The homotopy hypothesis is a statement which connects homotopy theory and (higher) category theory: a space up to weak homotopy equivalence is the same as an "infinity groupoid". This question was one of the starting point of Grothendieck's "Pursuing stack" in the early 80s. He gave a precise formulation of this conjecture, and while other formulations of the homotopy hypothesis are now well understood and are completely central to higher category theory, Grothendieck's formulation is still to this day an open problem.

In this talk I will gave an overview of the homotopy hypothesis, mostly focused on Grothendieck formulation and a survey of the progress that have been made toward a proof in the last 40 years.