On the Nature of the Sommerfeld-Brillouin Forerunners (or Precursors)

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When a light pulse with a well-defined leading edge enters a homogeneous, isotropic, and dispersive dielectric medium at $t = 0$, it gets distorted and attenuated as it makes its way through the medium; see Fig.1(a). At a large distance x_0 from the point of entry, the leading edge of the pulse arrives at time $t_0 = x_0/c$, where c is the speed of light in vacuum. Figure 1(b) shows that, in the vicinity of t_0, the light pulse has a weak amplitude but a high frequency; this is known as the first (or Sommerfeld) forerunner. A short while later, the character of the signal at $x = x_0$ changes; it now oscillates at a low frequency, albeit with a small amplitude, and is referred to as the second (or Brillouin) forerunner.[1] Eventually, the bulk of the signal arrives with the characteristic frequency of the incident beam. Sommerfeld has described the situation as follows: *"If we let white light fall perpendicularly on a dispersive plate, then the less refracted (and hence 'faster') components of the white light do not precede the more refracted (and hence 'slower') components, and the light is not red at the first instance of emergence. Instead, the wave front of each component propagates with the same velocity c through the plate, and each component contributes equally to the energy of the initially emerging light. These initially emerging forerunners do not show the colors of the components of which they are composed; instead, they have an ultraviolet wavelength determined by the dispersive power and thickness of the plate, and a very small intensity."*[1]

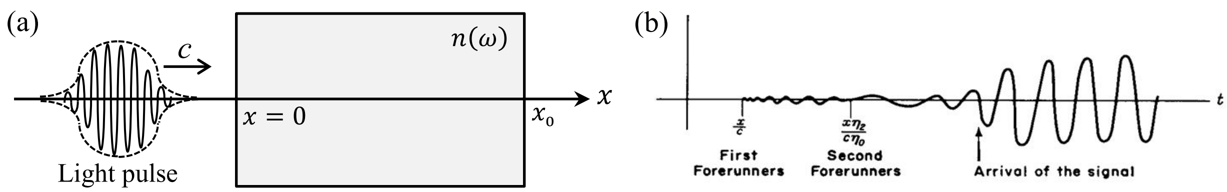

Fig.1. (a) A light pulse having a well-defined leading edge arrives at $t = 0$ at a dielectric slab of refractive index $n(\omega)$. (b) Evolution of the light amplitude at $x = x_0$ following its initial arrival at $t_0 = x_0/c$.[1]

The first forerunner thus appears to have the characteristics of a superoscillating signal, except that the incident pulse, due to its sharp leading-edge, is *not* a bandlimited waveform. In an effort to determine the circumstances under which the Sommerfeld forerunner could be considered a superoscillator, we extend the Sommerfeld-Brillouin theory to bandlimited incident beams. The single-oscillator Lorentz model of the refractive index used in this work has a pair of poles and zeros in the complex ω-plane, as shown in Fig.2.

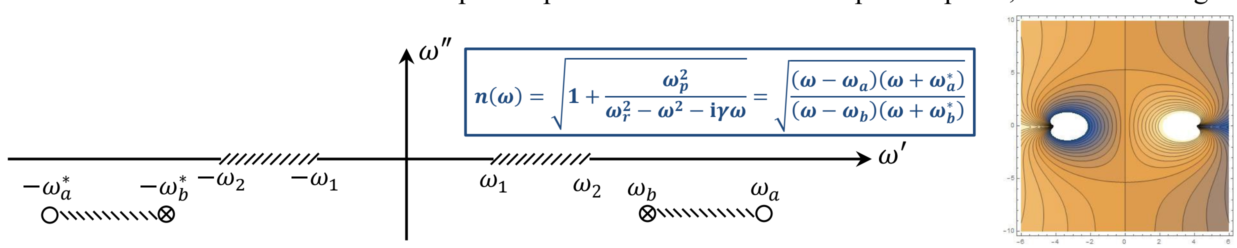

Fig.2. Complex-plane depiction of the zeros, $\omega_a, -\omega_a^*$, and poles, $\omega_b, -\omega_b^*$, of the refractive index $n(\omega)$ of the single-oscillator Lorentz model. The short line-segments connecting each pole to its adjacent zero are branch-cuts. In the expression of $n(\omega)$, ω_p is the plasma frequency, ω_r is the resonance frequency, and γ is the damping coefficient. The bandlimited spectrum $\mathcal{E}(\omega)$ of the incident beam is confined to the range $\omega_1 \le |\omega| \le \omega_2$, and the incident spectrum is Hermitian, that is, $\mathcal{E}(-\omega) = \mathcal{E}^*(\omega)$. The inset shows several steepest-descent contours in the ω-plane, which are used to evaluate $E(x_0, t)$ via saddle-point approximation.

The (complex) E-field amplitude at a distance $x = x_0$ from the entrance facet is given by

$$E(x_0, t) = \int_{\omega_1}^{\omega_2} \mathcal{E}(\omega) \exp\{i\omega[n(\omega)(x_0/c) - t]\} \,d\omega \; = \int_{\omega_1}^{\omega_2} \mathcal{E}(\omega) \exp\{it[\zeta\omega n(\omega) - \omega]\} \,d\omega,$$

where $\zeta = x_0/(ct) \le 1$. The spectral profile $\mathcal{E}(\omega)$ of the incident E-field is bandlimited, as it is confined to the interval $\omega_1 \le |\omega| \le \omega_2$. We shall discuss the use of saddle-point approximation to evaluate $E(x_0, t)$.

1. Léon Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York (1960).