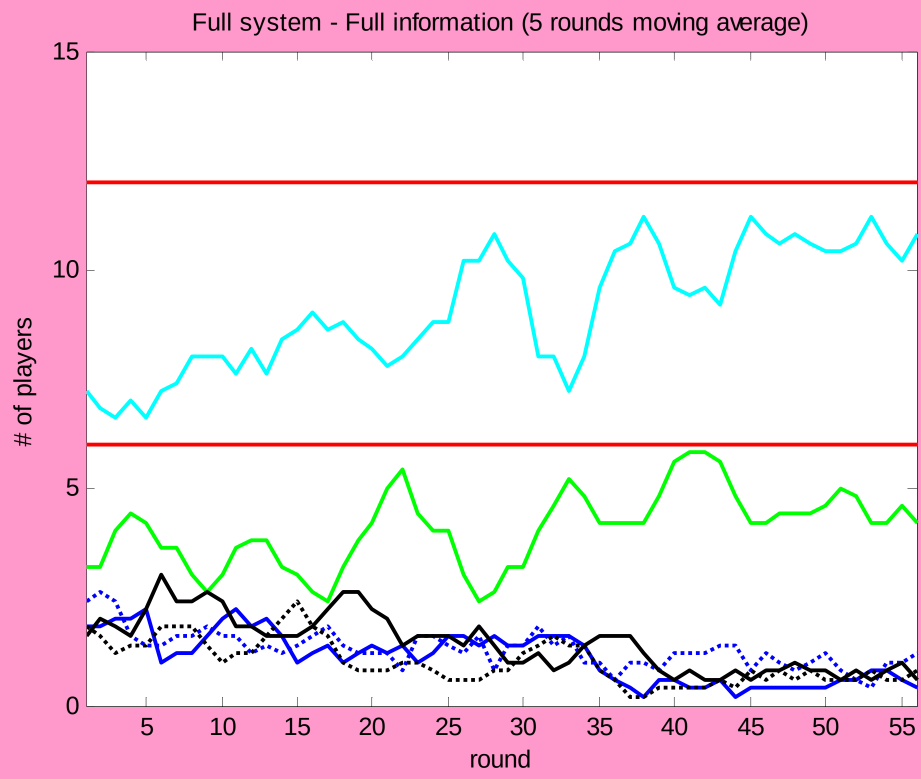
Game B

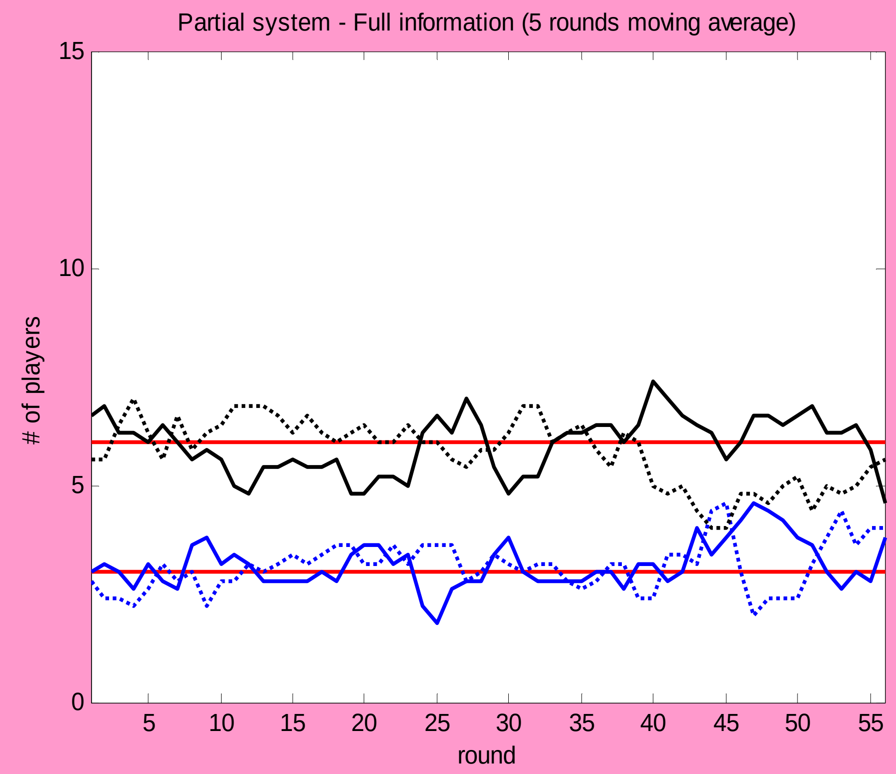
Route Choice by Game Type and Route

Condition: Complete Information

Session 1

Equilibrium predictions | **Game A: 12, 6, 0, 0, 0,0**
Game B: 6, 6, 3, 3





Game A **Game B**

Route Choice by Game Type and Route

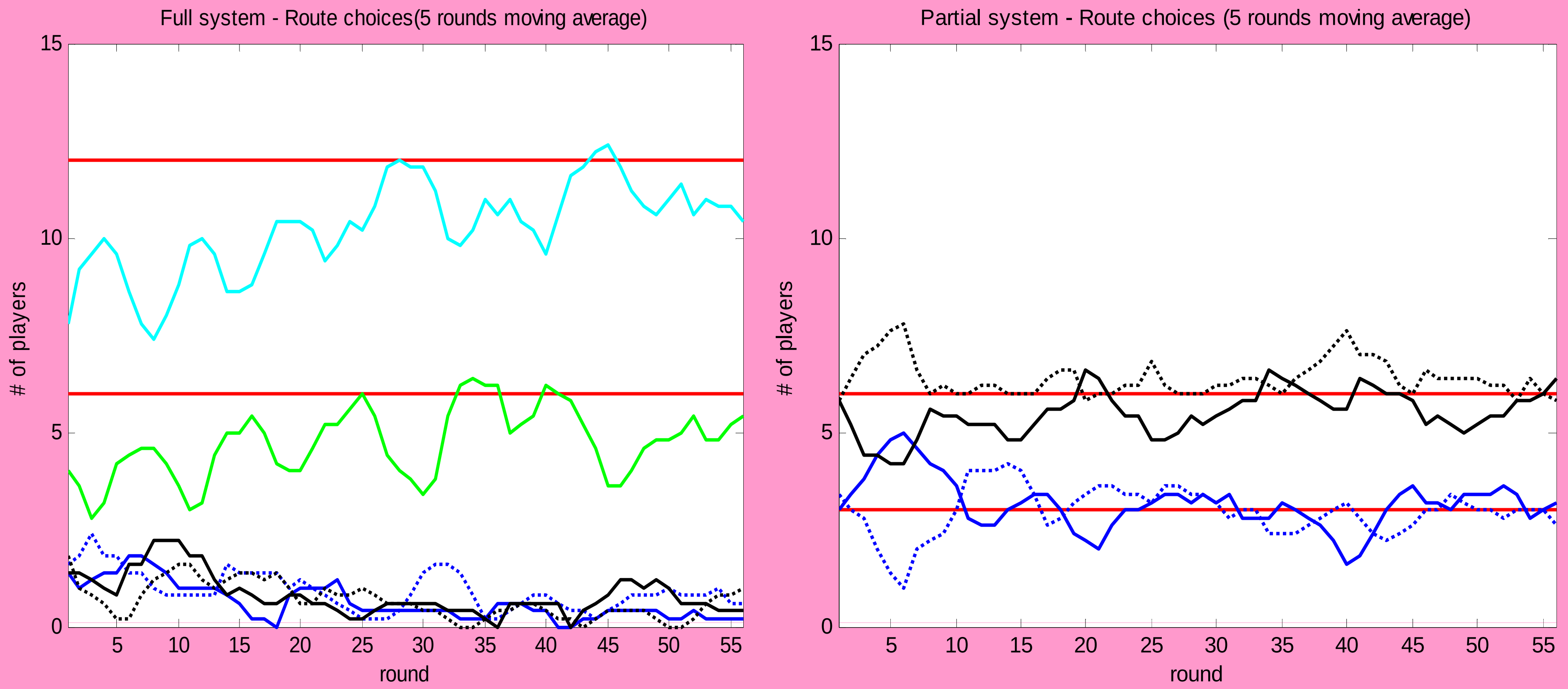
Condition: Complete Information

Session 2

Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A

Game B

Route Choice by Game Type and Route

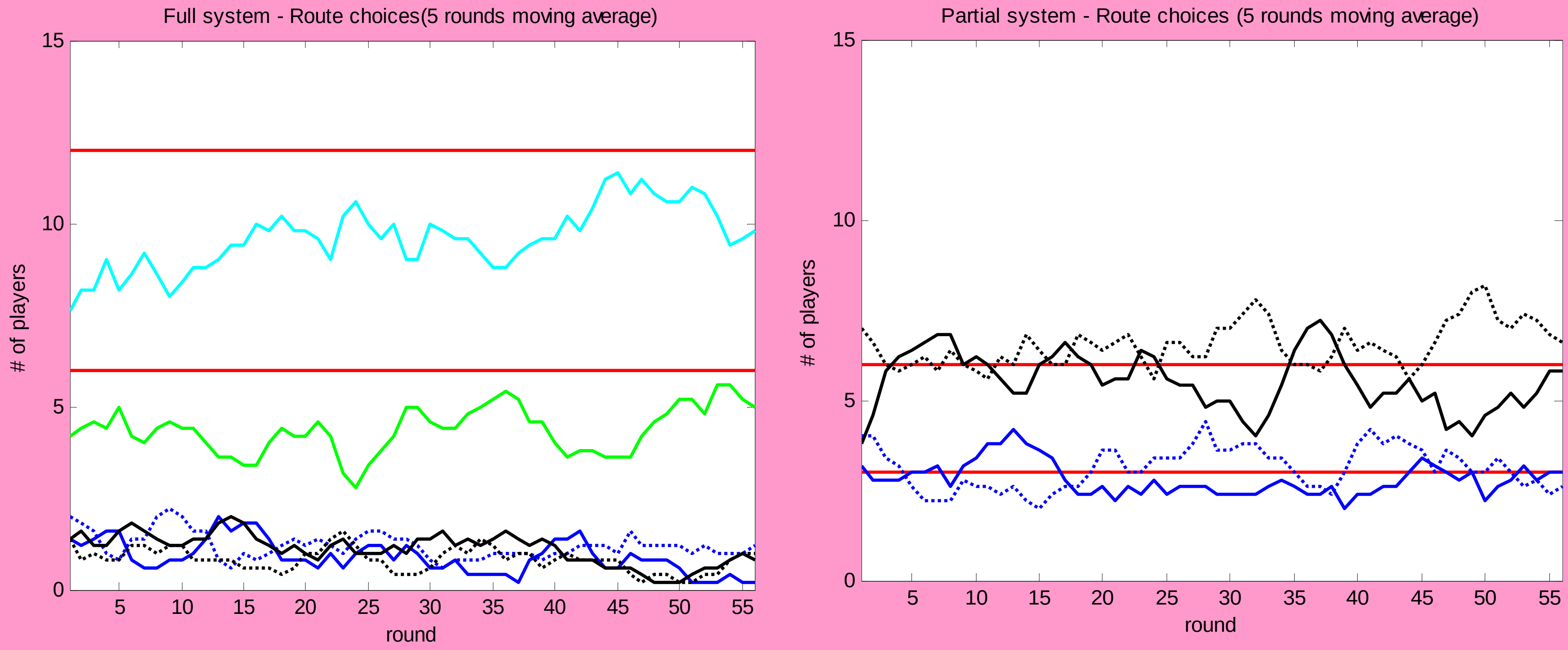
Condition: Complete Information

Session 3

Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A **Game B**

Route Choice by Game Type and Route

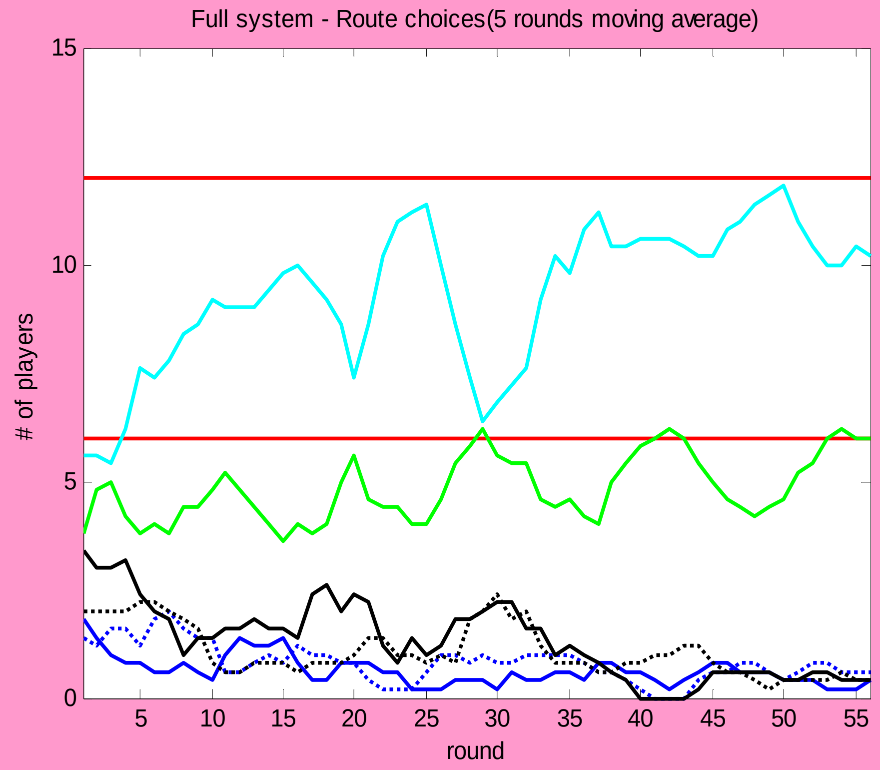
Condition: Complete Information

Session 4

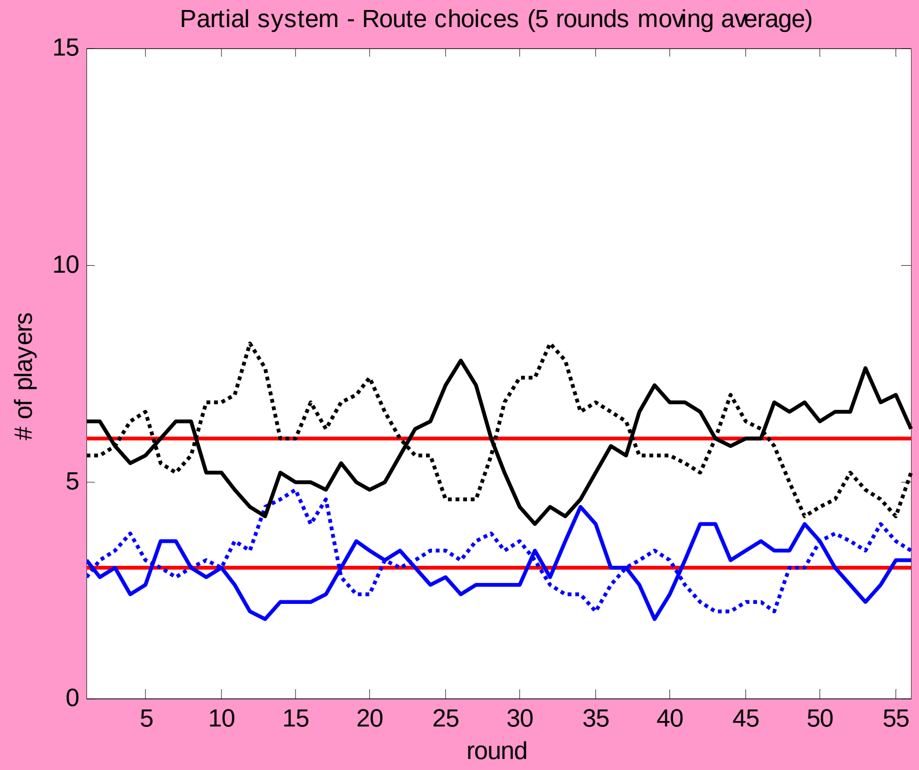
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A



Game B

Route Choice by Game Type and Route

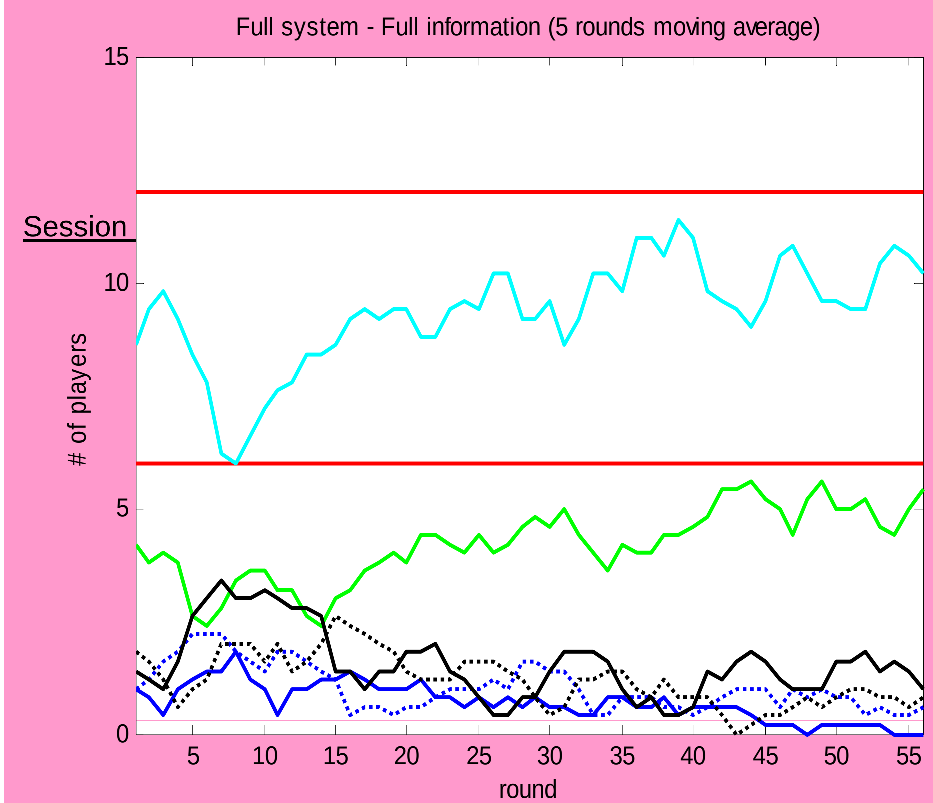
Condition: Complete Information

Session 5

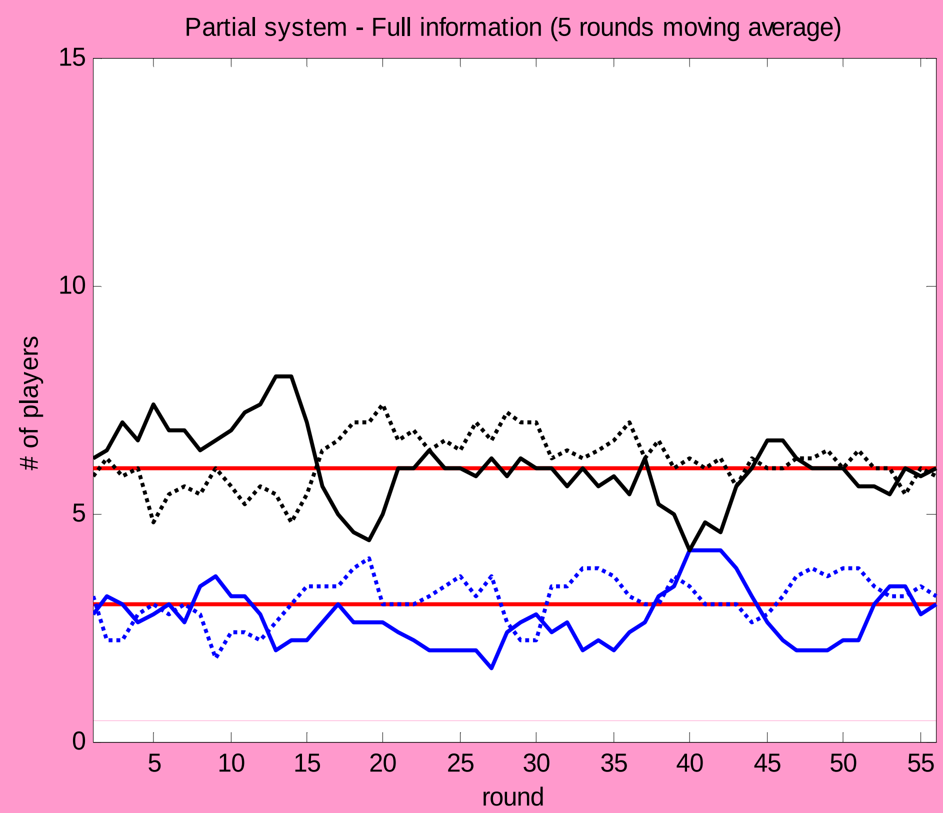
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A



Game B

Route Choice by Game Type and Route

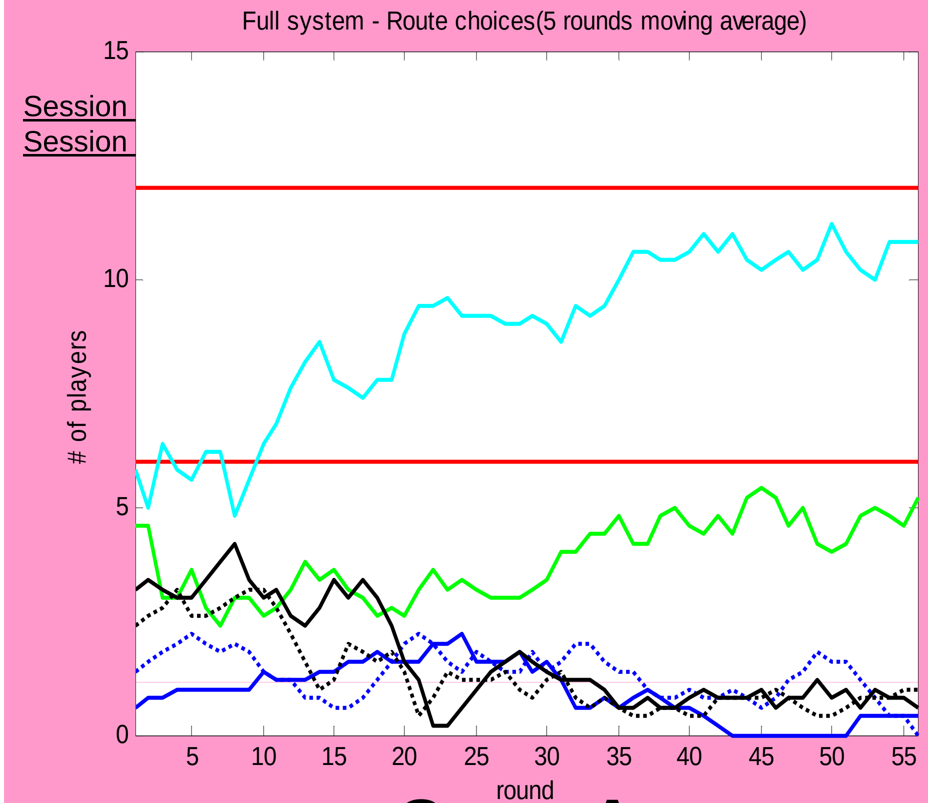
Condition: Incomplete Information

Session 1

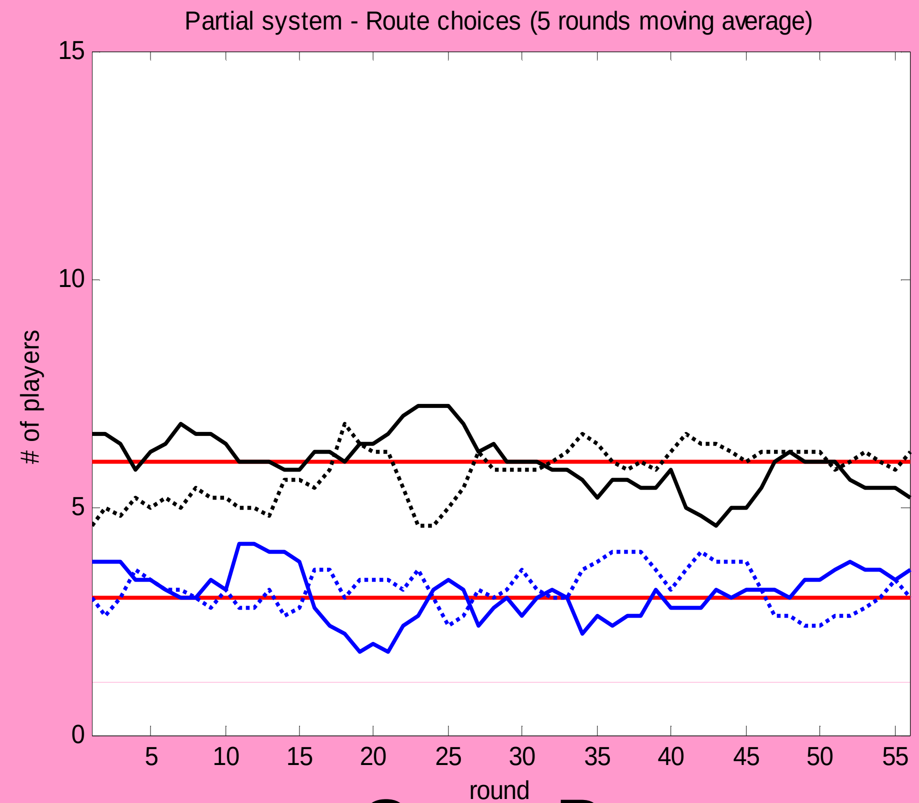
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A



Game B

Route Choice by Game Type and Route

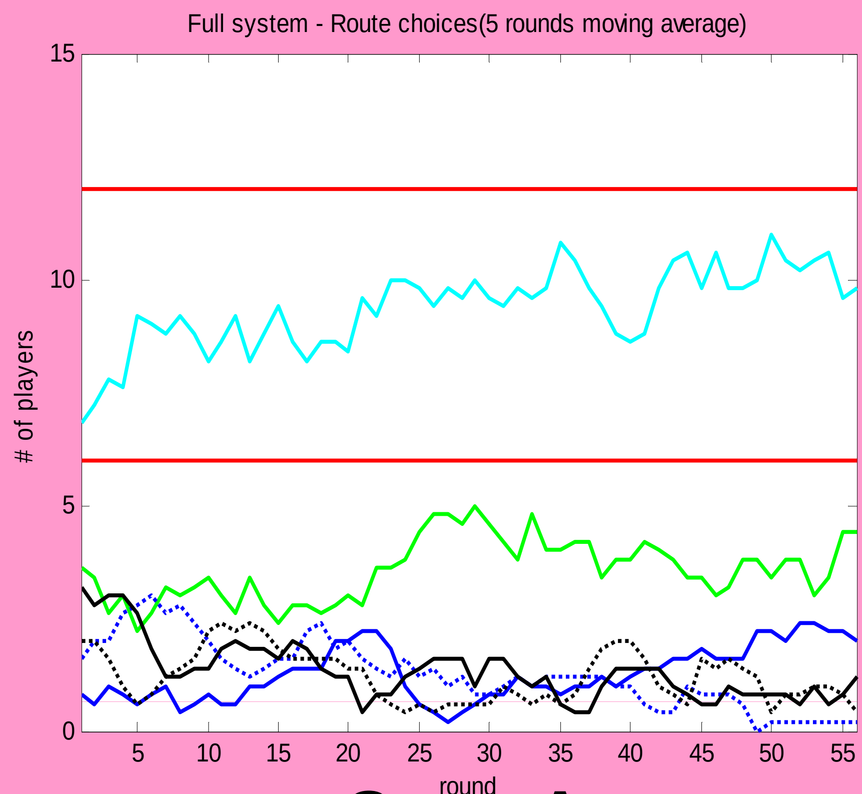
Condition: Incomplete Information

Session 2

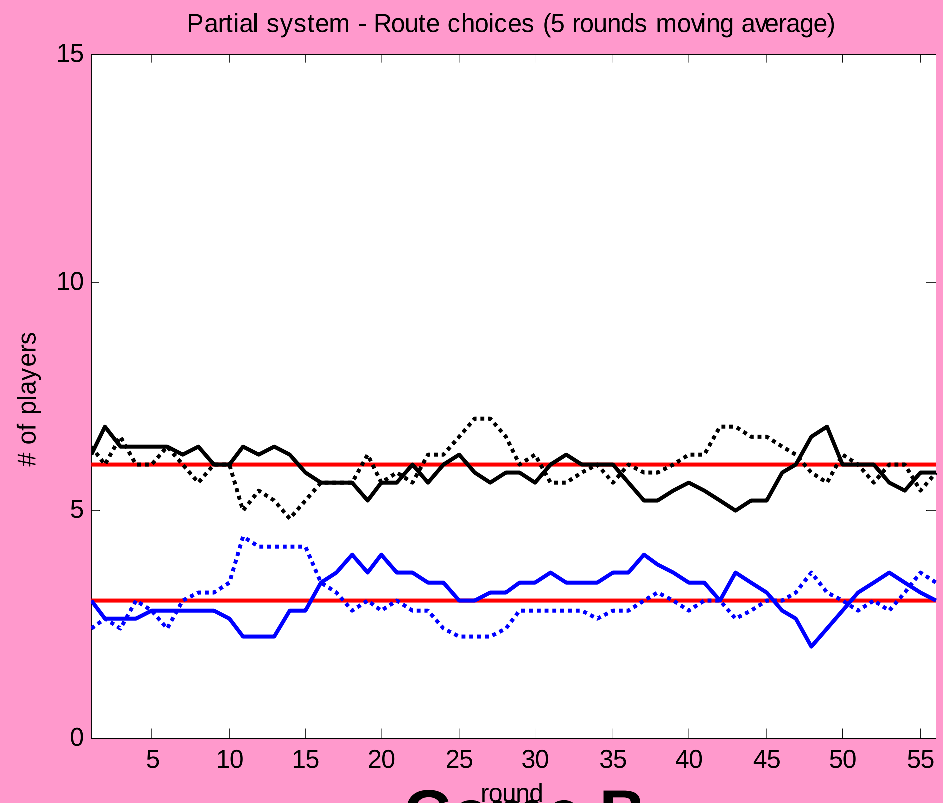
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A



Game B

Route Choice by Game Type and Route

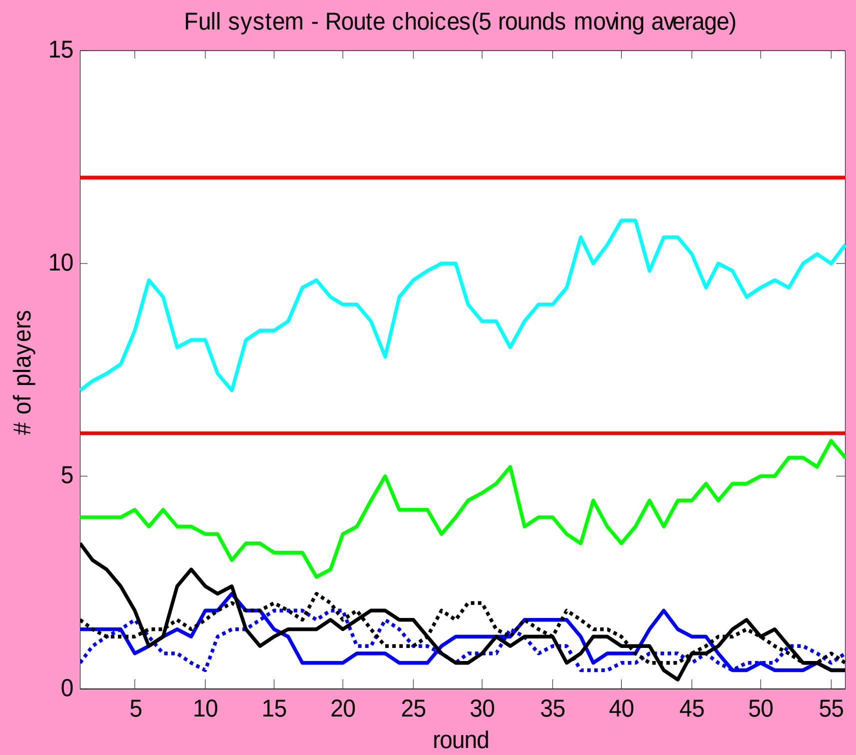
Condition: Incomplete Information

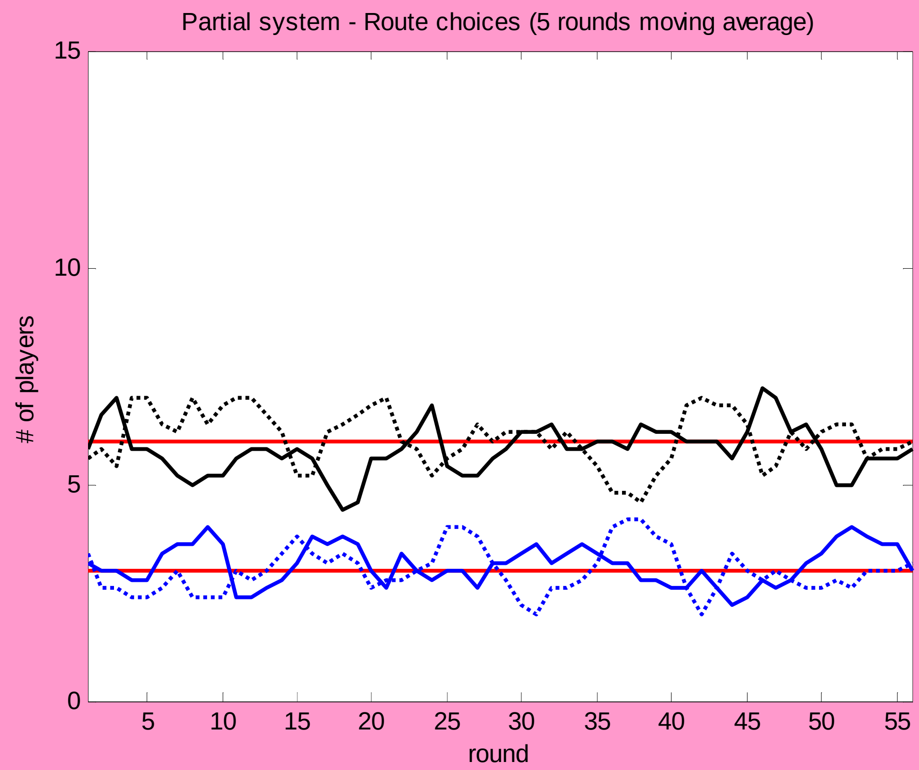
Session 3

Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3





Game A

Game B

Route Choice by Game Type and Route

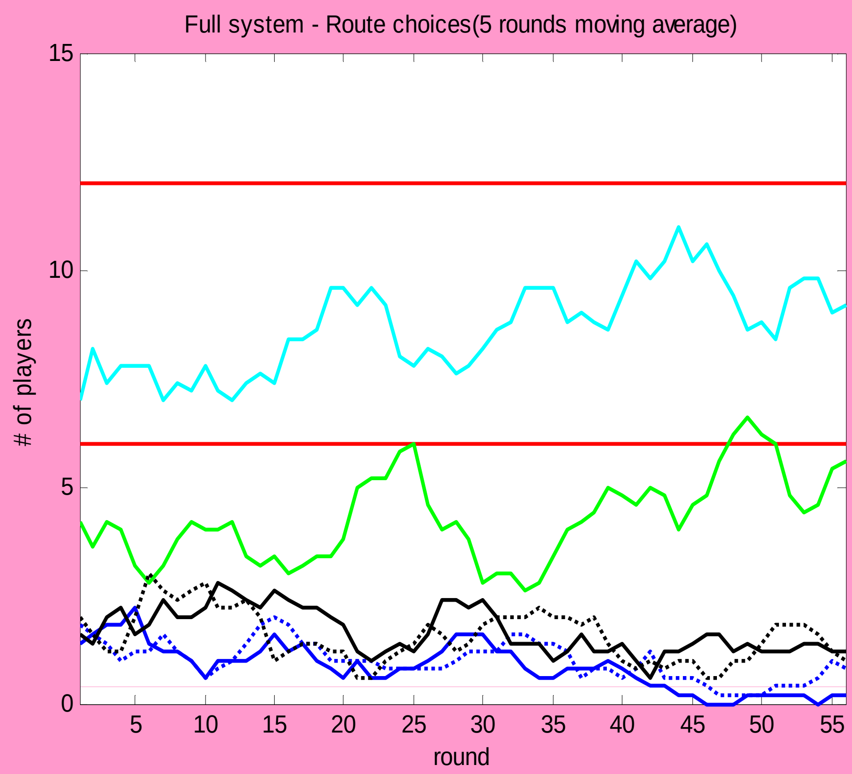
Condition: Incomplete Information

Session 4

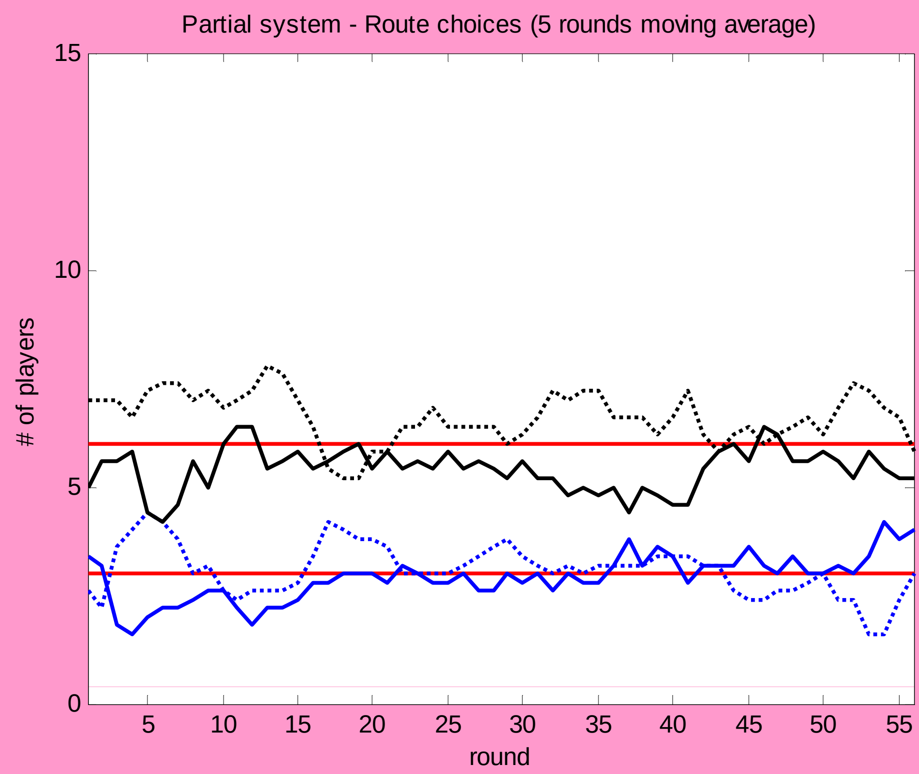
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Game A



Game B

Route Choice by Game Type and Route

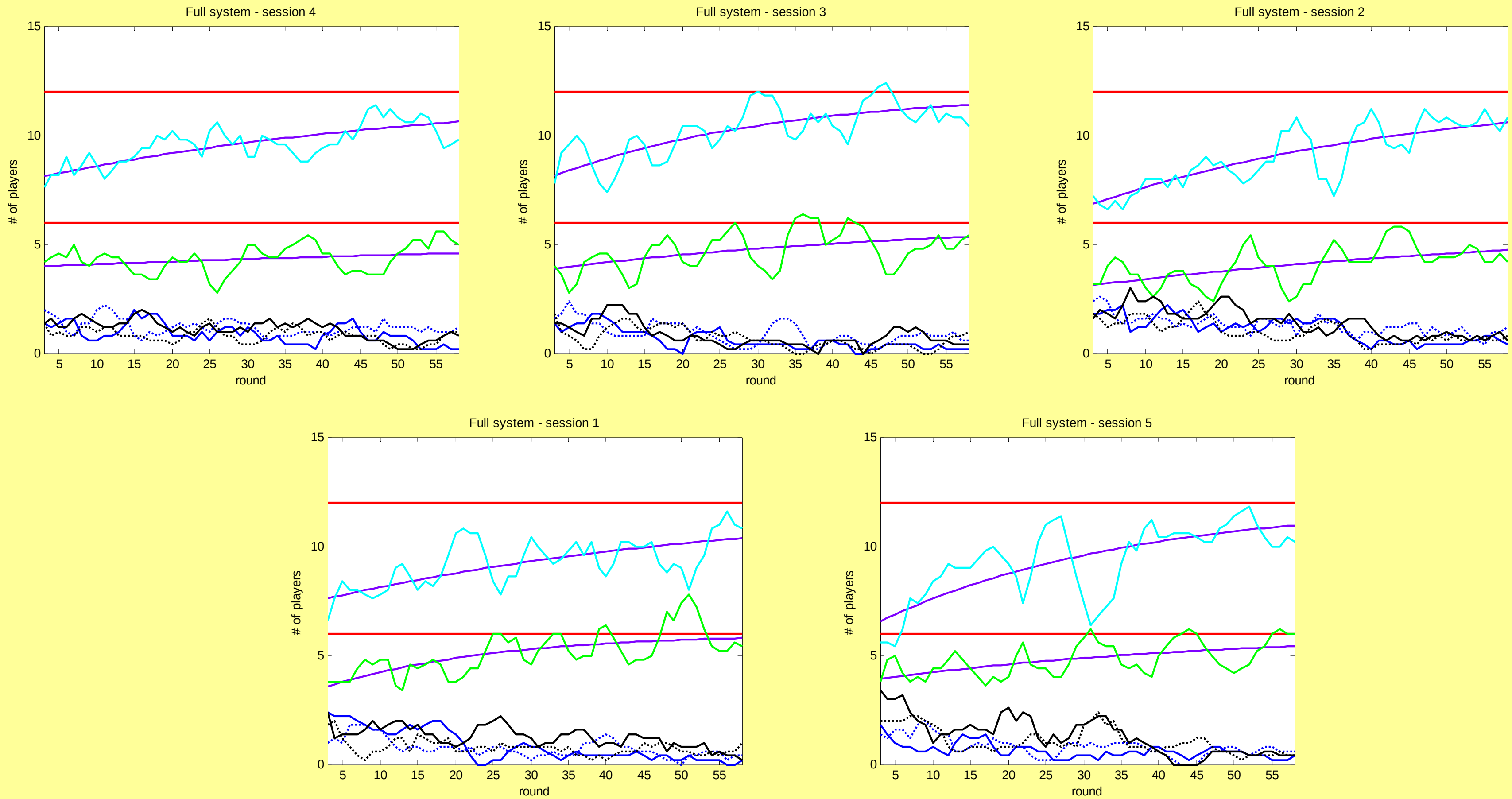
Condition: Incomplete Information

Session 5

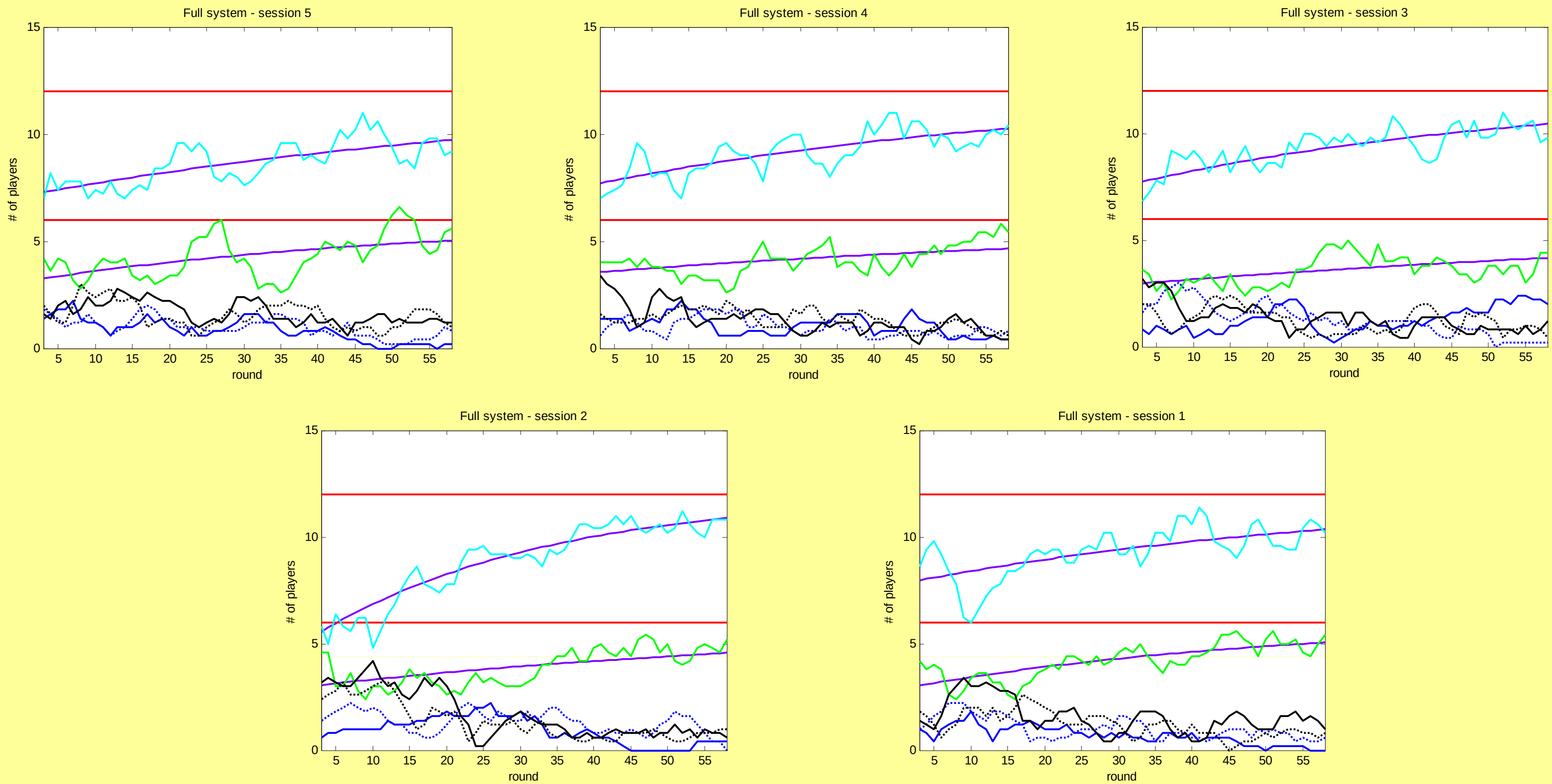
Equilibrium predictions

Game A: 12, 6, 0, 0, 0,0

Game B: 6, 6, 3, 3



Condition: Complete Information
Exponential functions fitted to routes O-B-F-D (top: 12) and O-A-E-D (bottom: 6) in the five individual sessions



Condition: Incomplete Information

Exponential functions fitted to routes O-A-F-D (top: 12) and O-A-E-D (bottom: 6) in the five individual sessions