

International Conference Complex Analysis and Operator Theory

Celebrating Daniel Alpay's 60th birthday

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Abstracts

# Generalized $Q$ -functions and Dirichlet-to-Neumann maps for elliptic differential operators

**Jussi Behrndt and Daniel Alpay**

Graz University of Technology, Steyrergasse 30, 8010 Graz, Austria

behrndt@tugraz.at

The classical concept of  $Q$ -functions associated to symmetric and self-adjoint operators due to M.G. Krein and H. Langer is extended in such a way that the Dirichlet-to-Neumann map in the theory of elliptic partial differential equations can be interpreted as a generalized  $Q$ -function. For couplings of uniformly elliptic second order differential expression on bounded and unbounded domains explicit Krein type formulas for the difference of the resolvents and trace formulas are obtained. This talk is based on the paper [1].

## References

- [1] D. Alpay and J. Behrndt, *Generalized  $Q$ -functions and Dirichlet-to-Neumann maps for elliptic differential operators*. J. Functional Analysis 257 (2009), 1666–1694.

# Minimal polynomials of quaternion matrices and Lagrange-Hermite type interpolation problems

Vladimir Bolotnikov

Department of Mathematics, The college of William and Mary  
Williamsburg, VA 23187-8795, USA

vladi@math.wm.edu

Any complex cyclic matrix  $A$  is similar to the companion matrix  $C(\boldsymbol{\mu}_A)$  of its minimal polynomial  $\boldsymbol{\mu}_A$ . Hence, the matrices

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 1 & \gamma_2 & 0 & \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & \gamma_n \end{bmatrix} \quad \text{and} \quad \mathcal{J} = \begin{bmatrix} \mathcal{J}_{r_1}(\lambda_1) & & 0 \\ & \ddots & \\ 0 & & \mathcal{J}_{r_k}(\lambda_k) \end{bmatrix},$$

(where  $J_r(\lambda)$  denotes the  $r \times r$  lower triangular Jordan block with diagonal entries equal  $\lambda$ ) are similar to the companion matrix  $C(f)$  if and only if  $\gamma_1, \dots, \gamma_n$  are all zeros of  $f$ , while  $\lambda_1, \dots, \lambda_k$  are all *distinct* zeros of  $f$  of respective multiplicities  $r_1, \dots, r_k$  that is, if and only if  $f$  admits respective representations

$$f = \boldsymbol{\rho}_{\gamma_1} \cdots \boldsymbol{\rho}_{\gamma_n} \quad \text{and} \quad f = \mathbf{lcm}(\boldsymbol{\rho}_{\lambda_1}^{r_1}, \dots, \boldsymbol{\rho}_{\lambda_k}^{r_k}), \quad \text{where} \quad \boldsymbol{\rho}_\gamma(z) := z - \gamma. \quad (1)$$

We will discuss quaternionic analogs of the latter observation, i.e., connections between various cyclic quaternion matrices and representations (1) for quaternion polynomials. Also we will show how these connections apply to Lagrange-Hermite interpolation problems over quaternions.

# Spectral theory for quaternionic operators

**Fabrizio Colombo**

Politecnico di Milano

fabrizio.colombo@polimi.it

In this talk we give an overview of the quaternionic spectral theory based on the notion of  $S$ -spectrum. We present the state of the art of the quaternionic version of the various functional calculi associated with slice hyperholomorphic functions. Moreover we discuss the spectral theorem for quaternionic (unbounded) normal operators using the notion of  $S$ -spectrum. An important motivation for studying the spectral theorem for quaternionic unbounded normal operators is given by the subclass of unbounded anti-self adjoint quaternionic operators which plays a crucial role in the quaternionic quantum mechanics.

# Two functional calculi based on slice hyperholomorphicity

Jonathan Gantner

Dipartimento di Matematica, Politecnico di Milano

Via E. Bornardi, 9 - 20133 Milano, Italy

jonathan.gantner@polimi.it

In recent years, the fundamental concepts of operator theory have been extended to linear operators on Banach and Hilbert spaces over the skew field of quaternions. The natural generalization of the holomorphic functional calculus has been developed [1] and it was even possible to prove the spectral theorem for normal quaternionic linear operators [2]. Two crucial steps in the development were the introduction of the  $S$ -spectrum, the correct notion of spectrum in this setting, and the identification of slice hyperholomorphicity as the notion of generalized holomorphicity that, in addition to its applications in Schur analysis [3], underlies quaternionic operator theory.

Using the theory of slice hyperholomorphic functions, we were able to generalize further classic results. This talk presents the Philips functional calculus for infinitesimal generators of strongly continuous groups of operators [4], which is based on the quaternionic Laplace-Stieltjes-transform. Moreover, we show how to construct fractional powers of quaternionic linear operators and that the famous Kato formula for the resolvents of fractional powers holds also true in the quaternionic setting [5].

## References

- [1] F. Colombo, I. Sabadini and D. C. Struppa. *Noncommutative Functional Calculus*. Birkhäuser, Basel, 2011.
- [2] D. Alpay, F. Colombo and D. P. Kimsey. The spectral theorem for quaternionic unbounded normal operators based on the  $S$ -spectrum. *J. Math. Phys.*, 57(2):023503, 2016.
- [3] D. Alpay, F. Colombo and I. Sabadini. *Slice Hyperholomorphic Schur Analysis*. Birkhäuser, Basel, 2017.
- [4] D. Alpay, F. Colombo, J. Gantner and D. P. Kimsey. Functions of the infinitesimal generator of a strongly continuous quaternionic group. *Anal. Appl. (Singap.)*, 2016, doi:10.1142/S021953051650007X.
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# Noncommutative functions and low dimensional algebras.

Bill Helton

Math. Dept. UC San Diego

helton@math.ucsd.edu

The talk concerns inequalities for functions having matrix variables. The functions are typically (noncommutative) polynomials or rational functions. A focus of much attention is the inequalities corresponding to convexity which in this noncommutative case corresponds directly to Linear Matrix Inequalities, LMIs. Such mathematics is central to linear systems problems which are specified entirely by a signal flow diagram and  $L^2$  performance specs on signals. Since systems problems seldom produce an LMI directly it is important to have a theory for changing variables to produce an LMI.

This talk will concern a subtopic: analytic changes of noncommuting variables which map one convex set to another. The remarkable thing is that for convex sets in  $g$  noncommuting variables there turns out to be a small class of such bianalytic maps parameterized by  $d$  dimensional algebras.

The work originates in trying to develop some theory for studying the matrix inequalities which are ubiquitous in linear engineering systems and control. Most of the work is done jointly by Meric Augat, J. William Helton, Igor Klep and Scott A. McCullough.

# **Developments in Publishing**

**Thomas Hempfling**

Springer International Publishing AG (Birkhäuser), Basel, Switzerland

email: [thomas.hempfling@birkhauser-science.com](mailto:thomas.hempfling@birkhauser-science.com)

Publishing is under continuous and sometimes fast development. In the last 15 years, tremendous changes mixed up the industry and the end-users of publishing products. For example, electronic publishing took over, abstracting and indexing services became (too?) dominant, with a lot of consequences for all involved parties. We try to give a short account, based on experiences in SpringerNature, on what has been established and which further developments are prepared for or have just been introduced to the market.

# Infinite networks and variation of conductance functions in discrete Laplacians

Palle Jorgensen and Feng Tian

Department of Mathematics  
University of Iowa  
Iowa City, IA 52242-1419, U.S.A.  
palle-jorgensen@uiowa.edu

For a given infinite connected graph  $G = (V, E)$  and an arbitrary but fixed conductance function  $c$ , we study an associated graph Laplacian  $\Delta_c$ ; it is a generalized difference operator where the differences are measured across the edges  $E$  in  $G$ ; and the conductance function  $c$  represents the corresponding coefficients. The graph Laplacian (a key tool in the study of infinite networks) acts in an energy Hilbert space  $\mathcal{H}_E$  computed from  $c$ . Using a certain Parseval frame, we study the spectral theoretic properties of graph Laplacians. In fact, for fixed  $c$ , there are two versions of the graph Laplacian, one defined naturally in the  $l^2$  space of  $V$ , and the other in  $\mathcal{H}_E$ . The first is automatically selfadjoint, but the second involves a Krein extension. We prove that, as sets, the two spectra are the same, aside from the point 0. The point zero may be in the spectrum of the second, but not the first.

We further study the fine structure of the respective spectra as the conductance function varies; showing now how the spectrum changes subject to variations in the function  $c$ . Specifically, we study an order on the spectra of the family of operators  $\Delta_c$ , and we compare it to the ordering of pairs of conductance functions. We show how point-wise estimates for two conductance functions translate into spectral comparisons for the two corresponding graph Laplacians; involving a certain similarity: We prove that point-wise ordering of two conductance functions  $c$  on  $E$ , induces a certain similarity of the corresponding (Krein extensions computed from the) two graph Laplacians  $\Delta_c$ . The spectra are typically continuous, and precise notions of fine-structure of spectrum must be defined in terms of equivalence classes of positive Borel measures (on the real line.) Our detailed comparison of spectra is analyzed this way.



**Singular integral operators induced by Bergman-Besov  
kernels on weighed Lebesgue classes on the ball**

**H. Turgay Kaptanoğlu**

Department of Mathematics, Bilkent University, Ankara  
<http://www.fen.bilkent.edu.tr/~kaptan/>

Although the boundedness of the Bergman-Besov projection operators from Lebesgue classes to Bergman-Besov spaces has been studied for several decades, the study of the boundedness of the same operators as singular integral operators between different Lebesgue classes are rather new. Some initial work was recently done by Cheng, Fang, Wang, Yu for the weighted Bergman operator on the unit disc and by Cheng, Hou, Liu for the Drury-Arveson operator. The methods they employed are somewhat sporadic and specific to the particular cases.

Our aim is to have global approach and cover all weighted Bergman-Besov kernel operators and weighted Lebesgue classes, and work on the unit ball of  $\mathbb{C}^n$ . We attempt to treat the different ranges of the parameters in a unified and systematic way. Our main tools are various new forms of the Schur test on integral operators that we have been developing for the boundedness of Bergman projections in our earlier works, growth estimates of the Bergman-Besov kernels, and precise inclusion relations between various Bergman-Besov spaces.

This is an ongoing project conducted jointly with A. Ersin Üreyen of Anadolu University, Eskişehir, Turkey.

# Has de Branges proved the Riemann Hypothesis?

**Eric Kvaalen**

consultant, Les Essarts-le-Roi, France

Eric\_Kvaalen hotmail com

In 2005 Professor Louis de Branges of Purdue University put a paper on the Internet entitled “A Proof of the Riemann Hypothesis”. [1] Although the paper does not actually prove the Riemann Hypothesis, it may be possible to use the results in order to do so. In this talk I will analyze this possibility and indicate what remains to be done.

## References

[1] L. de Branges A Proof of the Riemann Hypothesis. Unpublished, 2005, updated 2006.

<http://web.archive.org/web/20080409024036/>

<http://www.math.purdue.edu/~branges/riemannzeta.pdf>

(This links to the last version and is identical to earlier archived files starting from July 2006.)

# On White Noise Stochastic Analysis\*

David Levanony

Department of Electrical and Computer Engineering,  
Ben Gurion University, Beer Sheva, Israel

levanony@post.bgu.ac.il

**Abstract:** We construct and study a wide class of stationary increment Gaussian processes and their associated stochastic calculus. The underlying framework is set within the White Noise Space, a setting which allows to study a wide range of processes, among them, processes being not necessarily semi-martingales (e.g. the fractional Brownian motion), as well as their derivatives, understood as stochastic distributions. The Wick product is defined. A subsequent characterization of an associated Wick-Ito stochastic integral as a limit of Riemann sums is then shown to generalize the well known Ito and Skorohod integrals. The derivation of an Ito formula follows.

\* Talk based on joint works with Daniel Alpay, Haim Attia and Palle Jorgensen.

## References

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- [2] D. Alpay, H. Attia and D. Levanony, On the characteristics of a class of Gaussian processes within the white noise space setting. *Stochastic Processes and their Applications* 120 (2010), 1074-1104.
- [3] D. Alpay, P. Jorgensen and D. Levanony, A class of Gaussian processes with fractional spectral measures. *J. of Functional Analysis* 261 (2011), 507-541.
- [4] D. Alpay, H. Attia and D. Levanony, White noise based stochastic calculus associated with a class of Gaussian processes. *Opuscula Mathematica* 32-3 (2012), 401-422.

**Convex Invertible Cones**  
**in Dissipative Linear System Theory**  
**or**  
**my lesson from  $R - L - C$  circuits**

**Izchak Lewkowicz**

Elect. & Comp. Eng. dept. Ben-Gurion University, Be'er-Sheba, Israel.  
email: izchak@bgu.ac.il

Convex Invertible Cones (CICs in short) of matrices or of rational functions, are sets which are closed under (i) positive scaling, (ii) summation and (iii) inversion (whenever well defined). CICs are well defined over any real unital algebra.

In the context of driving point impedance of  $R - L - C$  electrical circuits, (i) positive scaling means transformer ratio, (ii) summation takes the form of series connection and (iii) inversion is interpreted as impedance/admittance duality.

With this motivation, over the years we (some of the works were collaborative, see sample below) found that, although not well studied, CICs appear in numerous places. Roughly, the message is that wherever there is continuous-time dissipativity, one should look for a hidden underlying CIC structure.

In this talk we offer a sample list, and make interconnections among the items. Needless to say, the details given are as long as time permits.

Many of these items open the door to research questions such as analysis of rational functions of several non-commuting variables.

**References**

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- [2] D. Alpay and I. Lewkowicz, The Positive Real Lemma and Construction of all Realizations of Generalized Positive Rational Functions, Sys. Cont. Lett., Vol. 60, pp. 985-993, 2011
- [3] D. Alpay and I. Lewkowicz, Convex Cones of Generalized Positive Rational Functions and Nevanlinna-Pick Interpolation, Lin. Alg.& Appl., Vol. 438, pp. 3949-3966, 2013.

# Scale shift for discrete-time signals and systems

Mamadou Mboup

CReSTIC - UFR SEN, University of Reims, Moulin de la Housse, France  
Mamadou.Mboup@univ-reims.fr

Joint work with Daniel Alpay

A wide class of causal discrete time-invariant linear systems can be given in terms of convolution in the form

$$y_n = \sum_{m=0}^n h_{m-n} u_m, \quad n = 0, 1, \dots, \quad (2)$$

where  $(h_n)$  is the impulse response and where the input sequence  $(u_m)$  and output sequence  $(y_m)$  are requested to belong to some pre-assigned sequences spaces. The  $\mathcal{Z}$  transform of the sequence  $(h_n)$ , that is  $\hat{h}(\zeta) = \sum_{n=0}^{\infty} \zeta^n h_n$ , is called the *transfer function* of the system, and there are deep relationships between properties of  $\hat{h}$  and of the system; see D. Alpay [1] for a survey. Analogs of systems of the form (2) when both  $h_n$  and  $u_n$  are Gaussian random variables, which belong to the white noise space, or more generally to the Kondratiev space, have been studied recently by D. Alpay and D. Levanony [2]. The pointwise product  $h_{m-n} u_m$  in (2) is then replaced by the Wick product (this allows in particular to have a Gaussian output). Recall that the Wick product is a convolution when expressed in terms of the Hermite functions (see [2]). So, using the Hermite transform, one can define a generalized transfer function, which is a function analytic in  $\zeta$  and in a countable number of other variables that take into account the randomness.

In the present talk we discuss an extension of (2) in a different direction, and develop an approach to discrete-time signals and systems with a multi scale-invariant property.

## References

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- [2] D. Alpay and D. Levanony. Linear stochastic systems: a white noise approach. *Acta Applicandae Mathematicae*, 2009.

# de Branges Spaces on Compact Riemann Surfaces and a Beurling-Lax Type Theorem

Daniel Alpay, Ariel Pinhas and Victor Vinnikov

Department of Mathematics, Ben-Gurion University of the Negev,  
Beer Sheva, Israel  
arielp@math.bgu.ac.il

The theory of de Branges-Rovnyak spaces of analytic functions allows to prove Beurling-Lax type theorems in the one complex variable framework when leaving the setting of the Hardy space. It is also known that one can prove de Branges structure theorem using the notion of single-operator colligation.

We study de Branges spaces of analytic sections of the line bundle  $L_{\tilde{\zeta}} \otimes \Delta$  on a compact Riemann surface  $X$ . Here  $L_{\tilde{\zeta}}$  is flat line bundle on  $X$  with multipliers corresponding to  $\zeta \in J(X)$  and  $\Delta$  is a square root of the canonical line bundle. In the first part, we introduce the counterpart of de Branges structure theorem in the setting of real compact Riemann surfaces. We give a characterization for reproducing kernel Hilbert spaces of sections with reproducing kernel of the form

$$K_T(p, q) = \frac{\vartheta[\tilde{\zeta}](\bar{q} - p)}{i\vartheta[\tilde{\zeta}](0)E(p, \bar{q})} - T(p)\overline{T(q)} \frac{\vartheta[\zeta](\bar{q} - p)}{i\vartheta[\zeta](0)E(p, )}. \quad (3)$$

Here  $T$  is  $(\zeta, \tilde{\zeta})$ -contractive line bundles mapping and  $\vartheta[\zeta]$  is the theta function with characteristic  $\zeta$ . To do so, we embed a pair of model's multiplication operators (corresponding to two real meromorphic functions  $y_1$  and  $y_2$  generating the field of meromorphic functions  $\mathcal{M}(X)$ ) in Livsic's commutative two-operator vessel.

In the second part, we present a Beurling type theorem on finite bordered Riemann surfaces. We consider a closed subspace  $H$  of the corresponding Hardy space which is invariant under a pair of multiplication operators  $\mathcal{M}_{\frac{1}{y_1(u)-\alpha}}$  and  $\mathcal{M}_{\frac{1}{y_2(u)-\beta}}$ . Then the orthogonal complement is invariant under the resolvent operators  $R_{\alpha}^{y_1}$  and  $R_{\beta}^{y_2}$ . Furthermore we show that the structure identity holds in our setting. Then, applying the structure theorem, the reproducing kernel of  $H^{\perp}$  is of the form (3) and a Beurling's type theorem follows.

# Toward an extension of Schur analysis to the slice hyperholomorphic setting

Irene Sabadini

Politecnico di Milano

irene.sabadini@polimi.it

Schur analysis can nowadays be seen as a collection of results pertaining to Schur functions treated in various setting e.g. one and several complex variables, compact Riemann surfaces just to name a few. It has also been treated in the quaternionic case, by means of the so-called Fueter regular functions. Recently, we started the study of Schur functions in the framework of the so-called slice hyperholomorphic functions of a quaternionic variable, see [1] and references therein. Using this class of functions we can introduce reproducing kernel Hilbert spaces, define Schur multipliers and describe their realizations. Indeed, slice hyperholomorphic functions allow to write realizations in terms of a suitable resolvent, the so called S-resolvent operator and we use this operator to extend several results from the complex case to the quaternionic case.

[1] D. Alpay, F. Colombo, I. Sabadini, *Slice Hyperholomorphic Schur Analysis*, Operator Theory: Advances and Applications, vol. 256, Springer, 2017.

## **Hilbert formulas for the Cauchy-Cimmino singular integrals**

**Baruch Schneider and J. Bory Reyes, R. Abreu Blaya, M. A. Pérez-de la Rosa**

Department of Mathematics, Izmir University of Economics, 35330 Izmir, Turkey

email: baruch.schneider@ieu.edu.tr

The Cimmino system offers a natural and elegant generalization to four-dimensional case of the Cauchy-Riemann system of first order complex partial differential equations. Recently, it has been proved that many facts from the holomorphic function theory have their extensions onto the Cimmino system theory. In the present talk we discuss some analogues of the Hilbert formulas on the unit 3-sphere and on the 3-dimensional space for the theory of Cimmino system.



# The Geometry of finite unions of segments

Ahmed Sebbar

Institut de Mathématiques de Bordeaux, Université de Bordeaux, France

Ahmed.Sebbar@math.u-bordeaux1.fr

Chapman University

sebbar@chapman.edu

We consider  $I$  a finite union of segments of the real line and we use a difference operators version of the Burchnell-Chaundy theorem to obtain a Jacobi matrix  $J$  whose spectrum  $\sigma$  is  $I$ . Under some generic arithmetic conditions, we express the coefficients of  $J$  by the Riemann theta function of the hyperelliptic curve associated to  $I$ . This enables us to obtain the logarithmic capacity of  $\sigma$ . We also study the variation-degeneration of  $\sigma$ . Some of these results were obtained with Thérèse Falliéro a few years ago.

# **An introduction to the Aharonov-Berry superoscillations**

**Daniele C. Struppa**

Chapman University

struppa@chapman.edu

Superoscillatory sequences arise naturally from the theory of weak measurements in quantum physics, as well as from classical optics. In this talk I will give the basic ideas on how superoscillations arise and I will study a problem posed by Aharonov, namely the question of whether superoscillatory behavior remains when superoscillatory initial data are expanded according to the Schrodinger equation. I will discuss in details a few concrete examples. The talk is based on a series of joint papers with Aharonov, Bunyi, Colombo, Gantner, Sabadini, and Tollaksen.

# **Algebraic Analysis of Abelian Instantons and Singularities of Yang-Mills Equations**

**Adrian I. Vajiac**

Chapman University

avajiac@chapman.edu

This paper shows how to apply some ideas from algebraic analysis to the study of the anti-self-dual Abelian Yang-Mills equations in the context of the gauge theory for electromagnetism and a study of the singularities of the Yang-Mills equations. In analogy with what has already been done for the Maxwell's equations, we use techniques based on the algebraic structure of the systems of interest, to obtain results on the regularity of the solutions, as well as on the natural compatibility conditions that such systems must satisfy; these conditions often take the form of conservation laws. The main computational software package we use is CoCoA. This work is in collaboration with F. Colombo, I. Sabadini, D.C. Struppa, and M.B. Vajiac.

# Harmonic Maps, Riemann-Hilbert Problems and Virasoro Actions

Mihaela B. Vajiac

Chapman University

mbvajiac@chapman.edu

In this joint work with K. Uhlenbeck, we discuss the actions of a half-Virasoro algebra on the space of (2+0) harmonic maps into Lie groups. This is generated by a natural action on the frames which defines this infinitesimal action of a complex half-Virasoro algebra on the space of harmonic maps from a simply connected domain in to the Lie group  $SU(n)$ . This is related to other known examples of Virasoro actions on integrable systems. A similar calculation on the space-time (1+1) harmonic maps yields formulas generated by John Schwarz.

## References

- [1] K. Uhlenbeck, M.Vajiac, Virasoro Actions and Harmonic Maps, Journal of Differential Geometry p. 327-341, Volume 8, number 2, October 2008

# Polynomial interpolation: discrete analytic case

Dan Voloĳ

Math. Dept., Kansas State University, Manhattan KS 66506, USA

danvolok@math.ksu.edu

The notion of discrete analyticity, originally introduced by J. Ferrand in [2], has received much attention recently, in particular because of the applications to probability and statistical physics; see e.g. [3]. When extending the results of the classical complex analysis to the discrete setting, much difficulty is caused by the fact that discrete analyticity is not preserved, in general, by the usual point-wise product of functions. For instance, the polynomials  $1, z, z^2$  are discrete analytic on the integer lattice in the complex plane but  $z^3$  is not.

In this talk we shall discuss interpolation properties of discrete analytic polynomials, making use of an explicit construction introduced by D. Alpay et al in [1].

## References

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# Averaging integral kernels in complex geometry (from archimedean towards non-archimedean case)

Alain Yger

Institut de Mathématiques, Université de Bordeaux, Talence, France  
yger@math.u-bordeaux.fr

Averaging Cauchy kernel in multivariate complex leads to the realization of so-called Bochner-Martinelli type currents. Such averaging fits well within a geometric frame, namely the realization and manipulation of hermitian bundles over (non necessarily smooth) analytic spaces. I will in particular formulate in these terms two classical procedures involved in cohomological (improper) intersection theory : Segre's and Vogel-Stückrad approaches. Part of the material I will present here is a joint work with Mats Andersson, Hakån Samuelsson and Elizabeth Wulcan (for what concerns local aspects, [1]), which has been since continued as work in progress together with Dennis Eriksson (beside the previous coauthors) (still unpublished yet). I will also present joint results obtained with A. Vidras involving averaged multivariate residue calculus. I will then sketch the notion of analytic space in the non-archimedean setting (Berkovich space), focus on the fundamental idea of tropicalization and explain how such currential type methods inspired by complex analytic geometry (involving tools such as  $(p, q)$ -differential calculus, Bedford-Taylor multiplicative process, etc.) could be transposed from the analytic (archimedean) complex geometry towards the non-archimedean analytic setting. This will rely on results due to Antoine Ducrot and Antoine Chambert-Loir, Walter Gubler [2], and my former students Farhad Babae ([3], with June Huh) and Ibrahima Hamidine [4].

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