The Attack and Defense of Weakest-Link Networks

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Abstract

We experimentally test the qualitatively different equilibrium predictions of two theoretical models of attack and defense of a weakest-link network of targets. In such a network, the attacker's objective is to successfully attack at least one target and the defender's objective is to defend all targets. The models differ in how the conflict at each target is modeled — specifically, the lottery and auction contest success functions (CSFs). Consistent with equilibrium in the auction CSF model, attackers utilize a stochastic "guerrilla-warfare" strategy, which involves randomly attacking at most one target with a random level of force. Inconsistent with equilibrium in the lottery CSF model, attackers use the "guerrilla-warfare" strategy and attack only one target instead of the equilibrium "complete-coverage" strategy that attacks all targets. Consistent with equilibrium in both models, as the attacker's valuation increases, the average resource expenditure, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

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1. Introduction

In many network applications, such as cyber-security, electrical power grids, or oil pipeline systems, the failure of any individual component in the network may be sufficient to disable the entire network. In the case of a system of dikes on the perimeter of an island, Hirshleifer (1983) coined the term weakest-link to describe this type of intra-network complementarity among components. [1] In addition to networks with physically linked components, political considerations may, also, create a situation in which physically disjoint components are connected by a form of weakest-link complementarity in preferences. For example, a single terrorist spectacular may allow a terrorist to influence its target audience.[2] This paper experimentally examines two models of attack and defense of a weakest-link network of targets that differ with respect to the choice of contest success function (CSF), i.e. the mapping from the two players' resource allocations to a target into their probabilities of winning the target, used to model the conflict at each target.

In the attack and defense of a weakest-link network, the nature of the CSF is a key determinant of equilibrium behavior. We focus on two CSFs, lottery and auction, which are two special cases of the general ratio-form contest success function $x_A^r/(x_A^r + x_D^r)$ where x_A and x_D are the attacker's and defender's allocations of force, respectively, and the parameter $r > 0$ is inversely related to the level of noise, or randomness, in the determination of the winner of the conflict (conditional on the players' allocations). In the lottery CSF $r = 1$, a situation in which the outcome of the conflict at each target has a relatively high amount of noise. The auction CSF

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[1] Applications of the weakest-link structure include: organizational performance that depends on the weakest-link (Kremer, 1993); internet security (Moore et al., 2009); and package auctions in which the objective of some bidders is to obtain all of the goods while for other bidders the objective is to obtain only one good (Milgrom, 2007).
[2] As stated in the *Joint House-Senate Intelligence Inquiry into September 11*, 2001 (US Congress, 2002), terrorists need to be successful only once to kill Americans and demonstrate the inherent vulnerabilities they face.

corresponds to the limiting case where $r = \infty$, a situation in which there is no noise (i.e., the player that allocates the higher level of force wins).[3]

In Clark and Konrad (2007) the conflict at each target of the weakest-link attacker-defender game features the lottery CSF. They assume that the exogenous noise generated is independent across targets and demonstrate the existence of a pure-strategy "complete-coverage" equilibrium in which all targets are attacked and defended.[4] In contrast, Kovenock and Roberson (2017) show that, in all equilibria of the game with the auction CSF, the attacker utilizes a stochastic "guerrilla-warfare" mixed strategy, which involves randomly attacking at most one target – where each target is equally likely to be the one that is attacked – with a random level of force.[5] Conversely, the defender uses a mixed strategy that stochastically covers all of the targets, allocating a random level of force to each target. This results in a correlation structure of endogenous noise that makes all multiple target attacks payoff dominated by a single target attack.[6] In this paper, we complete the characterization of equilibrium in the lottery CSF version of the game by showing that equilibrium is unique and test the implications of these two models in a laboratory experiment. We employ a two-by-two design that investigates the impact of the CSF (lottery versus auction) and the relative valuation of the attacker's prize (low versus high) on the behavior of attackers and defenders.

The results of our experiment support the theoretical prediction that, under the auction CSF, attackers use a stochastic "guerrilla-warfare" strategy of attacking at most one target, and

[3] However, as noted in the case of a single two-player contest with linear costs − by Baye et al. (1994) and Alcalde and Dahm (2010) − there exist equilibria that are payoff equivalent to the $r = \infty$ case whenever $r > 2$. Ewerhart (2017a) has demonstrated that, in fact, in this environment any Nash equilibrium is payoff and revenue equivalent to the all-pay auction. Thus, the auction CSF case with $r = \infty$ is a relevant theoretical benchmark for all $r > 2$.
[4] For the attacker, this prediction holds for all parameter configurations. For the defender, this prediction holds if the ratio of the attacker's valuation of success to the defender's valuation of success is below a certain threshold.
[5] See also the related papers Dziubiński and Goyal (2013, 2017).
[6] For almost all configurations of the players' valuations of winning, one of the two players drops out with positive probability by allocating zero resources to each target, with the identity of the dropout determined by a measure of asymmetry in the conflict that takes into account both the ratio of the players' valuations and the number of targets.

defenders use a stochastic "complete-coverage" strategy in which all targets are defended. In contrast, under the lottery CSF, instead of the pure-strategy Nash-equilibrium "complete-coverage" strategy, the expenditures of both the attackers and defenders are distributed over the entire strategy space. In fact, under the lottery CSF, attackers utilize a "guerrilla-warfare" strategy of attacking at most one target almost 45% of the time, instead of using a "complete-coverage" strategy, which is observed less than 30% of the time.

Consistent with predictions, under both CSFs, as the attacker's valuation increases, the attacker's resource expenditure increases and the defender's expenditure decreases. As a result, the attacker's probability of winning and the average payoff also increase. However, under both CSFs, both players' average resource expenditures exceed their respective theoretical predictions, as is common in other contest experiments (Dechenaux et al., 2015).

The rest of the paper is organized as follows. In Section 2, we provide a brief review of the multi-battle contest literature. Section 3 presents a theoretical model of the attack and defense game. Section 4 describes the experimental design, procedures and hypotheses. Section 5 reports the results of our experiment and Section 6 concludes.

2. Literature Review

Most of the existing theoretical work on multi-battle contests features symmetric objectives.[7] However, in applications such as cyber-security and terrorism, objectives are asymmetric with success for the attacker requiring the destruction of at least one target and

[7] For a survey see Kovenock and Roberson (2012). Recent theoretical work on multi-battle/Blotto-type games includes extensions such as: asymmetric players (Roberson, 2006; Hart, 2008; Weinstein, 2012; Dziubiński, 2013; Macdonell and Mastronardi, 2015), non-constant-sum variations (Szentes and Rosenthal, 2003; Kvasov, 2007; Hortala-Vallve and Llorente-Saguer, 2010, 2012; Roberson and Kvasov, 2012; Ewerhart, 2017b), alternative definitions of success (Golman and Page, 2009; Tang et al., 2010; Rinott et al., 2012), and political economy applications (Laslier, 2002; Laslier and Picard, 2002; Roberson, 2008; Bierbrauer and Boyer, 2016; Boyer et al. 2017; Thomas, 2017). There is also an extensive theoretical literature on dynamic multi-battle contests. See, for instance, Harris and Vickers (1987), Klumpp and Polborn (2006), Konrad and Kovenock (2009), and Gelder (2014).

successful defense requiring the preservation of all targets (Sandler and Enders, 2004). This structural asymmetry is the focus of the two weakest-link attacker-defender games (Clark and Konrad, 2007; Kovenock and Roberson, 2017) that we test experimentally.

To the best of our knowledge, our study is the first to examine behavior in weakest-link attacker-defender games utilizing both the lottery CSF and the auction CSF. Although most of the existing experimental studies focus on single-battle contests, there is a growing interest in multi-battle contests.[8] Experimental studies on multi-battle contests have examined how different factors such as budget constraints (Avrahami and Kareev, 2009; Arad and Rubinstein, 2012), objective functions (Duffy and Matros, 2017), information (Horta-Vallve and Llorente-Saguer, 2010), contest success functions (Chowdhury et al., 2013), focality (Chowdhury et al., 2016), and asymmetries in resources and battlefields (Arad, 2012; Holt et al., 2015; Montero et al., 2016, Duffy and Matros, 2017) impact individual behavior in contests. For a recent survey of the experimental literature on contests see Dechenaux et al. (2015).

Consistent with the previous studies in which allocations are not budget constrained, we find significant over-expenditure relative to the Nash equilibrium predictions under both the lottery CSF and the auction CSF. However, our most surprising result is that the theoretical prediction that attackers use a "guerrilla-warfare" strategy under the auction CSF is also observed under the lottery CSF. This is surprising because almost all multi-battle contest experiments in the literature find strong qualitative support for the theoretical predictions, even if the precise quantitative predictions are refuted. A potential explanation as to why attackers use a "guerrilla-warfare" strategy under the lottery CSF is that subjects may find it natural to concentrate resources on just one target since one successful attack is enough to win. Such a

[8] There is also a growing experimental literature on dynamic multi-battle contests. See, for instance, Deck and Sheremeta (2012, 2015), Mago et al. (2013), Mago and Sheremeta (2016, 2017), and Gelder and Kovenock (2017).

heuristic strategy also explains why individual behavior is so close to the theoretical predictions under the auction CSF.

3. The Game of Attack and Defense

The model is formally described as follows. Two risk neutral players, an attacker A and a defender D, simultaneously allocate resources across n targets. The players' one-dimensional resource expenditures for target i, denoted x_A^i and x_D^i for A and D respectively, are non-negative and are mapped into their respective probabilities of winning target i by the general ratio-form, or Tullock, CSF (Tullock, 1980). Thus, player D wins target i with probability

$$p_D^i\left(x_A^i, x_D^i\right) = \begin{cases} \frac{\left(x_D^i\right)^r}{\left(x_A^i\right)^r + \left(x_D^i\right)^r} & \text{if } x_A^i > 0 \\ 1 & \text{otherwise} \end{cases}, \tag{1}$$

and player A wins target i with probability $1 - p_D^i\left(x_A^i, x_D^i\right)$. For the lottery CSF $r = 1$, and for the auction CSF $r = \infty$.[9]

The attacker and the defender have asymmetric objectives. The defender's objective is to successfully defend all n targets in the network, in which case he receives a "prize" of value v_D. The expected payoff of D conditional on the expenditure $\left(x_A^i, x_D^i\right)$ is:

$$E(\pi_D) = \left(\prod_{i=1}^n p_D^i\left(x_A^i, x_D^i\right)\right)v_D - \sum_{i=1}^n x_D^i. \tag{2}$$

The attacker's objective is to successfully attack at least one target, in which case he receives a prize of value v_A. The corresponding expected payoff of A is:

$$E(\pi_A) = \left(1 - \prod_{i=1}^n p_D^i\left(x_A^i, x_D^i\right)\right)v_A - \sum_{i=1}^n x_A^i. \tag{3}$$

[9] For the auction CSF game, if the players allocate the same level of the resource to a target, it is assumed that the defender wins the target. However, a range of tie-breaking rules yields similar results. A detailed description of the theoretical model can be found in Clark and Konrad (2007) for the lottery CSF and Kovenock and Roberson (2017) for the auction CSF.

Clark and Konrad (2007) derive a Nash equilibrium for the lottery CSF ($r = 1$). We complete the characterization of equilibrium by showing that equilibrium is unique (see Online Appendix A).

***Proposition* 1:**

(i) If $v_D \geq (n-1)v_A$, then there exists a unique Nash equilibrium, which is in pure strategies. In equilibrium, player A allocates $x_A^* = \frac{v_A^2 v_D^n}{(v_A+v_D)^{n+1}}$ to every target and player D allocates $x_D^* = \frac{v_A v_D^{n+1}}{(v_A+v_D)^{n+1}}$ to every target.

(ii) If $v_D < (n-1)v_A$, then there exists a unique Nash equilibrium, which is in mixed strategies. In equilibrium, player A allocates $x_A^* = \frac{(n-1)^{n-1}}{n^{n+1}} v_D$ to each target and player D randomizes by allocating $x_D^* = \frac{(n-1)^n}{n^{n+1}} v_D$ to every target with probability $q^* = \frac{v_D}{(n-1)v_A}$ and 0 to every target with the probability $1 - q^*$.

Proposition 1 can be summarized as follows. If the ratio of the defender's valuation to the attacker's valuation exceeds a threshold, $v_D \geq (n-1)v_A$, then, in equilibrium, the defender uses a pure strategy that defends all targets with the same level of resources, $x_D^* > 0$. However, if $v_D < (n-1)v_A$, then the defender's equilibrium expected payoff is zero and, in equilibrium, the defender engages in the conflict — by allocating $x_D^* > 0$ to each target — with probability $q^* = \frac{v_D}{(n-1)v_A}$. With probability $1 - q^*$, the defender "surrenders" by allocating 0 to all n targets. In contrast, for all parameter configurations the attacker plays a pure strategy. Although the attacker's objective is to win at least one target, due to the decreasing returns to expenditure exhibited by the lottery CSF, the equilibrium strategy is to attack all targets with x_A^*.

Kovenock and Roberson (2017) characterize properties of the set of Nash equilibria for the auction CSF ($r = \infty$). They show that all equilibria are in mixed strategies, where a mixed strategy is an n-variate joint-distribution function. That paper completely characterizes the set of equilibrium payoffs and univariate marginal distributions, which are unique for all parameter configurations. These results are summarized as follows:

***Proposition* 2:**

(i) If $v_D \geq nv_A$, then with probability $1 - \frac{nv_A}{v_D}$ player A allocates 0 to every target. With the remaining probability, $\frac{nv_A}{v_D}$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $[0, v_A]$. To each and every target, player D allocates a random level of the resource drawn from a uniform distribution over the interval $[0, v_A]$. The players' sets of equilibrium univariate marginal distribution functions are unique, and for each target j are given by: $F_A^j\left(x_A^j\right) = 1 - \frac{v_A}{v_D} + \frac{x_A^j}{v_D}$ and $F_D^j\left(x_D^j\right) = \frac{x_D^j}{v_A}$, respectively, over the interval $[0, v_A]$.

(ii) If $v_D < nv_A$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $\left[0, \frac{v_D}{n}\right]$. With probability $1 - \frac{v_D}{nv_A}$ player D allocates 0 to every target. With the remaining probability, $\frac{v_D}{nv_A}$, player D allocates to each target a random level of resources drawn from a uniform distribution over the interval $\left[0, \frac{v_D}{n}\right]$. The players' sets of equilibrium univariate marginal distribution functions for every target are unique, and for each target j are given by: $F_A^j\left(x_A^j\right) = 1 - \frac{1}{n} + \frac{x_A^j}{v_D}$ and $F_D^j\left(x_D^j\right) = 1 - \frac{v_D}{nv_A} + \frac{x_D^j}{v_A}$, respectively, over the interval $\left[0, \frac{v_D}{n}\right]$.

It is important to note that, although there are multiple equilibria in this game, there exists a unique set of equilibrium univariate marginal distribution functions. Kovenock and Roberson

(2017) also show that the equilibrium joint distribution functions exhibit several distinctive properties. For example, in all equilibria of the auction CSF game, the attacker allocates a strictly positive amount to at most one target while the defender allocates a strictly positive amount to either all targets or to none of them. This particular property provides a striking contrast with equilibrium in the lottery CSF game (see *Proposition 1*) in which the attacker allocates, to every target, a strictly positive amount.

4. Experimental Design, Procedures, and Hypotheses

4.1. Experimental Design

Table 1 summarizes the experimental design. We employ a two-by-two design, by varying the CSF (*Lottery* versus *Auction*) and the relative valuation of the attacker's prize (*Low* versus *High*). In all four treatments, there are four targets and two players (attacker and defender). The experimental instructions, shown in Online Appendix B, used a context neutral language.[10]

In the *Lottery-Low* and *Lottery-High* treatments the probability that a player wins a given target is equal to the ratio of that player's allocation of resources to the target to the sum of both players' allocations to that target. In all treatments, the defender's valuation of defending all targets is $v_D = 200$ experimental francs. The attacker's valuation of successfully attacking at least one target is $v_A = 40$ francs in the *Lottery-Low* treatment and $v_A = 80$ francs in the *Lottery-High* treatment.[11] For the parameter configuration in the *Lottery-Low* treatment, Proposition 1 part (i) applies and in the pure-strategy equilibrium the attacker allocates 3.2 tokens to each

[10] For example, players were called participants and targets were called boxes.
[11] We chose these parameter valuations to ensure that: (a) the four treatments cover both parts of both propositions, and (b) each subject faced a non-trivial allocation problem in which both the attacker and the defender had a substantial chance of winning some targets.

target and the defender allocates 16.1 tokens to each target. For the parameter configuration in the *Lottery-High* treatment, Proposition 1 part (ii) applies and in equilibrium the attacker allocates 5.3 tokens to each target and the defender allocates 15.8 tokens to every target with probability 0.83 and 0 tokens to every target with probability 0.17.

In the *Auction-Low* and *Auction-High* treatments the winner of each target is determined by the auction CSF, but the remaining features of the model (v_D, v_A, and n) are the same. From Proposition 2 part (i), in any equilibrium of the *Auction-Low* treatment: (a) the attacker utilizes a mixed-strategy that attacks no targets with probability 0.2 and, with probability 0.8, chooses exactly one target to attack at random and stochastically allocates between 0 and 40 tokens to that target, according to a uniform distribution, and (b) the defender randomizes according to a joint distribution function that stochastically allocates between 0 and 40 tokens to each target according to a uniform marginal distribution. In the *Auction-High* treatment, Proposition 2 part (ii) applies. In any equilibrium: (a) the attacker randomly chooses one of the targets to attack and stochastically allocates between 0 and 50 tokens to that target according to a uniform distribution and (b) the defender employs a mixed strategy in which, with probability 0.375 he engages in no defensive efforts and, with probability 0.625, the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50, to each target.

4.2. Procedures

The experiment was conducted at the Vernon Smith Experimental Economics Laboratory. The computerized experimental sessions were run using z-Tree (Fischbacher, 2007). A total of 96 subjects participated in eight sessions, summarized in Table 2. All subjects were Purdue University undergraduate students who participated in only one session of this study. Some students had participated in other economics experiments that were unrelated to this research.

Each experimental session had 12 subjects and proceeded in two parts, corresponding to the lottery and auction treatments.[12] Each subject played for 20 periods in the *Lottery-Low* (*Auction-Low*) treatment and 20 periods in the *Lottery-High* (*Auction-High*) treatment. The sequence was varied so that half the sessions had the *Lottery-High* (*Auction-High*) treatment first, and half had the *Lottery-Low* (*Auction-Low*) treatment first.

In the first period of each treatment subjects were randomly and anonymously assigned for the first 10 periods and then changed their assignment for the last 10 periods.[13] Subjects of opposite assignments were randomly re-paired each period to form a new two-player group. Each period, each subject allocated a non-negative number of tokens to each of the 4 targets such that the sum of allocated tokens was weakly less than that subject's valuation. Subjects were informed that all allocated tokens were forfeited. After all subjects made their allocations, the computer displayed the following information: attacker allocation, defender allocation, which targets they won, and individual earnings for the period. In the *Lottery-High* and *Lottery-Low* treatments, the winner was chosen according to the lottery CSF, independently across targets. In

[12] Risk aversion preferences were also elicited, along the lines of Holt and Laury (2002). We found no interesting patterns between risk attitudes and behavior in weakest-link contests and omit discussion of this issue.

[13] Role switching avoids any social preferences, i.e., subjects who were assigned as disadvantaged attackers knew that they would also play the role of the advantaged defenders, and induces better learning, since subjects have an opportunity to learn strategies in the game in both roles.

the *Auction-High* and *Auction-Low* treatments, the player who allocated more tokens to a particular target was chosen as the winner of that target.[14]

After completing all 40 decision periods (two treatments), 4 periods were randomly selected for payment (2 periods for each treatment). The sum of the total earnings for these 4 periods was exchanged at the rate of 26 tokens = $1. Additionally, all players received a participation fee of $20 to cover potential losses. On average, subjects earned $25 each, ranging from $11 to $36, and this was paid in cash. Each experimental session lasted about 80 minutes.

4.3. Hypotheses

Our experiment tests five hypotheses motivated by the theoretical predictions. The first hypothesis addresses the comparative static properties of equilibrium in terms of a change in the attacker's valuation.[15] The next two describe equilibrium predictions concerning behavior in the *Lottery-Low* and *Lottery-High* treatments. The final two hypotheses describe equilibrium predictions concerning behavior in the *Auction-Low* and *Auction-High* treatments.

Hypothesis 1: Under the lottery and auction CSF, as the attacker's valuation increases from 40 to 80, the average resource allocation, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

Hypothesis 2: In the *Lottery-Low* and *Lottery-High* treatments the attacker uses a "complete-coverage" strategy, which involves allocating a strictly positive and identical level of the resource across all targets.

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[14] When both players allocated the same amount to a given target, the computer selected the defender as the winner of that target.

[15] Although the comparative statics results are framed in terms of a change in the attacker's valuation, due to invariance of preferences with respect to affine transformations of utility, the theoretical benchmark would also apply to a decrease in the unit cost of resource expenditure of the attacker.

Hypothesis 3: In the *Lottery-Low* treatment the defender uses a "complete-coverage" strategy. In the *Lottery-High* treatment the defender allocates a strictly positive and identical level of the resource across all targets with positive probability, and a zero level of the resource with the remaining probability.

Hypothesis 4: In the *Auction-Low* and *Auction-High* treatments the attacker uses a stochastic "guerrilla-warfare" strategy, which involves allocating a random level of the resource to at most one target.

Hypothesis 5: In the *Auction-Low* treatment the defender uses a stochastic "complete-coverage" strategy, which involves allocating positive random levels of the resource to all of the targets. In the *Auction-High* treatment the defender follows a stochastic "complete-coverage" strategy with positive probability and also allocates a zero level of the resource to every target with positive probability.

5. Results

5.1. Aggregate Behavior

Table 3 summarizes the average allocation of tokens, the probability of winning, and the average payoff by the attacker and the defender in each treatment. Consistent with *Hypothesis 1*, when the attacker's valuation increases from 40 to 80, the average allocation of tokens by the attacker increases from 4.4 to 7.8 under the lottery CSF, and it increases from 4.4 to 7.7 under the auction CSF. The average allocation of tokens by the defender decreases from 24.4 to 15.8 under the auction CSF, but does not decrease under the lottery CSF (19.4 versus 19.3). To support these conclusions we estimate panel regressions, reported in the top panel of Table 4, where the dependent variable is the allocation to any given target and the independent variables are a treatment dummy-variable (*High*), a period trend (*Period*), and a constant (*Constant*). The model includes a random effects error structure, with the individual subject as the random effect, to account for the multiple allocation decisions made by individual subjects over the course of the experiment. The standard errors are clustered at the session level to account for session effects. The treatment dummy-variable is significant in all regressions (p-values < 0.01), except the one where we compare the behavior of the defender in the *Lottery-High* and *Lottery-Low* treatments.

Also, consistent with *Hypothesis 1*, the attacker's probability of winning in the *Lottery-High* treatment (0.68) is higher than his probability of winning in the *Lottery-Low* treatment (0.51), and the probability of winning in *Auction-High* (0.68) is higher than the probability of winning in *Auction-Low* (0.33). The middle panel of Table 4 reports the regression results from a random effects probit model. From that estimation, we see that, for both the auction and lottery CSFs, the attacker's (defender's) probability of winning is higher (lower) in the high attacker valuation treatment (p-values < 0.01).

Finally, consistent with *Hypothesis 1*, from the estimation reported in the bottom panel of Table 4 we see that the defender's (attacker's) payoff in the *Lottery-Low* and *Auction-Low* treatments is higher (lower) than in the *Lottery-High* and *Auction-High* treatments, where the treatment dummy-variable is significant in all regressions (p-values < 0.01 for all except the defender in the *Lottery-High* to *Lottery-Low* comparison which has p-value < 0.05).

Result 1: Consistent with the prediction of *Hypothesis 1*, under the lottery and auction CSF, as the attacker's valuation increases, the average allocation of tokens, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

Although the comparative static predictions of the theory are supported by our experiment, there is significant over-expenditure of resources by both player types in all treatments. In the *Lottery-Low* treatment, the attacker allocates on average 4.4 tokens, instead of the predicted 3.2, and in the *Lottery-High* treatment, the attacker allocates 7.8 tokens, instead of 5.3. The relative magnitude of over-expenditure by the defender is similar: 19.4 tokens instead of 16.1 and 19.3 tokens instead of 13.1. The range of average over-expenditure is 21%-47%. Over-expenditure is also observed in the *Auction-High* and *Auction-Low* treatments; however, the magnitude is around 10%-22%.[16] As a result of significant over-expenditure, in all treatments both player types receive lower payoffs than predicted (see Table 3).

Significant over-expenditure in our experiment is consistent with previous experimental findings on all-pay auctions and lottery contests (Davis and Reilly, 1998; Potters et al., 1998; Gneezy and Smorodinsky, 2006; Sheremeta and Zhang, 2010; Price and Sheremeta, 2011; 2015). Suggested explanations for over-expenditure include bounded rationality (Sheremeta, 2011; Chowdhury et al., 2014), utility of winning (Sheremeta, 2010; Cason et al., 2012, 2017), other-regarding preferences (Fonseca, 2009; Mago et al., 2016), judgmental biases (Shupp et al., 2013), and impulsive behavior (Sheremeta, 2016).[17] The same arguments can be made to explain over-expenditure in our experiment.

5.2. Behavior of Attackers under the Lottery CSF

Next, we examine attacker behavior under the lottery CSF, where, in equilibrium, the attacker employs a uniform allocation of tokens across targets. Contrary to *Hypothesis 2*, within

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[16] A standard Wald test, conducted on estimates of panel regression models, rejects the hypothesis that the average expenditures under the lottery CSF are equal to the predicted theoretical values in Table 3 (all p-values < 0.05). Under the auction CSF we can reject the null hypothesis only for the defender (p-value < 0.05).

[17] For a detailed review of possible explanations for the over-expenditure phenomenon see Sheremeta (2013, 2016).

each target the allocation of tokens is highly dispersed. Figure 1 displays, by treatment and player type, the empirical univariate marginal cumulative distribution functions of tokens to an arbitrary target.[18] Instead of placing a mass point at 3.2 in the *Lottery-Low* treatment and 5.3 in the *Lottery-High* treatment, the attacker's resources are distributed between 0 and 50.

Another inconsistency with *Hypothesis 2* is that, instead of a strictly positive token allocation for each target, the attacker places mass at 0 (see Figure 1). Table 5 shows properties of the strategies used by subjects in the *Lottery-Low* and *Lottery-High* treatments. The attacker frequently uses a "guerrilla-warfare" strategy that attacks at most one target (44% in the *Lottery-Low* treatment and 46% in the *Lottery-High* treatment). A strategy of "complete coverage," allocating a positive amount to all four targets, is used only 24% of time in the *Lottery-Low* treatment and 32% in the *Lottery-High* treatment.

Result 2: Contrary to the prediction of *Hypothesis 2*, in the *Lottery-Low* and *Lottery-High* treatments, the attacker's allocation of tokens to each target is highly dispersed. Instead of using the "complete-coverage" strategy, the attacker uses the "guerrilla-warfare" strategy that allocates a positive bid to one or fewer targets in more than 50% of the periods.

The fact that the attacker's resource allocation to each target is highly dispersed is consistent with previous experimental studies documenting high variance of individual expenditures in lottery contests (Davis and Reilly, 1998; Potters et al., 1998; Chowdhury et al., 2014). Several explanations have been offered for this behavior based on the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al., 2013). These could also explain the pattern observed in our experiment.

[18] We combined the distribution of tokens to each of the 4 targets into one target, since marginal distributions to each target are identical across targets.

A more novel finding of our study is the use of a “guerrilla-warfare” strategy by the attacker. This is inconsistent with the unique Nash equilibrium in the attack and defense game under the lottery CSF. Also, it appears unlikely that attackers adjust their strategy away from equilibrium due to suboptimal behavior on the part of defenders because, as we discuss later, defenders behave in accordance with the theoretical predictions.

A likely explanation why attackers use a “guerrilla-warfare” strategy is that subjects may find it natural to concentrate resources on the necessary number of targets needed for victory (one in our case). Although such a strategy is not optimal, it is an appealing focal point (Schelling, 1960). It has been well documented in the experimental literature that subjects naturally gravitate towards focal points even when it is not necessarily in their best interest (Roth, 1985; Crawford et al., 2008, Chowdhury et al. 2016). Such a heuristic strategy can also explain why individual behavior is so close to the theoretical predictions under the auction CSF (as we discuss below).

5.3. Behavior of Defenders under the Lottery CSF

Our results concerning the behavior of the defender support *Hypothesis 3*. In particular, theory predicts that in the *Lottery-Low* treatment the defender makes a strictly positive, and uniform, allocation of tokens across targets. Table 5, indicates that, supporting *Hypothesis 3*, the defender allocates tokens to all targets in the *Lottery-Low* treatment 92% of the time. In the *Lottery-High* treatment, theory predicts that the defender covers all of the targets with probability 0.83 and none of the targets with probability 0.17. Consistent with this prediction, the data indicate that the defender covers all of the targets in the *Lottery-High* treatment 84% of the time and none of the targets 12% of the time. However, contrary to *Hypothesis 3*, instead of a uniform allocation across targets, the defender's resources are distributed between 0 and 50 (see Figure 1).

Result 3: Consistent with the prediction of *Hypothesis 3*, in the *Lottery-Low* treatment, the defender uses a "complete-coverage" strategy by defending all targets and in the *Lottery-High* treatment there is a very high incidence of "complete coverage," but with the "no coverage" strategy (allocating zero to every target) the next most frequent strategy. Contrary to the prediction, instead of a uniform allocation across targets, allocations are dispersed over the interval [0, 50].

As in the case of attackers, the relatively high dispersion of the allocations of defenders is consistent with previous experimental findings, and could be explained by the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al., 2013).

5.4. Behavior of Attackers under the Auction CSF

Next, we look at attacker behavior under the auction CSF. Theory predicts that in the *Auction-Low* and *Auction-High* treatments, the attacker employs a stochastic "guerrilla-warfare" strategy, which involves allocating a random level of the resource to at most one target. Figure 2 displays the empirical univariate marginal cumulative distribution function of the resource allocation to a target and indicates that, in the aggregate, the attacker's behavior is consistent with this prediction. The stochastic "guerrilla-warfare" strategy is characterized by a significant mass point at 0 for the attacker, which is very close to the predicted value (0.75 versus 0.80 in the *Auction-Low* treatment and 0.67 versus 0.75 in the *Auction-High* treatment).[19] Similarly,

[19] In calculating the empirical mass points at 0 (Figures 1 and 2), we use an allocation of less than 1 token as an approximation of 0. This approximation is used because the tie-breaking rule favors defenders, and therefore it may encourage attackers to place a very small allocation in some targets in order to reduce the tie-breaking disadvantage. However, even if we use only 0 allocations to compute mass points at 0, we still get results that are close to the

from Table 6, we see that the attacker allocates tokens to at most one target 89% of the time in the *Auction-Low* treatment and 81% of the time in the *Auction-High* treatment. These findings provide substantial support for *Hypothesis 4*.

Result 4: Consistent with the prediction of *Hypothesis 4*, in the *Auction-Low* and *Auction-High* treatments, the attacker uses a stochastic "guerrilla-warfare" strategy, which involves allocating a random level of the resource to at most one target.

5.5. Behavior of Defenders under the Auction CSF

We find that defender behavior is also consistent with *Hypothesis 5*. In particular, theory predicts that in the *Auction-Low* treatment the defender uses a stochastic "complete-coverage" strategy that allocates a strictly positive level of resources to each target with probability one. The data indicate that the defender covers all of the targets 87% of the time (see Table 6). Moreover, consistent with the theoretical prediction, in the *Auction-Low* treatment the defender's resources are uniformly distributed between 0 and 40 (see Figure 2). Similarly, in the *Auction-High* treatment defender behavior is consistent with the theoretical prediction that with probability 0.375 the defender engages in no defensive efforts and with probability 0.625 the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50, to each target. The data indicate that the defender covers all four targets 62% of the time, three targets 2%, two targets 2%, one target 4%, and zero targets 30% of the time (see Table 6).[20] Moreover, the defender's allocations are uniformly distributed between 0 and 50 (see Figure 2).

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theoretical predictions (in the *Auction-Low* and *Auction-High* treatments, for example, the mass points at 0 for the attackers are 0.6 and 0.5).

[20] The fact that the defender allocates 0 resources to all four targets 30% of the time could, potentially, be due to subjects changing role assignments after 10 periods and a period 1-10 attacker continuing to behave as an attacker during periods 11-20. However, our results are robust to restricting data to the first 10 periods.

Result 5: Consistent with the prediction of *Hypothesis 5*, in the *Auction-Low* treatment, the defender uses a stochastic "complete-coverage" strategy that involves allocating random positive levels of the resource to all of the targets. In the *Auction-High* treatment there is a high incidence of "complete-coverage," but the "no-coverage" strategy is employed almost a third of the time.

6. Conclusions

This study experimentally investigates behavior in a game of attack and defense of a weakest-link network under two benchmark contest success functions: the auction CSF and the lottery CSF. We find that the auction CSF's theoretical prediction that the attacker uses a "guerrilla-warfare" strategy and the defender uses a "complete-coverage" strategy is observed under both the auction and lottery CSFs. This is inconsistent with Nash equilibrium behavior under the lottery CSF. However, such behavior is consistent with a simple heuristic strategy of focusing only on the necessary number of targets needed for victory (one in our case).

A common explanation for the empirical finding that "periods of high terrorism" seem to be relatively infrequent (Enders, 2007) is that terrorists face a resource constraint, and therefore they cannot constantly attack all of the targets. Our experiment provides evidence for an alternative explanation. Infrequent "periods of high terrorism" may simply be the result of asymmetric objectives and strategic interactions between the attackers and defenders within a weakest-link contest environment.

References

Alcalde, J., & Dahm, M. (2010). Rent Seeking and Rent Dissipation: a Neutrality Result. Journal of Public Economics, 94, 1-7.

Arad, A. (2012). The Tennis Coach Problem: A Game-Theoretic and Experimental Study. B.E. Journal of Theoretical Economics, 12.

Arad, A., & Rubinstein, A. (2012). Multi-Dimensional Iterative Reasoning in Action: The Case of the Colonel Blotto Game. Journal of Economic Behavior and Organization, 84, 571-585.

Avrahami, J., & Kareev, Y. (2009). Do the Weak Stand a Chance? Distribution of Resources in a Competitive Environment. Cognitive Science, 33, 940-950.

Baye, M.R., Kovenock, D., & de Vries, C.G. (1994). The Solution to the Tullock Rent-Seeking Game when R > 2: Mixed-Strategy Equilibria and Mean Dissipation Rates. Public Choice, 81, 363-380.

Bierbrauer, F.J., & Boyer, P.C. (2016). Efficiency, Welfare, and Political Competition. Quarterly Journal of Economics, 131, 461-518.

Boyer, P.C., Konrad, K.A., & Roberson, B. (2017). Targeted Campaign Competition, Loyal Voters, and Supermajorities. Journal of Mathematical Economics, 71, 49-62.

Cason, T.N., Sheremeta, R.M., & Zhang, J. (2012). Communication and Efficiency in Competitive Coordination Games. Games and Economic Behavior, 76, 26-43.

Cason, T.N., Sheremeta, R.M., & Zhang, J. (2017). Asymmetric and Endogenous Communication in Competition between Groups. Experimental Economics, forthcoming.

Chowdhury, S.M., Kovenock, D., & Sheremeta, R.M. (2013). An Experimental Investigation of Colonel Blotto Games. Economic Theory, 52, 833-861.

Chowdhury, S.M., Sheremeta, R.M., & Turocy, T.L. (2014). Overbidding and Overspreading in Rent-Seeking Experiments: Cost Structure and Prize Allocation Rules. Games and Economic Behavior, 87, 224-238.

Chowdhury, S.M., Kovenock, D., Rojo Arjona, D., & Wilcox, N.T. (2016). Focality and Asymmetry in Multi-Battle Contests. Chapman University, Economic Science Institute Working Paper No. 16-16

Clark, D.J., & Konrad, K.A. (2007). Asymmetric Conflict: Weakest Link against Best Shot. Journal of Conflict Resolution, 51, 457-469.

Crawford, V.P., Gneezy, U., & Rottenstreich, Y. (2008). The Power of Focal Points is Limited: Even Minute Payoff Asymmetry May Yield Large Coordination Failures. American Economic Review, 98, 1443-1458.

Davis, D., & Reilly, R. (1998). Do Many Cooks Always Spoil the Stew? An Experimental Analysis of Rent Seeking and the Role of a Strategic Buyer. Public Choice, 95, 89-115.

Dechenaux, E., Kovenock, D., & Sheremeta, R.M. (2015). A Survey of Experimental Research on Contests, All-Pay Auctions and Tournaments. Experimental Economics, 18, 609-669.

Deck, C., & Sheremeta, R.M. (2012). Fight or Flight? Defending Against Sequential Attacks in the Game of Siege. Journal of Conflict Resolution, 56, 1069-1088.

Deck, C., & Sheremeta, R.M. (2015). Tug-of-War in the Laboratory. Chapman University, Economic Science Institute Working Paper No. 15-14.

Duffy, J., & Matros, A. (2017). Stochastic Asymmetric Blotto Games: An Experimental Study. Journal of Economic Behavior & Organization, 139, 88-105.

Dziubiński, M. (2013). Non-Symmetric Discrete General Lotto Games. International Journal of Game Theory, 42, 801-833.

Dziubiński, M., & Goyal, S. (2013) Network Design and Defence. Games and Economic Behavior, 79, 30-43.
Dziubiński, M., & Goyal, S. (2017). How do You Defend a Network? Theoretical Economics, 12, 331-376.
Enders, W. (2007). Terrorism: An Empirical Analysis. In K. Hartley and T. Sandler, (Eds.), Handbook of Defense Economics. Elsevier: Amsterdam, pp. 815-866.
Ewerhart, C. (2017a). Contests with Small Noise and the Robustness of the All-Pay Auction. Games and Economic Behavior. 105, 195-211.
Ewerhart, C. (2017b). A "Fractal" Solution to the Chopstick Auction. Economic Theory, forthcoming, DOI: 10.1007/s00199-017-1052-1.
Fischbacher, U. (2007). z-Tree: Zurich Toolbox for Ready-Made Economic Experiments. Experimental Economics, 10, 171-178.
Fonseca, M.A. (2009). An Experimental Investigation of Asymmetric Contests. International Journal of Industrial Organization, 27, 582-591.
Gelder, A. (2014). From Custer to Thermopylae: Last Stand Behavior in Multi-Stage Contests. Games and Economic Behavior, 87, 442-466.
Gelder, A., & Kovenock, D. (2017). Dynamic Behavior and Player Types in Majoritarian Multi-Battle Contests. Games and Economic Behavior, 104, 444-455.
Gneezy, U., & Smorodinsky, R. (2006). All-Pay Auctions – An Experimental Study. Journal of Economic Behavior and Organization, 61, 255-275.
Golman, R., & Page, S.E. (2009). General Blotto: Games of Strategic Allocative Mismatch. Public Choice, 138, 279-299.
Harris, C., & Vickers, J. (1987). Racing with Uncertainty. Review of Economic Studies, 54, 1-21.
Hart, S. (2008). Discrete Colonel Blotto and General Lotto Games. International Journal of Game Theory, 36, 441-460.
Hirshleifer, J. (1983). From Weakest-Link to Best-Shot: the Voluntary Provision of Public Goods. Public Choice, 41, 371-386.
Holt, C.A. & Laury, S.K. (2002). Risk Aversion and Incentive Effects. American Economic Review, 92, 1644-1655.
Holt, C.A., Kydd, A., Razzolini, L., & Sheremeta, R.M. (2015). The Paradox of Misaligned Profiling Theory and Experimental Evidence. Journal of Conflict Resolution, 60, 482-500.
Hortala-Vallve, R., & Llorente-Saguer, A. (2010). A Simple Mechanism for Resolving Conflict. Games and Economic Behavior, 70, 375-391.
Hortala-Vallve, R., & Llorente-Saguer, A. (2012). Pure-Strategy Nash Equilibria in Non-Zero-Sum Colonel Blotto Games. International Journal of Game Theory, 41, 331-343.
Klumpp, T., & Polborn, M.K. (2006). Primaries and the New Hampshire Effect. Journal of Public Economics, 90, 1073-1114.
Konrad, K.A., & Kovenock, D. (2009). Multi-Battle Contests. Games Economic Behavior, 66, 256-274.
Kovenock, D., & Roberson, B. (2012). Conflicts with Multiple Battlefields. In Michelle R. Garfinkel and Stergios Skaperdas, (Eds.), Oxford Handbook of the Economics of Peace and Conflict. Oxford University Press, New York.
Kovenock, D., & Roberson, B. (2017). The Optimal Defense of Networks of Targets. Chapman University, Economic Science Institute Working Paper No. 17-18.

Kremer, M. (1993). The O-Ring Theory of Economic Development. Quarterly Journal of Economics, 108, 551- 575.
Kvasov, D. (2007). Contests with Limited Resources. Journal of Economic Theory, 136, 738-748.
Laslier, J.F. (2002). How Two-Party Competition Treats Minorities. Review of Economic Design, 7, 297-307.
Laslier, J.F., & Picard, N. (2002). Distributive Politics and Electoral Competition. Journal of Economic Theory, 103, 106-130.
Macdonell, S., & Mastronardi, N. (2015). Waging Simple Wars: a Complete Characterization of Two-Battlefield Blotto Equilibria. Economic Theory, 58, 183-216.
Mago, S.D., & Sheremeta, R.M. (2016). New Hampshire Effect: Behavior in Sequential and Simultaneous Election Contests. Working Paper.
Mago, S.D., & Sheremeta, R.M. (2017). Multi-Battle Contests: An Experimental Study. Southern Economic Journal, forthcoming.
Mago, S.D., Savikhin, A.C., & Sheremeta, R.M. (2016). Facing Your Opponents: Social Identification and Information Feedback in Contests. Journal of Conflict Resolution, 60, 459-481.
Mago, S.D., Sheremeta, R.M., & Yates, A. (2013). Best-of-Three Contest Experiments: Strategic versus Psychological Momentum. International Journal of Industrial Organization, 31, 287-296.
Milgrom, P. (2007). Package Auctions and Exchanges. Econometrica, 75, 935-965.
Montero, M., Possajennikov, A., Sefton, M., & Turocy, T.L. (2016). Majoritarian Blotto Contests with Asymmetric Battlefields: An Experiment on Apex Games. Economic Theory, 61, 55-89.
Moore, T., Clayton, R., & Anderson, R. (2009). The Economics of Online Crime. Journal of Economic Perspectives, 23, 3-20.
Potters, J.C., de Vries, C.G., & Van Linden, F. (1998). An Experimental Examination of Rational Rent Seeking. European Journal of Political Economy, 14, 783-800.
Price, C.R., & Sheremeta, R.M. (2011). Endowment Effects in Contests. Economics Letters, 111, 217-219.
Price, C.R., & Sheremeta, R.M. (2015). Endowment Origin, Demographic Effects and Individual Preferences in Contests. Journal of Economics and Management Strategy, 24, 597-619.
Rinott, Y., Scarsini, M., & Yu, Y. (2012). A Colonel Blotto Gladiator Game. Mathematics of Operations Research, 37, 574-590.
Roberson, B. (2006). The Colonel Blotto Game. Economic Theory, 29, 1-24.
Roberson, B. (2008). Pork-Barrel Politics, Targetable Policies, and Fiscal Federalism. Journal of the European Economic Association, 6, 819-844.
Roberson, B., & Kvasov, D. (2012). The Non-Constant-Sum Colonel Blotto Game. Economic Theory, 51, 397-433.
Roth, A.E. (1985). Game Theoretic Models of Bargaining, New York: Cambridge University Press, pp. 259-268.
Sandler, T., & Enders, W. (2004). An Economic Perspective on Transnational Terrorism. European Journal of Political Economy, 20, 301-316.
Schelling, T.C. (1960). The Strategy of Conflict. Cambridge, Massachusetts: Harvard University Press.

Sheremeta, R.M. (2010). Multi-Stage Elimination Contests: An Experimental Investigation. Games and Economic Behavior, 68, 731-747.
Sheremeta, R.M. (2011). Contest Design: An Experimental Investigation. Economic Inquiry, 49, 573-590.
Sheremeta, R.M. (2013). Overbidding and Heterogeneous Behavior in Contest Experiments. Journal of Economic Surveys, 27, 491-514.
Sheremeta, R.M. (2016). Impulsive Behavior in Competition: Testing Theories of Overbidding in Rent-Seeking Contests. Working Paper.
Sheremeta, R.M., & Zhang, J. (2010). Can Groups Solve the Problem of Over-Bidding in Contests? Social Choice and Welfare, 35, 175-197.
Shupp, R., Sheremeta, R.M., Schmidt, D., & Walker, J. (2013). Resource Allocation Contests: Experimental Evidence. Journal of Economic Psychology, 39, 257-267.
Szentes, B., & Rosenthal, R.W. (2003). Beyond Chopsticks: Symmetric Equilibria in Majority Auction Games. Games and Economic Behavior, 45, 278-295.
Tang, P., Shoham, Y., & Lin, F. (2010). Designing Competitions Between Teams of Individuals. Artificial Intelligence, 174, 749-766.
Thomas, C. (2017). N-dimensional Blotto Game with Heterogeneous Battlefield Values. Economic Theory, forthcoming, DOI: 10.1007/s00199-016-1030-z.
Tullock, G. (1980). Efficient Rent Seeking. In James M. Buchanan, Robert D. Tollison, Gordon Tullock, (Eds.), Toward a Theory of the Rent-Seeking Society. College Station, TX: Texas A&M University Press, pp. 97-112.
US Congress (2002). Joint Inquiry into Intelligence Community Activities Before and After the Terrorist Attacks of September 11, 2001. Washington, DC: Senate Report No. 107-351, House Report No. 107-792.
Weinstein, J. (2012). Two Notes on the Blotto Game. B.E. Journal of Theoretical Economics, 12.

Table 1: Experimental Parameters and Theoretical Predictions

Treatment	Player	Value	Average Allocation	Expected Payoff	Probability of Winning
Lottery-Low	*A*	40	3.2	7.8	0.52
	D	200	16.1	32.2	0.48
Lottery-High	*A*	80	5.3	37.8	0.74
	D	200	13.1	0.0	0.26
Auction-Low	*A*	40	4.0	0.0	0.40
	D	200	20.0	40.0	0.60
Auction-High	*A*	80	6.3	30.0	0.69
	D	200	15.6	0.0	0.31

Average allocation in the *Auction-Low* and *Auction-High* treatments are calculated based on equilibrium mixed strategies.

Table 2: Experimental Sessions

Session Number	Design	Matching Protocol	Participants per Session	Periods per Treatment
1-2	*Lottery-Low → Lottery-High*	Strangers	12	20
3-4	*Lottery-High → Lottery-Low*	Strangers	12	20
5-6	*Auction-Low → Auction-High*	Strangers	12	20
7-8	*Auction-High → Auction-Low*	Strangers	12	20

Table 3: Average Allocation, Probability of Winning, and Payoff by Treatment

Treatment	Player	Value	Average Allocation		Probability of Winning		Expected Payoff	
			Predicted	Actual	Predicted	Actual	Predicted	Actual
Lottery-Low	*Attacker*	40	3.2	4.4 (2.5)	0.52	0.51 (0.50)	7.8	2.7 (18.6)
	Defender	200	16.1	19.4 (10.7)	0.48	0.49 (0.50)	32.2	20.6 (98.4)
Lottery-High	*Attacker*	80	5.3	7.8 (4.3)	0.74	0.68 (0.47)	37.8	23.6 (37.3)
	Defender	200	13.1	19.3 (13.0)	0.26	0.32 (0.47)	0.0	-14.1 (90.1)
Auction-Low	*Attacker*	40	4.0	4.4 (3.5)	0.40	0.33 (0.47)	0.0	-4.5 (16.8)
	Defender	200	20.0	24.4 (12.8)	0.60	0.67 (0.47)	40.0	36.2 (82.1)
Auction-High	*Attacker*	80	6.3	7.7 (4.6)	0.69	0.68 (0.47)	30.0	23.2 (33.5)
	Defender	200	15.6	15.8 (15.2)	0.31	0.32 (0.47)	0.0	1.7 (85.6)

Standard deviation in parentheses.

Table 4: Panel Estimation Testing *Hypothesis 1*

Treatments	*Lottery-Low* and *Lottery-High*		*Auction-Low* and *Auction-High*	
Player	*Attacker*	*Defender*	*Attacker*	*Defender*
Dependent variable	*Average Allocation*			
High	3.36***	-0.08	3.26***	-8.57***
[1 if high value]	(1.27)	(1.97)	(0.45)	(3.29)
Period	1.16**	4.86**	1.21**	6.48***
[inverse period trend]	(0.57)	(2.00)	(0.58)	(2.07)
Constant	4.19***	18.55***	4.22***	23.22***
	(0.44)	(1.87)	(0.48)	(1.65)
Dependent variable	*Probability of Winning*			
High	0.47***	-0.48***	0.92***	-0.95***
[1 if high value]	(0.13)	(0.13)	(0.11)	(0.11)
Period	-0.20	0.20	-0.30	0.35
[inverse period trend]	(0.22)	(0.23)	(0.27)	(0.28)
Constant	0.06	-0.06	-0.40***	0.40***
	(0.07)	(0.07)	(0.05)	(0.05)
Dependent variable	*Expected Payoff*			
High	20.91***	-34.69**	27.70***	-34.48***
[1 if high value]	(2.22)	(17.22)	(2.00)	(10.60)
Period	-12.53***	-4.30	-9.07***	3.44
[inverse period trend]	(3.14)	(13.73)	(2.37)	(10.29)
Constant	4.98***	21.41*	-2.87	35.58***
	(0.84)	(11.49)	(1.85)	(6.45)
Observations	960	960	960	960

* significant at 10%, ** significant at 5%, *** significant at 1%. All models include a random effects error structure, with the individual subject as the random effect, to account for the multiple decisions made by the subject over the course of the experiment. The standard errors are clustered at the session level to account for session effects.

Table 5: Strategies Used in the *Lottery-Low* and *Lottery-High* Treatments

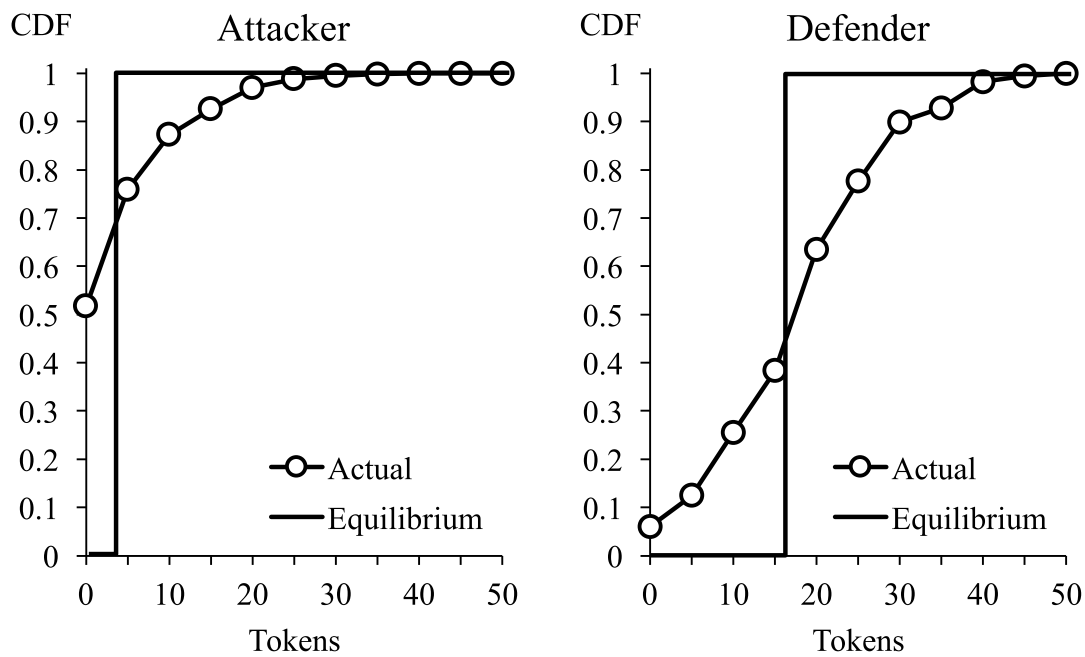
Treatment	Player	Frequency of Allocating Tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Lottery-Low	*Attacker*	0.10	0.44	0.14	0.08	0.24
	Defender	0.05	0.01	0.01	0.01	0.92
Lottery-High	*Attacker*	0.05	0.46	0.13	0.04	0.32
	Defender	0.12	0.01	0.01	0.02	0.84

Table 6: Strategies Used in the *Auction-Low* and *Auction-High* Treatments

Treatment	Player	Frequency of Allocating Tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Auction-Low	*Attacker*	0.28	0.61	0.04	0.01	0.06
	Defender	0.06	0.02	0.02	0.03	0.87
Auction-High	*Attacker*	0.11	0.70	0.05	0.02	0.12
	Defender	0.30	0.04	0.02	0.02	0.62

Figure 1: CDF of Tokens in the *Lottery-Low* and *Lottery-High* Treatments

The *Lottery-Low* treatment



The *Lottery-High* treatment

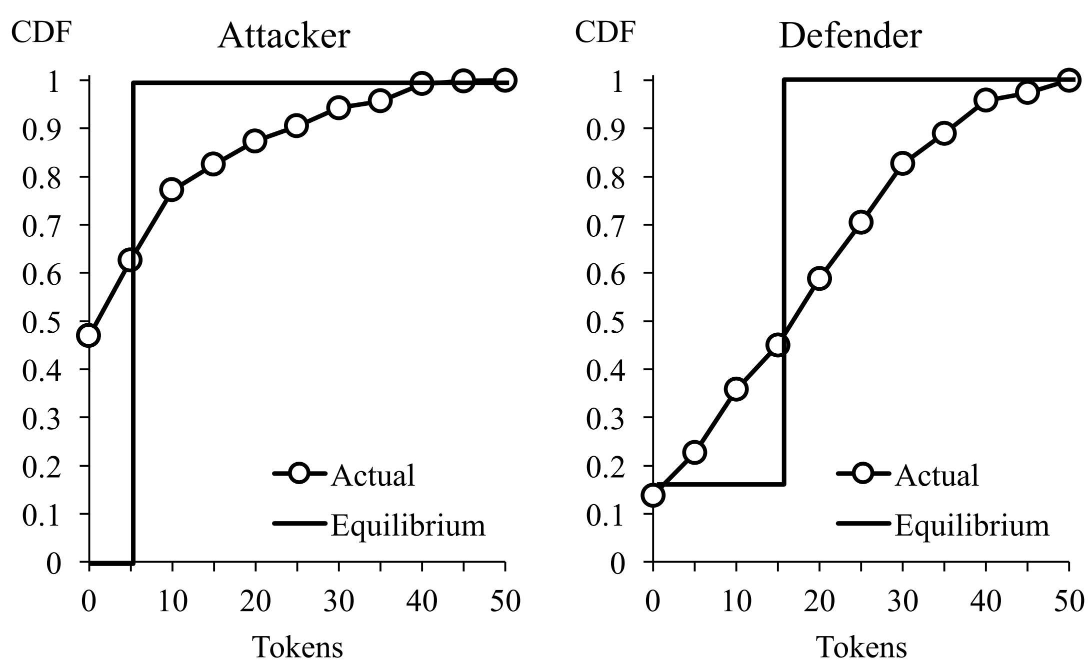
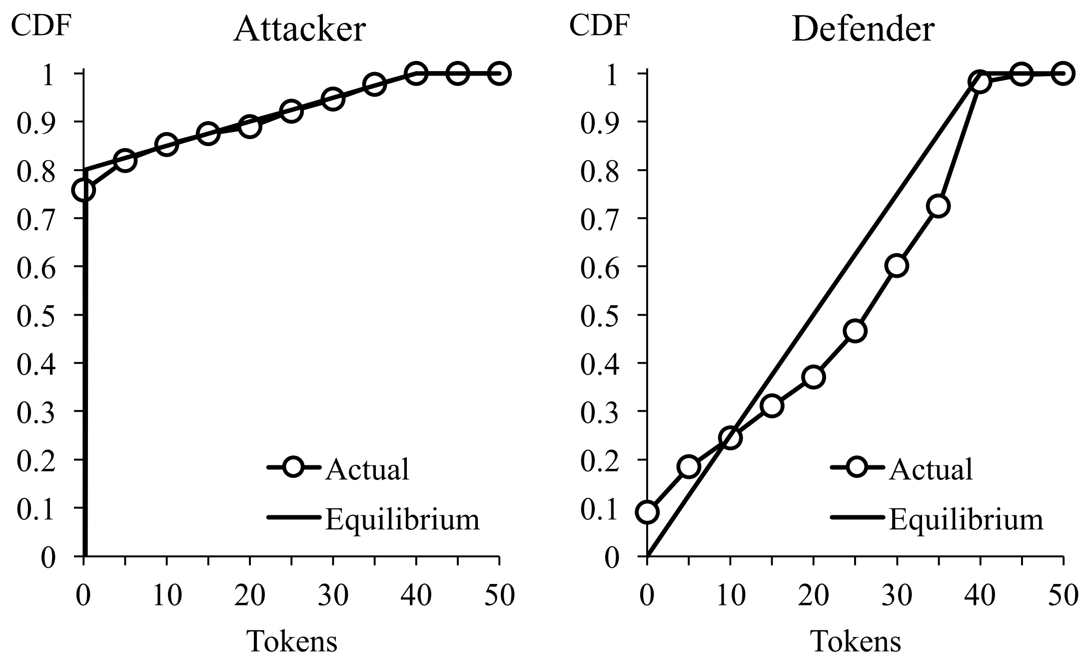
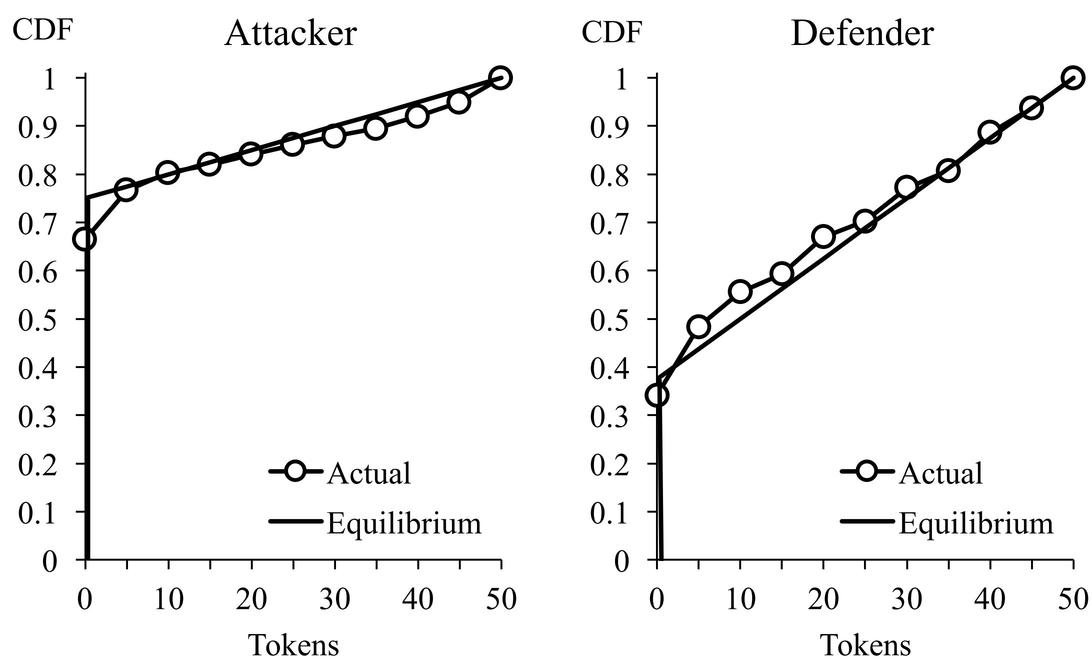


Figure 2: CDF of Tokens in the *Auction-Low* and *Auction-High* Treatments

The *Auction-Low* treatment



The *Auction-High* treatment



Appendix A – Uniqueness of Equilibrium in the Game of Attack and Defense with a Lottery CSF

In this appendix, we complete the characterization of equilibrium in the two-player simultaneous-move game of attack and defense with a set of contests with the lottery CSF in which the players have the objectives defined in equations (2) and (3), and where $p_D^i\left(x_A^i, x_D^i\right)$ is defined by equation (1) with $r = 1$,

$$p_D^i\left(x_A^i, x_D^i\right) = \begin{cases} \frac{x_D^i}{x_A^i + x_D^i} & \text{if } x_A^i + x_D^i > 0 \\ 1 & \text{otherwise} \end{cases} . \quad \text{(A1)}$$

We begin by defining additional notation and the concept of a uniform strategy. Let $\boldsymbol{x}_j \equiv \left\{x_j^i\right\}_{i=1}^n \in \mathbb{R}_+^n$ denote an arbitrary pure strategy (i.e. an allocation of forces across the n targets) for player j, where $x_j^i \in \mathbb{R}_+$ denotes player j's allocation to target i. Let P_j denote an arbitrary mixed-strategy (i.e., an n-variate joint distribution function) for player j, where for each $i \in \{1, \ldots, n\}$, P_j^i denotes player j's univariate marginal distribution for target i, which is defined by

$$P_j^i(x^i) = P_j\left(\{\infty\}_{i' \neq i}, x^i\right).$$

For a given (univariate) cumulative distribution function F, with support contained in $\mathbb{R}_+$, we define the mixed strategy[21] $P^{min}(\cdot\,|F)$ as the n-variate joint distribution function

$$P^{min}(\boldsymbol{x}|F) \equiv \min\{F(x^1), F(x^2), \ldots, F(x^n)\}.$$

If player j uses the mixed strategy $P^{min}(\cdot\,|F_j)$ and $\boldsymbol{x}$ is in the support of $P^{min}(\cdot\,|F_j)$, then $x_j^i = x$ for all $i \in \{1, \ldots, n\}$ and some $x \in \mathbb{R}_+$. Furthermore, $P_j^i(x) = F_j(x)$ for all $i \in \{1, \ldots, n\}$. That is the mixed strategy $P^{min}(\cdot\,|F_j)$ corresponds to a situation in which player j chooses $x \in \mathbb{R}_+$

[21] Note that P^{min} is the Fréchet-Hoeffding upper bound n-copula, and the properties of P^{min} are well known. See Nelsen (2006) and Schweizer and Sklar (1983) for further details.

according to F_j and sets $x_j^i = x$ for all $i \in \{1, \dots, n\}$. Note that under this notation, player j's pure strategy of allocating $x_j^i = x \in \mathbb{R}_+$ for all $i \in \{1, \dots, n\}$ may be written as $P^{min}(\cdot\,|F_j)$ where the distribution function F_j is degenerate, placing all mass on x.

***Definition 1*:** If player j uses a mixed strategy of the form $P^{min}(\cdot\,|F_j)$, then player j is said to use a *uniform strategy*. An equilibrium in which both players choose uniform strategies is called a *uniform equilibrium*.

An arbitrary strategy P_j, with the set of univariate marginal distributions $\{P_j^i\}_{i=1}^n$, is said to be distinct from $P^{min}(\cdot\,|F_j)$ if there exists a set of positive measure of $\boldsymbol{x} \in \mathbb{R}_+^n$ such that $P_j(\boldsymbol{x}) \neq P^{min}(\boldsymbol{x}|F_j) = \min\{F_j(x^1), F_j(x^2), \dots, F_j(x^n)\}$.

We now provide an outline of the proof, which consists of three parts. In part one, we show that all equilibria are interchangeable with the equilibrium stated in the corresponding case of Proposition 1. In part two, we state a slightly modified version of Lemma 1 of Clark and Konrad (2007) and, thereby, provide a condition — which is satisfied in equilibrium under the parameter restrictions of both parts (i) and (ii) of Proposition 1 — under which the best response to a uniform strategy is necessarily a uniform strategy. If any equilibrium is interchangeable with a uniform equilibrium and the best response to a uniform strategy is necessarily a uniform strategy, then it follows that all equilibria must be uniform equilibria. That is, parts one and two imply that every equilibrium is a uniform equilibrium. We conclude the proof by showing that the combination of part one and all equilibria are uniform can be used to establish uniqueness of equilibrium.

Beginning with the proof of part one (i.e., all equilibria are interchangeable with the equilibrium stated in the corresponding case of Proposition 1), let $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$ denote the uniform equilibrium stated in Proposition 1.[22] By way of contradiction, suppose that there exists a second (possibly non-uniform) mixed-strategy equilibrium (P_A, P_D) that is distinct from $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$. Let ι_A^i be an indicator function that takes a value of one in the event that player A wins the contest at target i. Because, by assumption, these are both equilibria we know that neither player has a payoff increasing deviation. Thus, for player D it must be the case that:

$$1 - \Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P_A, P_D\right) - \frac{\sum_{i=1}^n E_{P_D^i}(x)}{v_D} \geq 1 - \Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P_A, P^{min}(\cdot\,|F_D^*)\right) - \frac{nE_{F_D^*}(x)}{v_D} \tag{A2}$$

$$1 - \Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*)\right) - \frac{nE_{F_D^*}(x)}{v_D} \geq 1 - \Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P^{min}(\cdot\,|F_A^*), P_D\right) - \frac{\sum_{i=1}^n E_{P_D^i}(x)}{v_D} \tag{A3}$$

Similarly, for player A it must be that:

$$\Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P_A, P_D\right) - \frac{\sum_{i=1}^n E_{P_A^i}(x)}{v_A} \geq \Pr\left(\max_{i\in\{1,\ldots,n\}} \iota_A^i = 1 | P^{min}(\cdot\,|F_A^*), P_D\right) - \frac{nE_{F_A^*}(x)}{v_A} \tag{A4}$$

22 Recall that F_D^* is degenerate in case (i), where $v_D < (n-1)v_A$, with $\sum_{i=1}^n E_{F_D^*}(x_D^i) = nx_D^*$ and nondegenerate in case (ii), where $v_D \geq (n-1)v_A$, with $\sum_{i=1}^n E_{F_D^*}(x_D^i) = nq^*x_D^*$. Recall, also, that F_A^* is degenerate with $\sum_{i=1}^n E_{F_A^*}(x_A^i) = nx_A^*$ for both cases (i) and (ii) of Proposition 1.

$$\Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1 | P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*)\right) - \frac{nE_{F_A^*}(x)}{v_A} \tag{A5}$$

$$\geq \Pr\left(\max_{i\in\{1,\dots,n\}} \iota_A^i = 1 | P_A, P^{min}(\cdot\,|F_D^*)\right) - \frac{\sum_{i=1}^{n} E_{P_A^i}(x)}{v_A}$$

Taking the sum of (A2)-(A5), we have

$$2 - \frac{\sum_{i=1}^{n} E_{P_D^i}(x)}{v_D} - \frac{nE_{F_D^*}(x)}{v_D} - \frac{\sum_{i=1}^{n} E_{P_A^i}(x)}{v_A} - \frac{nE_{F_A^*}(x)}{v_A} \tag{A6}$$

$$\geq\ 2 - \frac{\sum_{i=1}^{n} E_{P_D^i}(x)}{v_D} - \frac{nE_{F_D^*}(x)}{v_D} - \frac{\sum_{i=1}^{n} E_{P_A^i}(x)}{v_A} - \frac{nE_{F_A^*}(x)}{v_A}$$

Note that if any of the equations (A2)-(A5) do not hold with equality, then (A6) cannot possibly hold with equality. Thus, because equation (A6) holds with equality, it follows that (A2)-(A5) must all hold with equality. But, if (A2)-(A5) all hold with equality, then this implies that P_D and $P^{min}(\cdot\,|F_A^*)$ are best-responses to each other and that P_A and $P^{min}(\cdot\,|F_D^*)$ are best-responses to each other. Thus, any arbitrary equilibrium (P_A, P_D) is interchangeable with $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$, and this completes the proof of part one.

Part two involves a slightly modified statement of Lemma 1 of Clark and Konrad (2007).

Lemma 1 (Clark and Konrad, 2007): If player $-j$ plays a uniform strategy $P^{min}(\cdot\,|F_{-j})$ such that $F_{-j}(0) \neq 1$ (i.e. player $-j$'s uniform strategy is not degenerate with all mass at 0), then each of player j's pure-strategy best responses to $P^{min}(\cdot\,|F_{-j})$ is a uniform strategy.

Given that player $-j$ plays a uniform strategy $P^{min}(\cdot\,|F_{-j})$ satisfying the Lemma 1 condition on F_{-j}, this slightly modified statement of Clark and Konrad's Lemma 1 follows directly from the proof of Lemma 1 in Clark and Konrad (2007). Note that Lemma 1 implies that in any mixed strategy best-response to a uniform strategy $P^{min}(\cdot\,|F_{-j})$, satisfying the Lemma 1 condition on F_{-j}, player j places probability one on a subset of the set of uniform pure-strategy best responses to F_{-j}. To summarize, the best response to a uniform strategy is necessarily a uniform strategy.

For both cases of Proposition 1, (i) $v_D \geq (n-1)v_A$ and (ii) $v_D < (n-1)v_A$, the proof that every equilibrium is a uniform equilibrium follows directly from the combination of steps one and two. If any equilibrium is interchangeable with the uniform equilibrium stated in Proposition 1 and the best response to a uniform strategy is necessarily a uniform strategy, then all equilibria are uniform equilibria. This completes the proof of part two.

We conclude the proof by showing that the combination of part one and all equilibria are uniform can be used to establish uniqueness of equilibrium. The following proof is for the case that $v_D \geq (n-1)v_A$. The remaining case that $v_D < (n-1)v_A$ follows along similar lines. By way of contradiction suppose that for $v_D \geq (n-1)v_A$ there exists a uniform equilibrium $(P^{min}(\cdot\,|F_A), P^{min}(\cdot\,|F_D))$ that is distinct from $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$, the Proposition 1 case (i) equilibrium. That is, there exists a uniform equilibrium $(P^{min}(\cdot\,|F_A), P^{min}(\cdot\,|F_D))$ such that for at least one player j there exists a set of positive measure of $x \in \mathbb{R}_+$ such that $F_j(x) \neq F_j^*(x)$. From part one the uniform equilibrium $(P^{min}(\cdot\,|F_A), P^{min}(\cdot\,|F_D))$ is interchangeable with $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$. Thus, $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D))$ is also an equilibrium and the fact that player D is best-responding to $P^{min}(\cdot\,|F_A^*)$ requires that for each $\boldsymbol{x}_D$ in the support of $P^{min}(\cdot\,|F_D)$, $x_D^i = x_D \in \mathbb{R}_+$ for all $i \in \{1, \dots, n\}$ and player D cannot increase his payoff by marginally changing his allocation x_D^i to any target i. That is, the first-order condition, with respect to x_D^i, must necessarily hold for each $\boldsymbol{x}_D$ in the support of $P^{min}(\cdot\,|F_D)$,

$$v_D\left(\prod_{i' \neq i} \frac{x_D}{x_D + x_A^*}\right)\left(\frac{x_A^*}{\left(x_D^i + x_A^*\right)^2}\right) - 1\Bigg|_{x_D^i = x_D} = v_D\left(\frac{x_D^{n-1} x_A^*}{\left(x_D + x_A^*\right)^{n+1}}\right) - 1 = 0. \quad \text{(A7)}$$

Note that equation (A7) coincides with equation (14) of Clark and Konrad (2007), which as they show in the proof of their Proposition 1 (provided in the Appendix of that paper) has a unique solution

$$x_D = x_D^* = \frac{v_A v_D^{n+1}}{(v_A+v_D)^{n+1}} \tag{A8}$$

Similarly, for player A and each $\boldsymbol{x}_A$ in the support of $P^{min}(\cdot\,|F_A)$, the corresponding first-order condition is given by

$$v_A\left(\prod_{i'\neq i}\frac{x_D^*}{x_D^*+x_A}\right)\left(\frac{x_D^*}{\left(x_D^*+x_A^i\right)^2}\right)-1\Bigg|_{x_A^i=x_A} = v_A\left(\frac{(x_D^*)^n}{\left(x_D^*+x_A\right)^{n+1}}\right)-1=0 \tag{A9}$$

which coincides with equation (10) of Clark and Konrad (2007) and as they show, equation (A9) has a unique solution

$$x_A = x_A^* = \frac{v_A^2 v_D^n}{(v_A+v_D)^{n+1}} \tag{A10}$$

Hence, we have a contradiction to the assumption that $(P^{min}(\cdot\,|F_A), P^{min}(\cdot\,|F_D))$ is distinct from $(P^{min}(\cdot\,|F_A^*), P^{min}(\cdot\,|F_D^*))$. Thus, for $v_D \geq (n-1)v_A$ the equilibrium stated in case (i) of Proposition 1 is the unique equilibrium, and this completes the characterization of equilibrium.

References

Nelsen, R. B. (2006), An Introduction to Copulas. Springer, New York.

Schweizer, B., and A. Sklar (1983), Probabilistic Metric Spaces. Elsevier Science Publishing Co., New York.

Appendix B – The *Lottery-Low* and *Lottery-High* treatments

GENERAL INSTRUCTIONS

This is an experiment in the economics of strategic decision making. Various research agencies have provided funds for this research. The instructions are simple. If you follow them closely and make careful decisions, you can earn an appreciable amount of money.

The experiment will proceed in three parts. Each part contains decision problems that require you to make a series of economic choices which determine your total earnings. The currency used in Part 1 of the experiment is U.S. Dollars. The currency used in Parts 2 and 3 of the experiment is francs. Francs will be converted to U.S. Dollars at a rate of **26** francs to **1** dollar. You have already received a **$20.00** participation fee. At the end of today's experiment, you will be paid in private and in cash. **12** participants are in today's experiment.

It is very important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation. At this time we proceed to Part 1 of the experiment.

INSTRUCTIONS FOR PART 1

In this part of the experiment you will be asked to make a series of choices in decision problems. How much you receive will depend partly on **chance** and partly on the **choices** you make. The decision problems are not designed to test you. What we want to know is what choices you would make in them. The only right answer is what you really would choose.

For each line in the table in the next page, please state whether you prefer option A or option B. Notice that there are a total of **15 lines** in the table but just **one line** will be randomly selected for payment. Each line is equally likely to be chosen, so you should pay equal attention to the choice you make in every line. After you have completed all your choices a token will be randomly drawn out of a bingo cage containing tokens numbered from **1 to 15**. The token number determines which line is going to be paid.

Your earnings for the selected line depend on which option you chose: If you chose option A in that line, you will receive **$1**. If you chose option B in that line, you will receive either **$3** or **$0**. To determine your earnings in the case you chose option B there will be second random draw. A token will be randomly drawn out of the bingo cage now containing twenty tokens numbered from **1 to 20**. The token number is then compared with the numbers in the line selected (see the table). If the token number shows up in the left column you earn $3. If the token number shows up in the right column you earn $0.

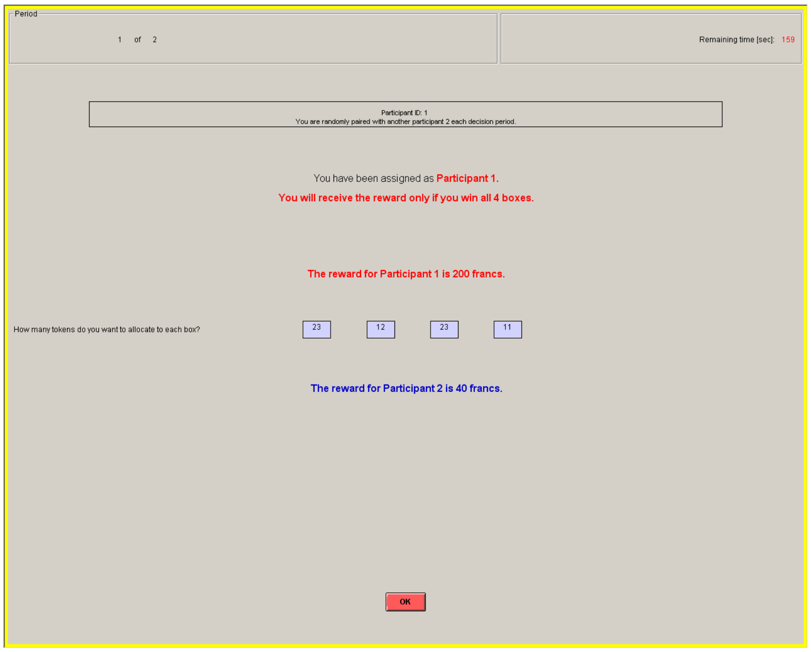
Are there any questions?

Decision no.	**Option A**	**Option B**		Please choose A or B
1	**$1**	**$3** never	**$0** if 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
2	**$1**	**$3** if 1 comes out of the bingo cage	**$0** if 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
3	**$1**	**$3** if 1 or 2	**$0** if 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
4	**$1**	**$3** if 1,2,3	**$0** if 4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
5	**$1**	**$3** if 1,2,3,4,	**$0** if 5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
6	**$1**	**$3** if 1,2,3,4,5	**$0** if 6,7,8,9,10,11,12,13,14,15,16,17,18,19,20	
7	**$1**	**$3** if 1,2,3,4,5,6	**$0** if 7,8,9,10,11,12,13,14,15,16,17,18,19,20	
8	**$1**	**$3** if 1,2,3,4,5,6,7	**$0** if 8,9,10,11,12,13,14,15,16,17,18,19,20	
9	**$1**	**$3** if 1,2,3,4,5,6,7,8	**$0** if 9,10,11,12,13,14,15,16,17,18,19,20	
10	**$1**	**$3** if 1,2,3,4,5,6,7,8,9	**$0** if 10,11,12,13,14,15,16,17,18,19,20	
11	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10	**$0** if 11,12,13,14,15,16,17,18,19,20	
12	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11	**$0** if 12,13,14,15,16,17,18,19,20	
13	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12	**$0** if 13,14,15,16,17,18,19,20	
14	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12,13	**$0** if 14,15,16,17,18,19,20	
15	**$1**	**$3** if 1,2, 3,4,5,6,7,8,9,10,11,12,13,14	**$0** if 15,16,17,18,19,20	

INSTRUCTIONS FOR PART 2

The second part of the experiment consists of **20 decision-making periods**. At the beginning of the first period, you will be randomly assigned either as **participant 1** or as **participant 2**. You will stay in the same role assignment for the **first 10 periods** and then change your role assignment for the **last 10 periods** of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form **a two-person group**. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Each period, both participants will choose how many tokens to allocate to **4 boxes** in order to receive a reward. Each token costs **1 franc**. The reward is worth **200 francs** to participant 1 and **40 francs** to participant 2. An example of a decision screen is shown below.



Participant 1 can allocate any number of tokens between **0** and **200** (including 0.1 decimal points) to each box. The total number of tokens in all boxes cannot exceed **200**. Similarly, participant 2 can allocate any number of tokens between **0** and **40** (including 0.1 decimal points). The total number of tokens in all boxes cannot exceed **40**.

The more tokens you allocate to a particular box, the more likely you are to win that box. The more tokens the other participant allocates to the same box, the less likely you are to win that box. Specifically, for **each token** you allocate to a particular box you will receive **10 lottery tickets**. At the end of each period the computer **draws randomly** one ticket among all the tickets purchased by you and the other participant in your group. The owner of the drawn ticket wins. Thus, your chance of winning a particular box is given by the number of tokens you allocate to that box divided by the total number of tokens you and the other participant allocate to that box.

$$\text{Chance of winning a box} = \frac{\text{Number of tokens you allocate to that box}}{\text{Number of tokens you allocate + Number of tokens the other participant allocates to that box}}$$

In case both participants allocate zero to the same box, the computer will randomly chose a winner of that box. Therefore, each participant has the same chance of winning the box.

Example of the Random Draw

This is a hypothetical example used to illustrate how the computer makes a random draw. Let's say participant 1 and participant 2 allocate their tokens to the 4 boxes in the following way.

Box	Participant 1	Participant 2	Chance of winning the box for Participant 1	Chance of winning the box for Participant 2
1	20.2	15	20.2/(20.2+15) = 0.57	15/(20.2+15) = 0.43
2	18.5	15	18.5/(18.5+15) = 0.55	15/(18.5+15) = 0.45
3	25	0	25/(25+0) = 1.00	0/(25+0) = 0.00
4	40	5	40/(40+5) = 0.89	5/(40+5) = 0.11
Total	103.7	35		

Participant 1 allocates 20.2 tokens to box 1, 18.5 tokens to box 2, 25 tokens box 3, and 40 tokens to box 4 (a total of 103.7 tokens). Participant 2 allocates 15 tokens to box 1, 15 tokens to box 2, 0 tokens to box 3, and 5 tokens to box 4 (a total of 35 tokens). Therefore, the computer will assign lottery tickets to participant 1 and to participant 2 according to their allocation of tokens.

For example, in box 1, the computer will assign 202 lottery tickets to participant 1 and 150 lottery tickets to participant 2. Then the computer will randomly draw one lottery ticket out of 352 (202+150). As you can see, participant 1 has a higher chance of winning box 1: 20.2/(20.2+15) = 0.57. Participant 2 has lower chance of winning box 1: 15/(20.2+15) = 0.43.

Similarly, in box 3, the computer will assign 250 lottery tickets to participant 1 and 0 lottery tickets to participant 2. Then the computer will randomly draw one lottery ticket out of 250 (250+0). As you can see, participant 2 has no chance of winning box 3: 0/(25+0) = 0.0. Therefore, participant 1 will win box 3 for sure: 25/(25+0) = 1.0.

YOUR EARNINGS

After both participants allocate their tokens and press the OK button, the computer will make a random draw for each box separately and independently. The random draws made by the computer will decide which boxes you win. Then the computer will assign a reward either to participant 1 or participant 2. **The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2.** The reward is worth **200** francs to participant 1 and **40** francs to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to the 4 boxes (each token costs 1 franc). Thus, the period earnings will be calculated in the following way:

If participant 1 receives the reward:

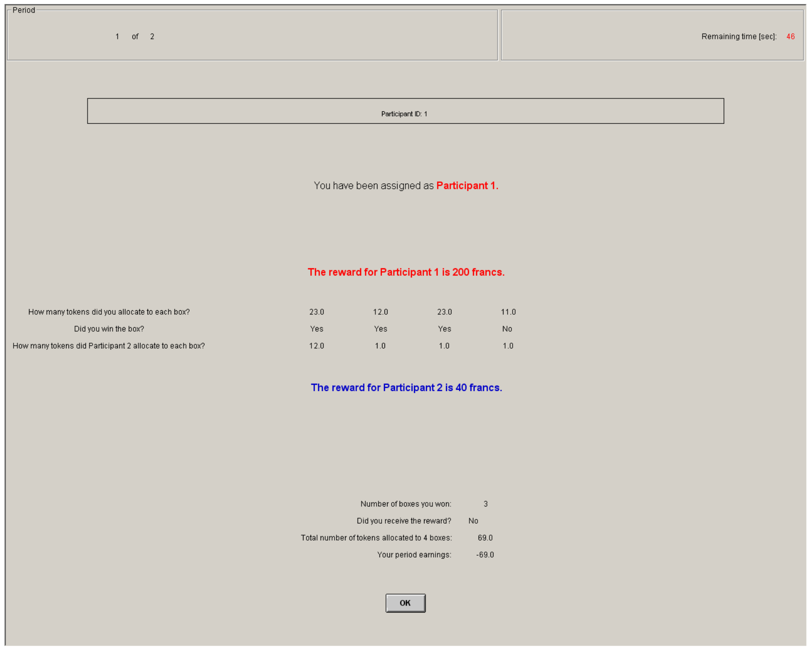
Participant 1's earnings = 200 – Tokens allocated to 4 boxes
Participant 2's earnings = 0 – Tokens allocated to 4 boxes

If participant 2 receives the reward:

Participant 1's earnings = 0 – Tokens allocated to 4 boxes
Participant 2's earnings = 40 – Tokens allocated to 4 boxes

Remember you have already received a **$20.00** participation fee (equivalent to **520 francs**). Depending on the outcome in a given period, you may receive either positive or negative earnings. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment. If the earnings are negative, we will subtract them from your participation fee. If the earnings are positive, we will add them to your participation fee.

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, whether you received the reward or not, and your period earnings are reported on the outcome screen as shown below. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading.



IMPORTANT NOTES

At the beginning of the first period, you will be randomly assigned either as participant 1 or as participant 2. You will stay in the same role assignment for the first 10 periods and then change your role assignment for the last 10 periods of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form a two-person group. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Both participants will choose how many tokens to allocate to 4 boxes. After both participants allocate their tokens, the computer will make a random draw for each box separately and independently. You can never guarantee that you will win a particular box. However, by increasing your allocation to that box, you can increase your chance of winning that box. The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to 4 boxes. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment.

INSTRUCTIONS FOR PART 3

The third part of the experiment consists of **20 decision-making periods**. The rules for Part 3 are exactly the same as the rules for Part 2. As in Part 2, at the beginning of the first period, you will be randomly assigned either as **participant 1** or as **participant 2**. You will stay in the same role assignment for the **first 10 periods** and then change your role assignment for the **last 10 periods** of the experiment. Each period you will be randomly re-paired with another participant of opposite assignment to form **a two-person group**. So, if you are participant 1, each period you will be randomly re-paired with another participant 2. If you are participant 2, each period you will be randomly re-paired with another participant 1.

Each period, both participants will choose how many tokens to allocate to **4 boxes** in order to receive a reward. Each token costs **1 franc**. The only difference from Part 2 is that in Part 3 the reward is worth **200 francs** to participant 1 and **80 francs** (instead of 40 francs) to participant 2. Participant 1 can allocate any number of tokens between **0** and **200** (including 0.1 decimal points) to each box. The total number of tokens in all boxes cannot exceed **200**. Similarly, participant 2 can allocate any number of tokens between **0** and **80** (including 0.1 decimal points). The total number of tokens in all boxes cannot exceed **80**.

After both participants allocate their tokens and press the OK button, the computer will make a random draw for each box separately and independently. The random draws made by the computer will decide which boxes you win. Then the computer will assign a reward either to participant 1 or participant 2. **The computer will assign a reward to participant 1 only if participant 1 wins all 4 boxes. Otherwise, the computer will assign the reward to participant 2.** The reward is worth **200** francs to participant 1 and **80** francs to participant 2. Regardless of who receives the reward, both participants will have to pay for the tokens they allocated to the 4 boxes (each token costs 1 franc).

At the end of each period, the allocation of your tokens, the allocation of the other participant's tokens, which boxes you win, whether you received the reward or not, and your period earnings are reported on the outcome screen. Once the outcome screen is displayed you should record your results for the period on your **Personal Record Sheet** under the appropriate heading. At the end of the experiment we will randomly select 1 out of the first 10 periods and 1 out of the last 10 periods of the experiment for actual payment. You will sum the total earnings for these two periods and convert them to a U.S. dollar payment.