

Riemann, Weierstrass and Tagaki functions

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Over the last seven months, an almost weekly seminar was held at Chapman University "in connexion" with Politecnico di Milano to explore the many facets of the Weierstrass function

$$W_{a,b}(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

together with the Riemann and Tagaki functions, given respectively by

$$R(x) = \sum_{n=1}^{\infty} \frac{\sin(n^2 \pi x)}{n^2}, \quad \tau(x) = \sum_{n=0}^{\infty} \frac{\langle\langle 2^n x \rangle\rangle}{2^n}$$

where $\langle\langle a \rangle\rangle$ is the distance of a to \mathbb{Z} . In this talk I will focus on two aspects: Lacunarity and modularity. The lacunarity will enable us to use probabilistic methods, in particular a famous theorem of Mary Weiss on the use, for lacunary trigonometric series, of the Law of the Iterated Logarithm of Kolmogoroff. The modularity will enable us to relate Riemann function and Weierstrass functions (for special values of a and b) to modular functions and to exhibit fast oscillations of $W_{a,b}(x)$.

M.V. Berry with Z.V. Lewis, and, with S. Morley-Short investigated superoscillations provided by the Weierstrass function.

This work is on progress, in collaboration with D.C. Struppa, Irene Sabadini and Fabrizio Colombo.