

An Alternating-Offers Model of Multilateral Negotiations*

Charles J. Thomas

Economic Science Institute & Argyros School of Business and Economics

Chapman University

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Abstract

I modify a standard infinite-horizon alternating-offers model of bilateral negotiations to analyze multilateral negotiations, an exchange process in which a buyer attempts to purchase from one of several sellers offering potentially different amounts of surplus to be split. Rival sellers constrain the terms of trade to the buyer's benefit in some cases, but not in others. I use the model to illustrate how the buyer's choice of conducting procurement via multilateral negotiations or an auction is affected by factors including the number of sellers, the extent of uncertainty when making the choice, and costs of participating in the procurement process.

1 Introduction

Bilateral negotiations play an important role in economic theory and practice, but negotiations often are multilateral in nature. For example, in procurement settings a firm or government might negotiate to purchase from one of several suppliers whose products differ on dimensions such as quality or goodness-of-fit. Likewise, in takeover contests there might be multiple potential acquirers who differ in their synergies or opportunity costs from completing the transaction. In high-end labor markets a job candidate might have several employers maneuvering for position. Compared to bilateral negotiations, such multilateral negotiations feature what many economists likely would consider to be basic elements of competition.

Multilateral negotiations have received relatively little academic attention despite their empirical relevance, perhaps because their strategic complexity has made them difficult to analyze with formal models. They feature elements of bilateral negotiations and auctions, both of which are potentially complex exchange mechanisms that have been the subject of significant amounts of academic research.¹ However, analyses of bilateral negotiations by definition tend to emphasize bilateral settings, with some exceptions described later that require one party to abandon current negotiations before beginning talks with a new potential trading partner, or that consider exogenously specified outside options rather than endogenously determined ones. Analyses of auctions focus on multilateral settings, but auctions lack the communication and interplay

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¹For surveys of the bargaining literature see Kennan and Wilson [1993], Ausubel, Cramton, and Deneckere [2002], and Serrano [2008], while for surveys of the auction literature see McAfee and McMillan [1987] and Milgrom [1989].

inherent to multilateral negotiations. Auctions also exhibit commitment on the part of the auctioneer not to renege on the rules of game, say by making counteroffers after receiving bids.

Incomplete information may be an important component of multilateral negotiations' strategic complexity, so in this paper I dispense with it to investigate whether existing models of bilateral negotiations with complete information can be usefully extended to accommodate multilateral negotiations. In Section 3 I describe related research that evaluates similar issues.

In Section 2 I modify the infinite-horizon alternating-offers model of bilateral negotiations developed in Rubinstein [1982] by adding additional sellers with whom the buyer negotiates simultaneously. The buyer wishes to trade with only one seller, and trade with different sellers can generate different amounts of surplus to be split. In some cases rival sellers constrain the terms of trade to the buyer's benefit, but in others they are irrelevant to the negotiated outcome. Results from the basic model illustrate how negotiated outcomes are affected by introducing additional sellers into a bilateral negotiation setting, by the similarity of sellers in terms of available surplus, and by the parties' relative bargaining strengths. One noteworthy finding is that the buyer can be better off when it becomes less patient in multilateral negotiations, which contrasts sharply with the effect of impatience in bilateral negotiations.

In Section 4 I investigate the important issue of institutional choice by extending the basic model to evaluate how the buyer's choice of conducting procurement via multilateral negotiations or an auction is affected by factors including the number of sellers, the extent of uncertainty when making the choice, and costs associated with participating in the procurement process. One interesting finding when sellers incur entry costs is that the buyer's preferred procurement mechanism can change multiple times within small ranges of those costs, which provides one explanation for the coexistence of auctions and multilateral negotiations across apparently similar buyers in narrowly defined markets.

More generally the model provides a tractable foundation for analyzing a variety of issues in strategic settings that feature multilateral negotiations. Section 5 describes some of those future research possibilities and provides concluding remarks, while the Appendix contains all proofs.

2 The Basic Model of Multilateral Negotiations

Consider a buyer that wishes to trade with at most one of N sellers. Trade between the buyer and seller i yields a commonly known surplus $V_i \geq 0$ to be split between the two parties, where $V_1 > 0$ and the sellers are indexed so that $V_1 \geq V_2 \geq \dots \geq V_N$. The buyer and each seller i have instantaneous rates of time preference $r_B > 0$ and $r_i > 0$. Unlike the V_i , the ranking of the r_i across sellers is not tied to the index i .

Trade is conducted as follows, where time is measured in discrete periods $t \in \{0, 1, 2, \dots\}$ that are of length $\Delta > 0$. In even-numbered period $t \in \{0, 2, 4, \dots\}$ each seller i makes a proposal $(s_{i,t}, V_i - s_{i,t}) \in [0, V_i]^2$ to the buyer that specifies the amount $s_{i,t}$ of the surplus V_i that seller i demands for itself, with the remainder $V_i - s_{i,t}$ that it offers to the buyer. The proposals are made simultaneously and are revealed publicly when received by the buyer, after which the buyer decides whether to accept one of the proposals. If the buyer accepts one of the proposals, then the negotiations conclude. If the buyer rejects all of the proposals, then in odd-numbered period $t + 1 \in \{1, 3, 5, \dots\}$ the buyer makes proposals $(b_{i,t+1}, V_i - b_{i,t+1}) \in [0, V_i]^2$ to each seller i that specify the amount $b_{i,t+1}$ of the surplus V_i that the buyer demands for itself, with the remainder $V_i - b_{i,t+1}$ that it offers to seller i . The proposals are made simultaneously and are revealed publicly when received by the sellers, after which the sellers simultaneously decide whether to accept their respective proposals. From the set of accepted proposal the buyer decides which transaction, if any, to consummate.

If the buyer trades with a seller that accepted its proposal, then the negotiations conclude. If no transaction occurs, then play continues to the next period.

A transaction that occurs in even-numbered period t between the buyer and seller i yields respective payoffs $(V_i - s_{i,t})e^{-r_B t \Delta}$ and $s_{i,t}e^{-r_i t \Delta}$, while a transaction that occurs in odd-numbered period t between the buyer and seller i yields respective payoffs $b_{i,t}e^{-r_B t \Delta}$ and $(V_i - b_{i,t})e^{-r_i t \Delta}$. Losing sellers' payoffs are 0. If no transaction occurs in any period, then each party's payoff is 0. For notational convenience define $\delta_k \equiv e^{-r_k \Delta}$, where $\delta_k \in (0, 1)$ is player k 's discount factor, for $k \in \{B, 1, 2, \dots, N\}$.

Three issues must be clarified regarding the acceptance of proposals. First, I allow the buyer not to consummate any transaction even if one or more sellers have accepted a proposal from the buyer. As will become apparent later, this assumption prevents the buyer from extracting the entire surplus V_1 from seller 1 by committing to engage in unfavorable trades. Second, in some period the buyer might accept one of multiple proposals that give it the same payoff. If so, I assume that from those proposals the buyer accepts the one from the seller with the lowest index. Third, in some period several sellers might accept the buyer's proposal. If trade occurs in such an instance, and if multiple proposals accepted by sellers give the buyer its highest payoff, I assume that from those proposals the buyer trades with the seller having the lowest index. The latter two tie-breaking assumptions address an equilibrium existence issue caused by an "openness" problem that arises because the players' allowable demands and offers are drawn from continuous sets.

The preceding model extends the bilateral negotiation model in Rubinstein [1982] by allowing one buyer to negotiate simultaneously with multiple sellers. The assumptions about proposal acceptance and the public nature of the proposals keep the model close in spirit to Rubinstein's model, while allowing for the multilateral aspect.

The players' strategies consist of proposals and accept/reject decisions at every decision point, all of which can depend on all prior moves that led to a specific point in the game. I solve for the game's pure strategy subgame perfect Nash equilibria (SPNE), and for simplicity I focus on *stationary* SPNE outcomes: SPNE outcomes supported by strategies for which one outcome occurs in all subgames that begin with the buyer making offers, and for which one outcome occurs in all subgames that begin with sellers making offers.² Let π_B^{bo} and π_i^{bo} denote the buyer's and seller i 's stationary SPNE payoffs in subgames that begin with offers from the buyer, and let π_B^{so} and π_i^{so} denote the buyer's and seller i 's stationary SPNE payoffs in subgames that begin with offers from sellers.

I modify the approach pioneered by Shaked and Sutton [1984] to characterize this game's stationary SPNE outcomes. Their method for solving the bilateral negotiation model from Rubinstein [1982] uses four variables. The first two variables are $\underline{\pi}_B$ and $\bar{\pi}_B$, which are the infimum and supremum of the set of the buyer's SPNE payoffs, for SPNE in subgames that begin with offers from the buyer. The second two variables are $\underline{\pi}_1$ and $\bar{\pi}_1$, which are the infimum and supremum of the set of seller 1's SPNE payoffs, for SPNE in subgames that begin with offers from the seller.

Shaked and Sutton [1984] characterize the extremal SPNE payoffs by demonstrating that the following four constraints must hold.

$$\underline{\pi}_B \geq V_1 - \delta_1 \bar{\pi}_1 \quad \bar{\pi}_B \leq V_1 - \delta_1 \underline{\pi}_1 \quad \underline{\pi}_1 \geq V_1 - \delta_B \bar{\pi}_B \quad \bar{\pi}_1 \leq V_1 - \delta_B \underline{\pi}_B$$

Roughly speaking, the first and third constraints tell a negotiator, "When you are making offers, you need

²Completely characterizing all SPNE outcomes has proven to be difficult. It appears that the stationary SPNE outcomes are the unique SPNE outcomes for three of the four parameter ranges listed in Theorem 1. In the fourth it appears there are multiple SPNE outcomes. I have characterized some of them, but have been unable to establish I have characterized all of them.

not give too much.” The second and fourth tell a negotiator, “When you are making offers, you cannot take too much.” Manipulating these constraints reveals that each must hold with equality, and that

$$\underline{\pi}_B = \bar{\pi}_B = \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \quad \text{and} \quad \underline{\pi}_1 = \bar{\pi}_1 = \left(\frac{1-\delta_B}{1-\delta_1\delta_B} \right) V_1.$$

Having established the SPNE payoffs are unique, one can show there is a unique SPNE.

The stationary SPNE outcomes with multilateral negotiations are derived by recognizing that versions of the preceding constraints continue to hold, except in particular circumstances in which the credible prospect of trading with seller 2 causes some modification to one or more of the constraints. In some cases these modifications involve introducing an even tighter constraint, and in others involve replacing a constraint with a looser one. Theorem 1 reveals that trade in any subgame occurs between the buyer and seller 1 in the subgame’s initial period.

Theorem 1 *In subgames that begin with the buyer making offers, in all stationary SPNE outcomes the payoffs for the buyer and seller 1 are*

$$\begin{aligned} \pi_B^{bo} &= \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 & \text{and} & \quad \pi_1^{bo} = \delta_1 \left(\frac{1-\delta_B}{1-\delta_1\delta_B} \right) V_1 & \text{if } V_2 \in \left[0, \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \right] \\ \pi_B^{bo} &= \frac{V_2}{\delta_B^2} & \text{and} & \quad \pi_1^{bo} = V_1 - \frac{V_2}{\delta_B^2} & \text{if } V_2 \in \left[\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1, \delta_B^2 V_1 \right] \\ \pi_B^{bo} &= V_1 & \text{and} & \quad \pi_1^{bo} = 0 & \text{if } V_2 \in (\delta_B^2 V_1, \delta_B V_1) \\ \pi_B^{bo} &= V_1 & \text{and} & \quad \pi_1^{bo} = 0 & \text{if } V_2 \in [\delta_B V_1, V_1]. \end{aligned}$$

In subgames that begin with sellers making offers, in all stationary SPNE outcomes the payoffs for the buyer and seller 1 are

$$\begin{aligned} \pi_B^{so} &= \delta_B \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 & \text{and} & \quad \pi_1^{so} = \left(\frac{1-\delta_B}{1-\delta_1\delta_B} \right) V_1 & \text{if } V_2 \in \left[0, \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \right] \\ \pi_B^{so} &= \frac{V_2}{\delta_B} & \text{and} & \quad \pi_1^{so} = V_1 - \frac{V_2}{\delta_B} & \text{if } V_2 \in \left[\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1, \delta_B^2 V_1 \right] \\ \pi_B^{so} &= \delta_B V_1 & \text{and} & \quad \pi_1^{so} = (1-\delta_B) V_1 & \text{if } V_2 \in (\delta_B^2 V_1, \delta_B V_1) \\ \pi_B^{so} &= V_2 & \text{and} & \quad \pi_1^{so} = V_1 - V_2 & \text{if } V_2 \in [\delta_B V_1, V_1]. \end{aligned}$$

The unique stationary SPNE payoffs with multilateral negotiations are more complex than the unique SPNE payoffs with bilateral negotiations, but the derivations of both outcomes share the fundamental concept that a party formulating a proposal must explicitly consider its counterpart’s payoff from rejecting that proposal. With bilateral negotiations the counterpart’s payoff from rejecting corresponds to incurring delay and making a proposal next period. With multilateral negotiations there is the additional influence of the buyer’s ability to trade with seller 2, which in several instances the buyer can use to dramatic effect.

Theorem 1’s characterization of the buyer’s and seller 1’s stationary SPNE payoffs demonstrates that whether seller 2 affects the negotiations depends on the relationship among V_1 , V_2 , δ_1 , and δ_B . Seller 2 is more likely to affect the negotiations as V_1 decreases or V_2 increases, because those changes make sellers 1 and 2 closer competitors in terms of what surplus they can offer the buyer. Seller 2 also is more likely to affect the negotiations as δ_1 increases or δ_B decreases, because those changes decrease the buyer’s relative bargaining strength and the portion of V_1 it would obtain if seller 2 were not present. Consequently, the surplus available from seller 2 is more likely to affect the negotiated outcome.

It is instructive to focus on the novel impact of adding additional sellers to a bilateral setting, which reveals that for low values of V_2 the multilateral negotiations' stationary SPNE payoffs are equivalent to those from Rubinstein's model of bilateral negotiations. Sellers 2 through N are strategically irrelevant to the negotiated outcome when the surpluses available from them are so low, so the negotiations between the buyer and seller 1 proceed as if those other sellers were not present. Figure 1 illustrates this range of V_2 , by plotting the buyer's SPNE payoffs in subgames beginning with offers from the buyer (panel (a)), and in subgames beginning with offers from the sellers (panel (b)). As V_2 increases from 0, for a range of V_2 the buyer's SPNE payoffs remain fixed at their levels from bilateral negotiations.

Figure 1 Here

For intermediate values of V_2 the multilateral negotiations have an unusual feature: when the sellers make offers, the buyer's payoff equals the net present value of what it expects to get next period ($\frac{V_2}{\delta_B}$ versus getting $\frac{V_2}{\delta_B}$ next period). Offering a player just enough to induce trade is a typical feature in bilateral negotiations. However, when the buyer makes offers, seller 1's payoff is less than the net present value of what it expects to get next period. Instead, seller 1's payoff is constrained by the requirement that the buyer not prefer to trade today with seller 2, which is equivalent to the net present value of the buyer's payoff from waiting until the next period (V_2 versus getting $\frac{V_2}{\delta_B}$ next period).

For high values of V_2 the multilateral negotiations' stationary SPNE outcome features the buyer extracting the entire surplus V_1 from seller 1 when the buyer makes offers to the sellers. This is possible because it is credible for the buyer to trade with seller 2 if seller 1 refuses the buyer's offer. Hence, it is as if the buyer can make a take-it-or-leave-it offer to seller 1, which enables the extraction of the entire surplus V_1 . For such values of V_2 , Figure 1 (b) shows that differences in the SPNE outcome emerge for different V_2 based on what seller 1 must offer when the sellers make offers to the buyer. When $\delta_B V_1 > V_2$, the buyer's payoff is $\delta_B V_1$: seller 1 offers the buyer just enough not to reject all offers and get V_1 next period, and trade with seller 2 is not a credible threat. When $\delta_B V_1 < V_2$, the buyer's payoff is V_2 : seller 1 offers the buyer just enough not to trade with seller 2, which is a credible threat rather than rejecting all offers and getting V_1 next period.

At this point it is worth returning to the modeling assumption that the buyer is not committed to trade whenever at least one seller accepts a proposal from the buyer. If the buyer were so committed, then with its proposals the buyer always could extract V_1 from seller 1. To see why, imagine that the buyer offered $\epsilon > 0$ to sellers 1 and 2. In any equilibrium, seller 2 accepts whenever seller 1 rejects, which forces seller 1 to accept rather than receive a payoff of 0. Letting $\epsilon \rightarrow 0$ completes the argument, which illustrates that such commitment effectively allows the buyer to always make seller 1 a take-it-or-leave-it offer. The problem is that the buyer may prefer not to trade with seller 2, if by doing so the buyer's payoff $V_2 - \epsilon$ is less than what the buyer might receive if the negotiations continued to the next period. In the high range of V_2 described in the preceding paragraph, the buyer actually prefers trading today with seller 2 rather than letting the negotiations continue, so there is no sense in which the buyer has somehow committed itself to a trade it would rather reject.

Figure 2 reveals that the equilibrium payoffs in Theorem 1 have some unusual properties, relative to the conventional wisdom that has been derived from bilateral negotiation models. As a function of the discount factor δ_B , the figure plots the buyer's stationary SPNE payoff in multilateral negotiations (the solid line) and the buyer's SPNE payoff in bilateral negotiations (the dashed line).

Figure 2 Here

Panel (a) considers subgames that begin with offers by the buyer. When the buyer is sufficiently impatient, with its offers it actually extracts the entire surplus V_1 from seller 1. This feat is possible because the buyer is so impatient that it can credibly commit to trade today with seller 2 if seller 1 rejects the buyer's offer.³ For intermediate values of δ_B the buyer does worse when it becomes more patient, which contrasts with the conventional wisdom about the effect of patience on bargaining outcomes. Finally, for sufficiently high values of δ_B the buyer's payoff is the same in multilateral and bilateral negotiations, because the buyer is sufficiently patient that it can protect its interests without resorting to the threat of trading with seller 2. Note that the buyer's payoff at the beginning of this range of δ_B is strictly less than its payoff for very low values of δ_B . Only as δ_B approaches 1 does the buyer's payoff return to the same level as when δ_B was small.

Panel (b) considers subgames that begin with offers by the sellers. For low values of δ_B seller 1 anticipates that the buyer will extract the entire surplus V_1 if the negotiations reach the next period. To end the negotiations in the current period and get a positive payoff, seller 1 therefore must offer the buyer at least $\delta_B V_1$. However, if $V_2 > \delta_B V_1$, then the threat from seller 2 requires seller 1 to ensure that the buyer's payoff is V_2 . Hence, for low values of δ_B the buyer's payoff is V_2 , and eventually switches to $\delta_B V_1$ as δ_B increases. Once again, for intermediate values of δ_B the buyer does worse when it becomes more patient, and for high values of δ_B the buyer's payoff is the same in multilateral and bilateral negotiations.

Corollary 1 evaluates the stationary SPNE outcomes from Theorem 1 as the time period between offers $\Delta \rightarrow 0$. Muthoo [1999, Ch. 3.2] suggests this limiting case is the appropriate one to consider, because a party making a counteroffer has incentives to do so quickly to reduce its cost of delay. The limiting values of the payoffs as $\Delta \rightarrow 0$ also do not depend on who makes the initial offers, so there is no first-mover advantage in the limit. Letting $\Delta \rightarrow 0$ simplifies matters by eliminating two of the relevant ranges for V_2 from Theorem 1, and the stationary SPNE payoffs have an intuitively appealing form.

Corollary 1 *As the time period between offers $\Delta \rightarrow 0$, in all stationary SPNE outcomes in all subgames, the payoffs for the buyer and seller 1 approach*

$$\begin{aligned} \pi_B^* &= \left(\frac{r_1}{r_1+r_B}\right) V_1 & \text{and} & \quad \pi_1^* = \left(\frac{r_B}{r_1+r_B}\right) V_1 & \text{if } V_2 \in \left[0, \left(\frac{r_1}{r_1+r_B}\right) V_1\right) \\ \pi_B^* &= V_2 & \text{and} & \quad \pi_1^* = V_1 - V_2 & \text{if } V_2 \in \left[\left(\frac{r_1}{r_1+r_B}\right) V_1, V_1\right]. \end{aligned}$$

These can be written more compactly as

$$\pi_B^* = \max \left[V_2, \left(\frac{r_1}{r_1+r_B}\right) V_1 \right] \quad \text{and} \quad \pi_1^* = \min \left[V_1 - V_2, \left(\frac{r_B}{r_1+r_B}\right) V_1 \right].$$

In the limit, a player's stationary SPNE payoff in a subgame does not depend on whether the buyer or the sellers are making offers.

Corollary 1 completely characterizes the buyer's and seller 1's payoffs in all stationary SPNE outcomes as the time period between offers $\Delta \rightarrow 0$. In those SPNE the division of the maximum surplus V_1 depends on whether seller 2 constrains the negotiations. If V_2 is large enough to constrain the negotiations, then the buyer's payoff is higher than it would be if seller 2 were not present, and seller 1's payoff is lower. This is direct evidence of the benefit to the buyer of bringing an additional seller into an initially bilateral

³As mentioned earlier, if the game's structure required the buyer always to trade according to its best accepted offer, then the buyer could always extract V_1 when making offers: Seller 2 will accept any offer giving it a strictly positive payoff whenever seller 1 rejects, so seller 1 could never reject an extremely unfavorable offer.

negotiation setting. If V_2 is small enough not to constrain the negotiations, then the buyer's and seller 1's payoffs are identical to those in Rubinstein's model. In fact, setting $V_2 = 0$ gives Rubinstein's bilateral negotiation model as a special case.

Comparative statics regarding the players' stationary SPNE payoffs, with respect to the discount rates, also depend on whether seller 2 constrains the negotiations. If V_2 is large enough to constrain the negotiations, then changes in the buyer's and seller 1's discount rates do not affect the parties' payoffs. For example, one special case of seller 2 constraining the negotiations is when $V_1 = V_2$, in which case the buyer gets all of the surplus and the discount rates are irrelevant. If V_2 is small enough not to constrain the negotiations, then standard comparative statics emerge: as the buyer becomes more patient it does strictly better in the negotiations and seller 1 does strictly worse, while the reverse holds as seller 1 becomes more patient.

One thing to notice from the results in Theorem 1 and Corollary 1 is that seller 2's patience is irrelevant to the negotiated outcome, even though the surplus V_2 available from seller 2 might play a role. All that matters is that seller 2 is ready to give the entire surplus V_2 to the buyer when making an offer, and is willing to accept any offer from the buyer. Therefore, seller 2's patience is irrelevant because its behavior is driven entirely by consideration of its payoff in the current period.

In Section 4 I use the unique limiting values of the stationary SPNE payoffs π_B^* and π_1^* to evaluate the buyer's choice of conducting procurement via multilateral negotiations or an auction. The limiting case seems appropriate given the points raised immediately prior to Corollary 1, and the unique stationary SPNE outcome seems to be natural one to consider. In Section 4 I also exploit the fact that π_B^* and π_1^* can be written as functions of the ratio of discount rates, $\frac{\tau_B}{\tau_1}$, which reflects the importance of relative bargaining strength, rather than absolute. Before proceeding to that analysis, I place this new model in context.

3 Relationship to Prior Research

Earlier research considers various extensions of bilateral negotiation models to address the idea of a buyer negotiating with multiple sellers. Below I describe representative examples that use models or ask questions similar to mine, for the purpose of comparing their structure and predictions with the model described in Theorem 1.^{4,5} The models that explicitly incorporate multiple sellers assume the sellers are identical, which conforms to the special case of my model in which $V_1 = V_2 = \dots = V_N$.

Shaked and Sutton [1984] model bilateral negotiations with complete information and alternating offers. The buyer bargains with one seller at a time, but can switch to a different seller after some commonly known length of time. Offers from one seller are void upon switching to another seller. The authors find that the presence of another seller constitutes a credible threat that permits the buyer to obtain greater surplus than if switching were impossible. The outcome as the switching delay approaches the length of a bargaining period is identical to the outcome if the buyer instead conducted a first-price auction between the two sellers. My model lets the buyer bargain simultaneously with multiple sellers, and the negotiations and an auction

⁴Other papers that share some similarities include Reinganum and Daughety [1991, 1992], Hendon and Tranaes [1991], Muthoo [1995], Chatterjee and Dutta [1998], and Marx and Shaffer [2010]. Two recent papers that consider multilateral negotiations with incomplete information are McAdams and Schwarz [2007] and Thomas [2011]. See Thomas [2011] for references to additional papers with incomplete information that consider some aspects of the multilateral negotiation problem analyzed there.

⁵For clarity's sake it is worth mentioning that these models and mine address an exchange problem that is distinct from the multilateral bargaining considered by authors such as Krishna and Serrano [1996], in which multiple players bargain to split one joint surplus. That setting emphasizes different issues and is inherently different from the one I am considering.

have the same outcome only if seller 2 constrains the negotiated outcome.⁶

Binmore, Shaked, and Sutton [1989] modify Rubinstein’s bilateral negotiation model so that the buyer can take up a commonly known and exogenously specified outside option whenever it rejects the seller’s offer. Shaked [1994] assumes the buyer can take up its outside option only after the seller rejects the buyer’s offer. Both models’ equilibrium payoffs share similarities with those in Theorem 1 if the payoff from the exogenous outside option equals V_2 . Despite the similarity of the three modeling approaches, there are at least two reasons to prefer Section 2’s model of endogenous outside options. First, the latter model illustrates how the outside option is determined, that the payoff from the outside option represents the *entire surplus* available from that alternative trade, and that the patience of the alternative trading partner is irrelevant. Second, applications like those in Sections 4 and 5 make sense only if the outside options result from decisions by strategic actors.

Vincent [1992] considers simultaneous negotiations by adding an additional seller to a bilateral negotiation setting with complete information in which only the sellers make offers. He finds that the buyer may not secure all gains from trade, due to the sellers’ ability to support collusive equilibria through the prospect of potentially an infinite number of bargaining periods. My model differs from his by allowing both the buyer and the sellers to make offers. This crucial distinction enables a seller to deviate profitably from collusive behavior by accepting an offer that ends the game, thereby avoiding retaliatory future punishment from rival sellers.

Finally, Chatterjee and Lee [1998] model bilateral negotiations with complete information in which the buyer can hold an offer from one seller while it incurs a cost to acquire an offer from another seller. The important distinction from my model is that the authors assume the competing offer is a draw from a commonly known and exogenously specified probability distribution, rather than being the outcome of strategic interaction with another seller.

4 Multilateral Negotiations Versus Auctions

In practice the buyer must choose a method for conducting procurement, and the reality of such decisions is one reason economists emphasize understanding the performance of different voluntary exchange mechanisms.⁷ In this section I use the unique stationary SPNE payoffs from Corollary 1 to directly compare two commonly used procurement mechanisms: multilateral negotiations and auctions.

Bulow and Klemperer [1996] analyze a buyer’s choice between negotiations and auctions in a setting with homogeneous products and incomplete information about sellers’ costs. They illustrate circumstances in which a buyer prefers a simple second-price auction with $N + 1$ sellers to an optimal mechanism with N sellers. The authors do not analyze a specific negotiation protocol, but let the optimal mechanism provide an upper bound on the buyer’s payoff from negotiating. Their results suggest that attracting more sellers can be more important than increasing the buyer’s bargaining strength.^{8,9}

Prompted in part by the observation that multilateral negotiations and auctions are used by apparently similar buyers in the same market, Thomas and Wilson [2002, 2005, 2012] use experiments to compare

⁶As it would in the special case of my model that conforms to theirs, in which $V_1 = V_2 = \dots = V_N$.

⁷For example, consider how the work by Chamberlin [1948] and Smith [1962] spurred research on the importance of institutions in determining economic outcomes.

⁸I do not address this issue in the analysis to follow, but one could do so. It is straightforward to generate examples that give the opposite conclusion, which suggests the importance of incomplete information in making such comparisons.

⁹More recently, Bulow and Klemperer [2009] consider this choice when the mechanisms involve costly entry but differ in their timing. Their auctions involve simultaneous entry decisions followed by price competition. Their negotiations involve sequential entry by sellers, where later sellers enter only if trade with earlier sellers is insufficiently attractive.

both exchange mechanisms. In settings with incomplete information they consider first-price and second-price auctions, homogeneous and differentiated products, and negotiations that differ in the buyer's ability to provide verifiable information to a seller about offers from rival sellers. The authors find that the mechanisms' outcomes are similar, and in some cases are statistically indistinguishable. They conduct experiments because no formally solved models were available that considered the sort of negotiations in which they were interested.

I evaluate the buyer's choice by extending Section 2's basic model in ways that make the institutional comparison nontrivial; it will be evident that in the basic model the buyer weakly prefers multilateral negotiations. This section's goal is to provide some simple extensions that yield theoretical and empirical insights, and that also suggest more complex or comprehensive extensions.

Three phases comprise the extended game: The *surplus-determination phase* consists of a move by Nature that determines the V_i draws according to some commonly known probability distribution. The *planning phase* consists of three choices that occur in a specific order: First, the buyer chooses publicly between conducting multilateral negotiations or a first-price auction. Second, the buyer chooses publicly which of the N sellers to invite to participate. Third, the invited sellers make simultaneous and public decisions whether to participate. The *procurement phase* consists of competition amongst the participating sellers. That competition is conducted via either the multilateral negotiations described in Section 2, or a first-price auction in which each seller i makes a simultaneous price offer $p_i \in [0, V_i]$ to the buyer. The buyer purchases from the seller whose offer gives the buyer its highest payoff, $V_i - p_i$, with ties handled in the same fashion as in the multilateral negotiations. It is straightforward to establish that the buyer's payoff in a first-price auction is the second-highest surplus from the set of participating sellers.¹⁰

The procurement phase occurs last in the extensions I consider below, but either the surplus-determination or the planning phase can occur first. The order of those two phases determines whether the buyer and sellers make their planning choices after or before they know the available surpluses.

4.1 Institutional Choice with Fixed V_i

Consider the case in which the surplus-determination phase occurs before the planning phase, and in which the buyer incurs an incremental cost of conducting multilateral negotiations versus an auction, $C^{mln} \in \mathbb{R}$. I assume the buyer invites all sellers, and that all invited sellers participate, because participation is costless. The only substantive difference from the basic model is the inclusion of the cost C^{mln} .¹¹

From Corollary 1 the buyer's payoff in multilateral negotiations is

$$\pi_B^{mln} = \pi_B^* - C^{mln} = \max \left[V_2, \left(\frac{r_1}{r_1 + r_B} \right) V_1 \right] - C^{mln},$$

while its payoff in a first-price auction is

$$\pi_B^{fpa} = V_2.$$

The following result is straightforward to obtain.

Proposition 1 *Consider the case in which the surplus-determination phase precedes the planning phase.*

¹⁰For example, see Kim and Che [2004]. In this complete information setting the buyer's payoff also is the same in a Dutch, English, or second-price auction.

¹¹One also could include costs incurred by the buyer for each seller that participates. In Subsection 4.3 I consider costs incurred by each seller that participates.

The buyer strictly prefers multilateral negotiations if and only if

$$C^{mln} < \max \left[0, \left(\frac{r_1}{r_1 + r_B} \right) V_1 - V_2 \right].$$

The buyer's relative preference for multilateral negotiations intensifies as C^{mln} decreases, as V_1 increases or V_2 decreases, and as $\frac{r_B}{r_1}$ decreases.

This simple extension of the basic model illustrates what factors make the buyer more inclined to use multilateral negotiations. Figure 3 shows the buyer's payoff in both institutions, as a function of the ratio of discount rates $\frac{r_B}{r_1}$, for some positive incremental cost of conducting multilateral negotiations, C^{mln} . Each panel shows a parameter change that increases the range of $\frac{r_B}{r_1}$ for which the buyer prefers multilateral negotiations. Panel (a) shows that decreasing C^{mln} has the obvious effect of increasing multilateral negotiations' attractiveness, because they become cheaper to conduct relative to an auction. Panel (b) shows that increasing V_1 increases the buyer's payoff from multilateral negotiations while leaving unchanged its payoff from a first-price auction. Increasing V_1 in multilateral negotiations increases the amount of surplus to be split between the buyer and seller 1, but in a first-price auction has no effect because the buyer's payoff is determined solely by V_2 . Panel (c) shows that decreasing V_2 lowers the buyer's payoff in both institutions, but to different extents. In a first-price auction decreasing V_2 reduces the buyer's payoff one-for-one, because the buyer's payoff is V_2 . In multilateral negotiations decreasing V_2 leaves the buyer's payoff unchanged in the original instances in which seller 2 is irrelevant to the negotiated outcome, decreases the buyer's payoff less than one-for-one in a range of cases in which seller 2 initially constrained the negotiations, and otherwise decreases the buyer's payoff one-for-one. The net effect makes multilateral negotiations more attractive for a larger range of relative discount rates, $\frac{r_B}{r_1}$.

Figure 3 Here

Finally, each panel of Figure 3 illustrates that multilateral negotiations are relatively more attractive as $\frac{r_B}{r_1}$ decreases, which is a crucial element of the buyer's choice between multilateral negotiations and an auction. Reducing $\frac{r_B}{r_1}$ reflects an increase in the buyer's bargaining strength relative to seller 1, and by conducting an auction the buyer forgoes any bargaining advantage it might have.

4.2 Institutional Choice with Random V_i and Costless Participation

Now consider the case in which the planning phase precedes the surplus-determination phase. The available surpluses are unknown when the buyer chooses the procurement mechanism and which sellers to invite, and when the sellers make their participation decisions. For simplicity I assume the V_i are independently and identically drawn from $U[0, 1]$. I also assume $r_i = r_S$ for each seller i , because otherwise the details of the institutional choice depend in a straightforward but tedious way on the values of the r_i and the realized V_i . Finally, I assume the buyer invites all sellers, and that all invited sellers participate, because participation is costless.

The buyer's payoff in multilateral negotiations depends on the highest and second-highest realized V_i , while its payoff in a first-price auction depends on the second-highest realized V_i . To calculate the buyer's expected payoffs one therefore must consider distributions of the highest and second-highest order statistics of N draws from $U[0, 1]$. Denoting those order statistics by $V_{(1)}$ and $V_{(2)}$, the distribution of $V_{(1)}$ is

$$G_1(v) \equiv \Pr(V_{(1)} \leq v) = v^N,$$

the distribution of $V_{(2)}$ conditional on the value of $V_{(1)}$ is

$$G_2(v | w) \equiv \Pr(V_{(2)} \leq v | V_{(1)} = w) = \frac{v^{N-1}}{w^{N-1}},$$

and the unconditional distribution of $V_{(2)}$ is

$$G_2(v) \equiv \Pr(V_{(2)} \leq v) = v^N + N(1-v)v^{N-1}.$$

Corollary 1 gives the buyer's payoff for any particular realization of the V_i , so the buyer's expected payoff in multilateral negotiations is

$$\begin{aligned} \pi_B^{mln} &= \left[\int_0^1 \left\{ \int_0^{v_{(1)}} \max \left[v_{(2)}, \left(\frac{r_S}{r_S + r_B} \right) v_{(1)} \right] G_2'(v_{(2)} | v_{(1)}) dv_{(2)} \right\} G_1'(v_{(1)}) dv_{(1)} \right] - C^{mln} \\ &= \frac{N-1 + \left(\frac{r_S}{r_S + r_B} \right)^N}{N+1} - C^{mln}. \end{aligned}$$

Similarly, the buyer's expected payoff in a first-price auction is

$$\begin{aligned} \pi_B^{fpa} &= \int_0^1 v_{(2)} G_2'(v_{(2)}) dv_{(2)} \\ &= \frac{N-1}{N+1}. \end{aligned}$$

Proposition 2 *Consider the case in which the planning phase precedes the surplus-determination phase, and the V_i are iid draws from $U[0, 1]$. The buyer strictly prefers multilateral negotiations if and only if*

$$C^{mln} < \frac{\left(\frac{r_S}{r_S + r_B} \right)^N}{N+1}.$$

The buyer's relative preference for multilateral negotiations intensifies as C^{mln} decreases, as N decreases, and as $\frac{r_B}{r_S}$ decreases.

Proposition 2 illustrates that the buyer tends to prefer multilateral negotiations for low values of C^{mln} , N , and $\frac{r_B}{r_S}$. Decreasing C^{mln} has the obvious effect of making multilateral negotiations relatively cheaper to conduct. Decreasing N increases the expected difference between the highest and second-highest surpluses. As Corollary 1 demonstrates, as that gap increases it is more likely that negotiations outperform an auction. Likewise, decreasing $\frac{r_B}{r_S}$ strengthens the buyer's bargaining position, an advantage that is forgone by conducting an auction.

One also can assess how changing the variance of the V_i draws affects the buyer's institutional choice, say by considering mean-preserving changes in the distribution. A formal analysis is messy, even with the V_i uniformly distributed, but the following informal argument demonstrates that increasing the variance increases the relative attractiveness of multilateral negotiations.

Suppose the V_i are independently and identically drawn from $U\left[\frac{1}{2} - \theta, \frac{1}{2} + \theta\right]$ for $\theta \in \left[0, \frac{1}{2}\right]$. If θ is sufficiently small such that $\frac{1}{2} - \theta \geq \left(\frac{r_S}{r_S + r_B}\right)\left(\frac{1}{2} + \theta\right)$, then seller 2 constrains the negotiations for any realizations of the V_i . That is, even for the highest possible value of $V_{(1)}$ ($= \frac{1}{2} + \theta$) and the lowest possible

value of $V_{(2)}$ ($= \frac{1}{2} - \theta$), $V_{(2)}$ exceeds the buyer's share of $V_{(1)}$ from bilateral negotiations. Consequently, the buyer's expected payoff is the same with multilateral negotiations or an auction, not counting any differential cost of conducting negotiations.¹² This informal analysis suggests that negotiations are more likely to be used when sellers are more likely to differ in their available surplus, such as when the buyer's tastes tend to be quite distinct for different sellers' products, or when there is significant cost variability across sellers. Multilateral negotiations outperform auctions in such circumstances because the buyer's bargaining ability strongly influences the negotiated outcome, and that ability is wasted by conducting an auction.

4.3 Institutional Choice with Random V_i and Costly Participation

Finally, consider extending the preceding subsection's analysis by having sellers incur a cost of participating before they learn the V_i draws, $C^{entry} \in \mathbb{R}^+$. This cost can reflect opportunity costs of designing prototypes, evaluating production costs, assessing the product's fit with the buyer's preferences, or formulating an initial proposal. I assume $C^{mln} = 0$ to focus attention on the uncertainty when decisions are made.

Costly entry has been considered in other oligopoly models, mainly to determine its effect on prices and welfare.¹³ A common finding is that the buyer may benefit from decreasing the size of the pool of potential entrants, which in my extended model amounts to inviting fewer than N sellers to participate.

As a first step I consider the sellers' participation decisions when the buyer invites all N sellers to participate, for each institution $x \in \{mln, fpa\}$. Let $\pi_S^x(n)$ denote a seller's expected payoff from participating in institution x when a total of n sellers participate.

If $C^{entry} < \pi_S^x(N)$, then each seller participates with probability 1 because it expects a positive net payoff even if all of its rivals participate. With the V_i drawn from $U[0, 1]$,

$$\pi_S^{mln}(N) = \frac{1 - \left(\frac{r_S}{r_S + r_B}\right)^N}{N(N+1)} \quad \text{and} \quad \pi_S^{fpa}(N) = \frac{1}{N(N+1)}.$$

Because $\pi_S^{mln}(N) < \pi_S^{fpa}(N)$, in multilateral negotiations sellers stop participating with probability 1 at a lower value of C^{entry} than in an auction.

If $C^{entry} > \pi_S^x(1)$, then each seller participates with probability 0 because it expects a negative net payoff even if none of its rivals participate. Once again exploiting the Uniform distribution of the V_i ,

$$\pi_S^{mln}(1) = \left(\frac{r_B}{r_S + r_B}\right) \left(\frac{1}{2}\right) \quad \text{and} \quad \pi_S^{fpa}(1) = \frac{1}{2}.$$

Because $\pi_S^{mln}(1) < \pi_S^{fpa}(1)$, in multilateral negotiations sellers stop participating entirely at a lower value of C^{entry} than in an auction. For later reference note that these values of C^{entry} are invariant to the number of invited sellers, because they are determined solely from a seller's expected payoff when it is the only participating seller.

Finally, if $\pi_S^x(N) \leq C^{entry} \leq \pi_S^x(1)$, then I consider strategies such that each seller participates with probability $\rho^x \in (0, 1)$. The entry probability depends on the number of invited sellers and the value of C^{entry} .¹⁴ The equilibrium value of ρ^x equates a seller's expected payoff from participating and not

¹²The informal analysis is messy for larger values of θ , because in calculating the buyer's expected payoff one must account for whether the realizations of the V_i are such that seller 2 constrains the negotiations.

¹³ For example, costly entry is examined in a Bertrand setting by Lang and Rosenthal [1991], in a Cournot setting by Dixit and Shapiro [1986], and in an auction setting by Levin and Smith [1994].

¹⁴While there exist asymmetric equilibria in which a subset of invited sellers participate with probability 0, I focus on

participating. Specifically, ρ^x solves

$$\left[\sum_{k=0}^{N-1} \binom{N-1}{k} (\rho^x)^k (1-\rho^x)^{n-1-k} \pi_S^x(k+1) \right] - C^{entry} = 0.$$

Once the entry probability is derived, the buyer's expected payoff is

$$\pi_B^x = \sum_{k=0}^N \binom{N}{k} (\rho^x)^k (1-\rho^x)^{n-1-k} \pi_B^x(k),$$

where $\pi_B^x(k)$ denotes the buyer's expected payoff from institution x when a total of k sellers participate.

Even with the simple expressions for $\pi_B^x(k)$ that precede Proposition 2 (substituting k for N as appropriate), the equilibrium values of ρ^x and π_B^x do not have convenient analytic expressions. However, the equilibrium value of ρ^x can be solved numerically, from which π_B^x is easily calculated.

Figure 4 reports π_B^{mln} and π_B^{fpa} as a function of C^{entry} , with panels (a)-(c) featuring different numbers of invited sellers. Each panel shows π_B^{mln} for three values of $\frac{r_B}{r_S}$, while π_B^{fpa} is invariant to the parties' discount rates.

Figure 4 Here

The first insight from Figure 4 is that the buyer strictly prefers multilateral negotiations for low entry costs, and strictly prefers a first-price auction for high entry costs. From Proposition 2 the buyer prefers multilateral negotiations over an auction for any specific number of participants when $C^{mln} = 0$, and both institutions' efficiency implies that the sellers prefer an auction for any specific number of participants. For sufficiently low entry costs, all invited sellers participate in both institutions. Therefore, the buyer prefers multilateral negotiations. As the entry cost increases further, sellers in multilateral negotiations participate less frequently than they do in an auction. The decreased participation harms the buyer directly in the negotiations because with fewer sellers the expected value of the highest surplus declines. Eventually this decline swamps the buyer's advantage from its bargaining ability. This effect is clearly seen in the extreme when C^{entry} is such that no sellers enter with negotiations, but some sellers might enter with an auction.

Finding that the buyer's preferred procurement method depends on entry costs illustrates an important reason for considering costly entry. Levin and Smith [1994] found no such effects in their analysis of costly entry with different auction formats, because the formats they considered all were revenue equivalent for any particular number of actual entrants. Hence, the sellers' entry probabilities were the same across institutions. In contrast, the fact that payoffs in multilateral negotiations and a first-price auction differ for any specific number of participating sellers causes the buyer's preference to depend on the level of C^{entry} .

The second insight from Figure 4 is that the value of C^{entry} at which the buyer's institutional choice changes increases as the buyer's relative bargaining strength diminishes. Extreme values of $\frac{r_B}{r_S}$ illustrate this point clearly. As $\frac{r_B}{r_S} \rightarrow 0$ the buyer prefers a first-price auction for any $C^{entry} > 0$, because the buyer's overwhelming bargaining ability ensures that no sellers participate if the buyer uses multilateral negotiations. As $\frac{r_B}{r_S} \rightarrow \infty$ multilateral negotiations and an auction become more similar for any specific number of participating sellers. Hence, the sellers' participation probabilities also get more similar, and the value of C^{entry} increases at which the buyer's preferred choice changes.

symmetric equilibria because of the strategic environment's symmetry.

Careful examination of Figure 4 reveals that the buyer’s expected payoff is higher with fewer invited sellers, for some values of C^{entry} . This result is consistent with findings mentioned earlier from Lang and Rosenthal [1991] and Levin and Smith [1994], and it stems from the interplay between the number of invited sellers and the sellers’ probability of participating. One implication of this finding is that the buyer’s ability to restrict the number of sellers it invites may eliminate the change in the buyer’s institutional choice that was highlighted above. If C^{entry} is such that the buyer prefers an auction to multilateral negotiations for a certain number of invited sellers, the buyer might still prefer multilateral negotiations after suitably reducing the number of invited sellers.

Figure 5 illustrates the buyer’s expected payoff when it optimally chooses how many sellers to invite, restricting attention to a setting in which there are $N = 10$ sellers available. Each panel reflects a different ratio of the discount rates, $\frac{r_B}{r_S}$. Each line in Figure 5 is the upper envelope from plotting the buyer’s expected payoff for each number of invited sellers for a particular institution, such as the three reported in Figure 4.

Figure 5 Here

The first insight from Figure 5 is that, even optimizing over the number of invited sellers, the buyer prefers multilateral negotiations for sufficiently low values of C^{entry} , and prefers a first-price auction for sufficiently high values of C^{entry} . For very low values of C^{entry} , all invited sellers participate even if all sellers are invited. In this case the buyer invites all sellers and prefers multilateral negotiations, as shown in Proposition 2. For values of $C^{entry} \in (\pi_S^{mln}(1), \pi_S^{fpa}(1))$, no sellers participate in multilateral negotiations, but participate with positive probability in a first-price auction. Consequently, for such values of C^{entry} the buyer prefers a first-price auction to multilateral negotiations.

The second insight from Figure 5 is that the specifics of the buyer’s institutional choice are murky for intermediate values of C^{entry} . While there obviously is a change at some point from preferring multilateral negotiations to preferring an auction, there can be multiple such changes as C^{entry} increases. For example, in panel (b) the buyer’s optimal choice is to invite 2 sellers and use multilateral negotiations when $C^{entry} = 0.14$, to invite 2 sellers and use a first-price auction when $C^{entry} = 0.17$, and to invite 1 seller and use multilateral negotiations when $C^{entry} = 0.23$. Once $C^{entry} > 0.25$ the buyer switches back to inviting 2 sellers and using a first-price auction.

Finally, one could assess how the effects of costly entry are influenced by the variance of the V_i draws. An informal analysis similar to that in Subsection 4.2 suggests that the two institutions are equivalent if the variance of the V_i draws is sufficiently low. Hence, entry costs would not affect the buyer’s institutional choice.

5 Conclusions and Future Research

This paper develops a model of multilateral negotiations with complete information by adding additional sellers to Rubinstein’s classic infinite-horizon alternating-offers model of bilateral negotiations. The model’s unique stationary SPNE outcomes involve the buyer trading immediately with the surplus-maximizing seller.

The presence of additional sellers in certain instances provides a measure of protection to the buyer that enables it to receive higher payoffs than it otherwise would. In other instances the additional sellers are irrelevant, because the surplus available from trading with them is too low to give the buyer a credible threat to trade with them if seller 1 is intransigent. In the limit as the time between offers goes to 0, these stationary outcomes have an intuitively appealing structure that is consonant with the SPNE outcome from

bilateral negotiations: The prospect of trade with seller 2 constrains the multilateral negotiations if and only if V_2 exceeds what the buyer would obtain in bilateral negotiations with seller 1.

I apply the model to the buyer's choice between conducting procurement via multilateral negotiations or an auction. As a general matter the buyer tends to prefer multilateral negotiations when the buyer is relatively patient, when the sellers' products are distinct, and when there are few sellers. In such settings the buyer's bargaining ability plays a significant role in the negotiations, and the buyer forgoes that ability by conducting procurement with an auction. However, the buyer prefers an auction if it is sufficiently costly for sellers to participate in the procurement process, because sellers' anticipated low payoffs in multilateral negotiations make them less likely to participate than if procurement were conducted with an auction. These differences in the buyer's institutional preference are maintained if the buyer can strategically limit the number of sellers it invites to participate, and a new one emerges: For intermediate entry costs the buyer can prefer either institution, and the preferred choice can change multiple times as the entry cost increases.

The analysis of the buyer's choice helps explain why multilateral negotiations and auctions are used by apparently similar buyers. In some instances the two institutions might coexist because each gives the buyer the same expected payoff, such as when there is little variation in the surpluses available from different sellers. In other instances buyers might have distinct preferences that depend on factors including variation in the cost of sellers' participation across buyers (say because of complexity in determining a seller's fit with the buyer's preferences), differences in the buyers' bargaining ability relative to the sellers', or differences in the buyers' costs of conducting multilateral negotiations.

The model illustrates a setting in which multilateral negotiations can be evaluated straightforwardly, and the following examples give a sense of the variety of ways in which it can be applied. One could use the model to evaluate horizontal mergers in markets where the strategic interaction among firms involves multilateral negotiations. The basic model is sufficiently simple that one could include merger-specific efficiencies or changes in bargaining positions, changes in product offerings from the merging firms or their rivals, or entry by new sellers. Likewise, one could consider collusion amongst sellers who repeatedly compete for the business of buyers. The model's emphasis on surplus allows consideration of cost differences across sellers, or of horizontal and vertical product differentiation. One also could consider incentives regarding dual-sourcing, sellers' investments to reduce costs or improve their products' goodness-of-fit with the buyer's preferences, the buyer's design of its purchasing requirements, or joint decisions between the buyer and each seller on relationship-specific investments. Finally, one could use the model as a starting point for designing experiments. In addition to comparing experimental outcomes to the model's predictions regarding parameters such as the available surpluses, the parties' discount rates, or the number of sellers, one also can assess whether a structured model of multilateral negotiations can reasonably explain the outcomes of more realistic unstructured negotiations.

6 Appendix

This appendix contains proofs of Theorem 1, Corollary 1, and Propositions 1 and 2. It begins with several lemmas that are used in the proof of Theorem 1.

Let π_k^{so} and π_k^{bo} respectively denote player k 's stationary SPNE payoffs in subgames that begin with offers from the sellers and from the buyer, for $k \in \{B, 1, \dots, N\}$. I will frequently refer to SPNE strategies by $\sigma \equiv \{\sigma_B, \sigma_1, \sigma_2, \dots, \sigma_N\}$, where each σ_k denotes player k 's strategy in the infinite-horizon game.

Lemma 1 *In subgames beginning with offers from the sellers, all stationary SPNE outcomes involve trade*

between the buyer and seller 1 in the subgame's initial period.

Proof of Lemma 1: In subgames beginning with offers from the sellers, I first show that all stationary SPNE outcomes involve trade between the buyer and seller 1. The proof involves three cases.

Case 1: In subgames beginning with offers from the sellers, suppose there exists a SPNE σ that involves trade between the buyer and a seller $k \neq 1$ for which $V_k = V_1$. Relabel the sellers so that trade involves seller 1.

Case 2: In subgames beginning with offers from the sellers, suppose there exists a SPNE σ that involves trade between the buyer and a seller k for which $V_k < V_1$. By following σ , the buyer's payoff is π_B^{so} , seller k 's payoff is π_k^{so} , and seller 1's payoff is 0. Note that $\pi_B^{so} + \pi_k^{so} \leq V_k < V_1$.

In the initial period of a subgame in which σ is to be followed, suppose seller 1 offers the buyer $V_1 - \epsilon > \pi_B^{so}$ for some $\epsilon > 0$. Such an ϵ must exist because $\delta_B < 1$ and $\pi_B^{so} < V_1$. Note also that $\pi_B^{so} \geq \delta_B \pi_B^{bo}$, because the buyer always can reject all offers and let play continue to the next period. The buyer strictly prefers accepting seller 1's offer, because the payoff $V_1 - \epsilon$ the buyer receives by accepting strictly exceeds both its payoff from accepting seller k 's offer (which is less than or equal to π_B^{so} , depending on whether σ specified a trade in the subgame's initial period), and its payoff from rejecting all offers and letting play continue to the next period (which is worth $\delta_B \pi_B^{bo}$ from the perspective of the subgame's initial period). Seller 1 strictly prefers offering $V_1 - \epsilon$, because doing so gives it a strictly higher payoff than from following σ , $\epsilon > 0$. Therefore, there cannot exist a SPNE σ according to which the buyer trade with some seller $k \neq 1$ for which $V_k < V_1$, in subgames beginning with offers from the sellers.

Case 3: In subgames beginning with offers from the sellers, suppose there exists a SPNE σ that involves no trade in any period. By following σ , $\pi_B^{so} = \pi_1^{so} = \pi_B^{bo} = \pi_1^{bo} = 0$. In the initial period of a subgame in which σ is to be followed, suppose seller 1 offers the buyer $V_1 - \epsilon > 0$ for some $\epsilon > 0$. The buyer strictly prefers accepting seller 1's offer, because the buyer's payoff from accepting strictly exceeds both its payoff from letting play continue to the next period, and its payoff from accepting any other seller's offer according to σ (both of which must be 0, else the buyer could not receive a payoff of 0 according to σ). Seller 1 strictly prefers offering $V_1 - \epsilon$, because doing so gives it a strictly higher payoff than from following σ , $\epsilon > 0$. Therefore, there cannot exist a SPNE σ according to which there is no trade in any period, in subgames beginning with offers from the sellers.

Cases 1-3 exhaust all possibilities other than trading with seller 1, and so in all subgames that begin with offers from the sellers, all stationary SPNE involve trade with seller 1.

I next show that all stationary SPNE outcomes involve trade in the subgame's initial period, in subgames beginning with offers from the sellers. Suppose strategy σ supports the stationary SPNE outcome with payoffs π_k^{so} and π_k^{bo} , but that trade does not occur in the subgame's initial period. Because of discounting, this delay in trading implies $\pi_1^{so} + \pi_B^{so} < V_1$. In the initial period of an arbitrary subgame in which σ is to be followed, suppose seller 1 offers the buyer $\pi_B^{so} + \epsilon$ for some $\epsilon > 0$ such that $\pi_1^{so} + \pi_B^{so} + \epsilon < V_1$. The buyer strictly prefers accepting seller 1's offer, because the payoff it receives by accepting strictly exceeds both its payoff from trading with another seller (each of whom are making offers according to σ), and its payoff from rejecting all offers and letting play continue to the next period (because $\pi_B^{so} + \epsilon > \pi_B^{so} \geq \delta_B \pi_B^{bo}$). Seller 1 strictly prefers offering $\pi_B^{so} + \epsilon$, because doing so gives it a strictly higher payoff than from following σ , $V_1 - (\pi_B^{so} + \epsilon) > \pi_1^{so}$, where π_1^{so} is its payoff from following σ . Therefore, there cannot exist a SPNE σ according to which trade does not occur in the subgame's initial period, in subgames beginning with offers from the sellers. ■

Lemma 2 *In subgames beginning with offers from the buyer, all stationary SPNE outcomes involve trade between the buyer and seller 1 in the subgame's initial period.*

Proof of Lemma 2: In subgames beginning with offers from the buyer, I first show that all stationary SPNE outcomes involve trade between the buyer and seller 1. The proof involves three cases.

Case 1: In subgames beginning with offers from the buyer, suppose there exists a SPNE σ that involves trade between the buyer and a seller $k \neq 1$ for which $V_k = V_1$. Relabel the sellers so that trade involves seller 1.

Case 2: In subgames beginning with offers from the buyer, suppose there exists a SPNE σ that involves trade between the buyer and a seller k for which $V_k < V_1$. By following σ , the buyer's payoff is π_B^{bo} , seller k 's payoff is π_k^{bo} , and seller 1's payoff is 0. Trade must occur in the initial period of an arbitrary subgame in which σ is to be followed, because by Lemma 1 trade must involve the buyer and seller 1 if play continues to the subgame's next period. Consequently, the payoffs from following σ are such that $\pi_B^{bo} + \pi_k^{bo} = V_k < V_1$.

If $\pi_B^{bo} > \delta_B \pi_B^{so}$, then the following offers give the buyer a strictly higher payoff than from following σ . The buyer offers V_j to each seller $j \neq 1, k$, and offers $\epsilon > 0$ to sellers 1 and k such that $V_1 - \epsilon > V_k - \epsilon \geq \pi_B^{bo} - \epsilon > \delta_B \pi_B^{so}$. The second inequality must hold because $V_k \geq \pi_B^{bo}$. If seller k accepts and seller 1 rejects, then the buyer strictly prefers trading with seller k rather than letting play continue to the next period, because $V_k - \epsilon > \delta_B \pi_B^{so}$. If seller 1 accepts, then the buyer strictly prefers trading with seller 1, because $V_1 - \epsilon > V_k - \epsilon > \delta_B \pi_B^{so}$. Consequently, in any equilibrium seller k accepts if seller 1 rejects, and hence seller 1 must accept the buyer's offer. Therefore, the specified offers give the buyer a strictly higher payoff than from following σ , which contradicts σ as a SPNE.

If $\pi_B^{bo} = \delta_B \pi_B^{so}$, then the following offers give the buyer a strictly higher payoff than from following σ . The buyer offers V_j to each seller $j \neq 1$, and offers $\delta_1 (V_1 - \pi_B^{so}) + \epsilon < V_1$ to seller 1, for some $\epsilon > 0$. If seller 1 accepts, then the buyer strictly prefers trading with seller 1 to trading with any other seller (which gives a payoff of 0) or to letting play continue to the next period. To see the latter point, by trading with seller 1 the buyer's payoff is $V_1 - \delta_1 (V_1 - \pi_B^{so}) - \epsilon = (1 - \delta_1) V_1 + \delta_1 \pi_B^{so} - \epsilon$. There are three cases to consider.

- If $\pi_B^{so} = V_1$, then by trading today the buyer's payoff is $V_1 - \epsilon > \delta_B V_1 = \pi_B^{bo}$ for sufficiently small $\epsilon > 0$. Thus, the deviation is profitable. (Need to argue that seller 1 accepts, here and for other cases.)
- If $0 < \pi_B^{so} < V_1$, then by trading today the buyer's payoff strictly exceeds $\pi_B^{so} - \epsilon = \frac{\pi_B^{bo}}{\delta_B} - \epsilon > \pi_B^{bo}$, for sufficiently small $\epsilon > 0$.
- If $0 = \pi_B^{so}$, then $\pi_B^{bo} = 0$, and by trading today the buyer's payoff is $(1 - \delta_1) V_1 - \epsilon > 0 (= \pi_B^{bo})$ for sufficiently small $\epsilon > 0$.

If the buyer offers seller 1 $\delta_1 (V_1 - \pi_B^{so}) + \epsilon$, the seller 1 strictly prefers accepting. By accepting seller 1 anticipates trading with the buyer this period and getting a payoff of $\delta_1 (V_1 - \pi_B^{so}) + \epsilon$, while by rejecting seller 1 expects at best a strictly lower payoff $\delta_1 (V_1 - \pi_B^{so})$. Therefore, making the specified offers gives the buyer a strictly higher payoff than from following σ , which contradicts σ as a SPNE.

Case 3: In subgames beginning with offers from the buyer, suppose there exists a SPNE σ that involves no trade in any period. In an arbitrary subgame in which σ is to be followed, play will reach the subgame's second period (in which sellers make offers to the buyer). By Lemma 1, trade will occur in the subgame's second period, which is a contradiction.

Cases 1-3 exhaust all possibilities other than trading with seller 1. Therefore, in all subgames beginning with offers from the buyer, all stationary SPNE involve trade with seller 1.

I next show that all stationary SPNE outcomes involve trade in the subgame's initial period, in all subgames beginning with offers from the buyer. Suppose strategy σ supports the stationary SPNE outcome with payoffs π_k^{bo} and π_k^{so} , but that trade does not occur in the subgame's initial period. Because of discounting, this delay in trading implies $\pi_1^{bo} + \pi_B^{bo} < V_1$. In the initial period of an arbitrary subgame in which σ is to be followed, suppose the buyer offers seller 1 $\pi_1^{bo} + \epsilon$ for some $\epsilon > 0$ such that $\pi_1^{bo} + \pi_B^{bo} + \epsilon < V_1$, while making offers to the remaining sellers according to σ . The buyer strictly prefers trading with seller 1 if seller 1 accepts this offer, because the buyer's payoff from doing so is $V_1 - (\pi_1^{bo} + \epsilon) > \pi_B^{bo}$. Hence, the buyer has no incentive to trade with any other seller, nor to reject all offers and let play continue to the next period (because $\pi_B^{bo} \geq \delta_B \pi_B^{so}$). Seller 1 strictly prefers accepting the buyer's offer, because its payoff from doing so strictly exceeds its payoff from rejecting the offer (which is at most π_1^{bo}). Therefore, the buyer strictly prefers offering $\pi_1^{bo} + \epsilon$ to seller 1 because doing so gives the buyer a strictly higher payoff than from following σ , which contradicts σ as a SPNE. ■

Lemma 3 *In all stationary SPNE outcomes, $\pi_1^{so} = V_1 - \max [V_2, \delta_B \pi_B^{bo}]$.*

Proof of Lemma 3: For a SPNE σ that supports the stationary payoffs π_k^{so} and π_k^{bo} , suppose $\pi_1^{so} < V_1 - \max [V_2, \delta_B \pi_B^{bo}]$. Consider the initial period of an arbitrary subgame beginning with offers from the sellers, and suppose seller 1 demands $\pi_1^{so} + \epsilon < V_1 - \max [V_2, \delta_B \pi_B^{bo}]$, for some $\epsilon > 0$. The buyer strictly prefers accepting this demand, because its payoff from doing so is $V_1 - (\pi_1^{so} + \epsilon) > \max [V_2, \delta_B \pi_B^{bo}]$, which strictly exceeds the highest payoff the buyer can get by rejecting seller 1's demand (which is either at most V_2 by trading with seller 2, or $\delta_B \pi_B^{bo}$ by rejecting all offers and letting play continue to the next period). Therefore, $\pi_1^{so} \geq V_1 - \max [V_2, \delta_B \pi_B^{bo}]$.

Now suppose that $\pi_1^{so} > V_1 - \max [V_2, \delta_B \pi_B^{bo}]$. In the initial period of an arbitrary subgame beginning with offers from the sellers, by Lemma 1 the buyer's payoff according to σ is $V_1 - \pi_1^{so} < \max [V_2, \delta_B \pi_B^{bo}]$. If $\delta_B \pi_B^{bo} \geq V_2$, then the buyer strictly prefers rejecting all offers and letting play continue to the next period, where from the initial period's perspective the buyer's payoff will be $\delta_B \pi_B^{bo}$. If $V_2 > \delta_B \pi_B^{bo}$, then seller 2 strictly prefers deviating from σ by offering the buyer $V_2 - \epsilon > \max [V_1 - \pi_1^{so}, \delta_B \pi_B^{bo}]$, for some $\epsilon > 0$. The buyer strictly prefers accepting seller 2's offer, because doing so gives the buyer a strictly higher payoff than either accepting seller 1's offer, or rejecting all offers and letting play continue to the next period. Therefore, $\pi_1^{so} \leq V_1 - \max [V_2, \delta_B \pi_B^{bo}]$.

The arguments in the two preceding paragraphs imply $\pi_1^{so} = V_1 - \max [V_2, \delta_B \pi_B^{bo}]$. ■

Lemma 4 *In all stationary SPNE outcomes, if $V_2 > \delta_B (V_1 - \pi_1^{so})$, then $\pi_B^{bo} = V_1$.*

Proof of Lemma 4: For a SPNE σ that supports the stationary payoffs π_k^{so} and π_k^{bo} , suppose that $V_2 > \delta_B (V_1 - \pi_1^{so})$, but that $\pi_B^{bo} < V_1$. In an arbitrary subgame beginning with offers from the buyer, suppose that in the subgame's initial period the buyer demands $V_1 - \frac{\epsilon}{2} > \pi_B^{bo}$ from seller 1 and $V_2 - \epsilon > \delta_B (V_1 - \pi_1^{so})$ from seller 2 for some $\epsilon > 0$, and demands 0 from all other sellers.

If seller 1 accepts the buyer's offer, then the buyer strictly prefers trading this period with seller 1, because doing so gives the buyer a strictly higher payoff than from trading this period with any other seller, and a strictly higher payoff than from letting play continue to the next period. If seller 2 accepts the buyer's offer and seller 1 rejects, then the buyer strictly prefers trading this period with seller 2, because doing so gives the buyer a strictly higher payoff than from trading this period with any other seller, and a strictly higher payoff than from letting play continue to the next period.

If seller 1 rejects the buyer's offer, then seller 2 strictly prefers accepting to rejecting the buyer's offer: the buyer will trade this period with seller 2, as noted in the preceding paragraph. Seller 2's payoff in this case is $\epsilon > 0$, which strictly exceeds its payoff from rejecting the buyer's offer.

Because of seller 2's incentives, if seller 1 rejects the buyer's offer, then seller 1's payoff is 0. If seller 1 accepts the buyer's offer, then seller 1's payoff is $\epsilon > 0$. Hence, seller 1 accepts the buyer's offer ϵ .

The buyer strictly prefers making these offers, because doing so gives the buyer a strictly higher payoff than from following σ , which contradicts σ as a SPNE. Therefore, it must be the case that $\pi_B^{bo} = V_1$. ■

Lemma 5 *In all stationary SPNE outcomes, if $V_2 < \delta_B (V_1 - \pi_1^{so})$, then $\pi_B^{bo} = V_1 - \delta_1 \pi_1^{so}$.*

Proof of Lemma 5: Consider a SPNE σ that supports the stationary payoffs π_k^{so} and π_k^{bo} , and for which $V_2 < \delta_B (V_1 - \pi_1^{so})$.

First, suppose that $\pi_B^{bo} < V_1 - \delta_1 \pi_1^{so}$. Consider the initial period of an arbitrary subgame that begins with offers from the buyer. Suppose that in the subgame's initial period the buyer follows σ , except it demands $\pi_B^{bo} + \epsilon$ from seller 1, for some $\epsilon > 0$, such that $\pi_B^{bo} + \epsilon < V_1 - \delta_1 \pi_1^{so}$. If seller 1 accepts the offer, then the buyer strictly prefers trading with seller 1, because its payoff from doing so strictly exceeds its payoff from trading with another seller or letting play continue to next period (which is at most π_B^{bo}). Seller 1 strictly prefers accepting the offer to rejecting it, because its payoff from doing so is $V_1 - (\pi_B^{bo} + \epsilon) > \delta_1 \pi_1^{so}$, where $\delta_1 \pi_1^{so}$ is the net present value of seller 1's payoff if play continues to the next period. The buyer strictly prefers making these offers, because doing so gives the buyer a strictly higher payoff than from following σ , which contradicts σ as a SPNE. Therefore, it must be the case that $\pi_B^{bo} \geq V_1 - \delta_1 \pi_1^{so}$.

Next, suppose that $\pi_B^{bo} > V_1 - \delta_1 \pi_1^{so}$. Consider the initial period of an arbitrary subgame that begins with offers from the buyer. By Lemma 2, trade takes place in the subgame's initial period, and seller 1's payoff is $\pi_1^{bo} = V_1 - \pi_B^{bo}$. If seller 1 deviates from σ by rejecting the buyer's equilibrium offer, and if the buyer does not trade with any other seller in the subgame's initial period, then seller 1's payoff is $\delta_1 \pi_1^{so} > \pi_1^{bo}$. If seller 1 rejects the buyer's offer, then the buyer will not trade with any other seller in the subgame's initial period. The highest payoff the buyer can get is V_2 by trading with seller 2, but by waiting until the next period the buyer gets a payoff of $\delta_B (V_1 - \pi_1^{so}) > V_2$. Therefore, it is profitable for seller 1 to reject the buyer's equilibrium offer in the subgame's initial period, which contradicts σ as a SPNE. Therefore, it must be the case that $\pi_B^{bo} \leq V_1 - \delta_1 \pi_1^{so}$.

The arguments in the two preceding paragraphs imply $\pi_B^{bo} = V_1 - \delta_1 \pi_1^{so}$. ■

Lemma 6 *In all stationary SPNE outcomes, if $V_2 = \delta_B (V_1 - \pi_1^{so})$, then $\pi_1^{so} = V_1 - \frac{V_2}{\delta_B}$.*

Proof of Lemma 6: Follows from simple algebra. ■

With the preceding results, the stationary equilibrium payoffs can be derived for each of four cases.

6.1 Case 1: $V_2 < \delta_B (V_1 - \pi_1^{so})$

Lemma 7 *If $V_2 < \delta_B (V_1 - \pi_1^{so})$, then*

$$\pi_B^{bo} = \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1 \quad \pi_1^{bo} = \delta_1 \left(\frac{1 - \delta_B}{1 - \delta_1 \delta_B} \right) V_1 \quad \pi_B^{so} = \delta_B \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1 \quad \pi_1^{so} = \left(\frac{1 - \delta_B}{1 - \delta_1 \delta_B} \right) V_1.$$

Proof of Lemma 7: From Lemma 3, $\pi_1^{so} = V_1 - \max[V_2, \delta_B \pi_B^{bo}]$. Substituting the expression for π_1^{so} into the condition for the Lemma requires $V_2 < \delta_B \max[V_2, \delta_B \pi_B^{bo}]$, so for the Lemma's condition to hold requires $\delta_B \pi_B^{bo} = \max[V_2, \delta_B \pi_B^{bo}]$.

From Lemma 5, $\pi_B^{bo} = V_1 - \delta_1 \pi_1^{so}$. Substituting the expression for π_1^{so} into the expression for π_B^{bo} yields

$$\pi_B^{bo} = V_1 - \delta_1 (V_1 - \delta_B \pi_B^{bo}) \iff \pi_B^{bo} = \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1.$$

Substituting the derived value for π_B^{bo} into the expression for π_1^{so} yields $\pi_1^{so} = \left(\frac{1 - \delta_B}{1 - \delta_1 \delta_B} \right) V_1$. By Lemmas 1 and 2, $\pi_1^{bo} = \delta_1 \left(\frac{1 - \delta_B}{1 - \delta_1 \delta_B} \right) V_1$ and $\pi_B^{so} = \delta_B \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1$. These are the desired results. ■

Lemma 8 *If $V_2 < \delta_B (V_1 - \pi_1^{so})$, then $V_2 < \delta_B^2 \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1$.*

Proof of Lemma 8: If $V_2 < \delta_B (V_1 - \pi_1^{so})$, then Lemma 7 shows that $\pi_1^{so} = \left(\frac{1 - \delta_B}{1 - \delta_1 \delta_B} \right) V_1$. Substituting that expression into the condition for the Lemma yields $V_2 < \delta_B^2 \left(\frac{1 - \delta_1}{1 - \delta_1 \delta_B} \right) V_1$, which is the desired result. ■

6.2 Case 2: $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$

Lemma 9 *If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$, then*

$$\pi_B^{bo} = V_1 \quad \pi_1^{bo} = 0 \quad \pi_B^{so} = V_2 \quad \pi_1^{so} = V_1 - V_2.$$

Proof of Lemma 9: From Lemma 3, $\pi_1^{so} = V_1 - \max[V_2, \delta_B \pi_B^{bo}]$, so the second condition for the Lemma implies $\pi_1^{so} = V_1 - V_2$. From Lemma 4, $\pi_B^{bo} = V_1$. By Lemmas 1 and 2, $\pi_1^{bo} = 0$ and $\pi_B^{so} = V_2$. These are the desired results. ■

Lemma 10 *If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$, then $V_2 \geq \delta_B V_1$.*

Proof of Lemma 10: If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$, then substituting the stationary SPNE payoffs derived in Lemma 9 into the conditions for the Lemma yields $V_2 > \delta_B V_2$ and $V_2 \geq \delta_B V_1$. The first constraint holds trivially, so the relevant constraint is the second one, which is the desired result. ■

6.3 Case 3: $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$

Lemma 11 *If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$, then*

$$\pi_B^{bo} = V_1 \quad \pi_1^{bo} = 0 \quad \pi_B^{so} = \delta_B V_1 \quad \pi_1^{so} = (1 - \delta_B) V_1.$$

Proof of Lemma 11: From Lemma 4, $\pi_B^{bo} = V_1$. From Lemma 3, $\pi_1^{so} = V_1 - \max[V_2, \delta_B \pi_B^{bo}]$, so the second condition for the Lemma implies $\pi_1^{so} = (1 - \delta_B) V_1$. By Lemmas 1 and 2, $\pi_1^{bo} = 0$ and $\pi_B^{so} = \delta_B V_1$. These are the desired results. ■

Lemma 12 *If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$, then $\delta_B^2 V_1 < V_2 < \delta_B V_1$.*

Proof of Lemma 12: If $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$, then substituting the stationary SPNE payoffs derived in Lemma 11 into the conditions for the Lemma yields $V_2 > \delta_B^2 V_1$ and $V_2 < \delta_B V_1$. These are the desired results. ■

6.4 Case 4: $V_2 = \delta_B (V_1 - \pi_1^{so})$

Lemma 13 *If $V_2 = \delta_B (V_1 - \pi_1^{so})$, then*

$$\pi_B^{bo} = \frac{V_2}{\delta_B^2} \quad \pi_1^{bo} = V_1 - \frac{V_2}{\delta_B^2} \quad \pi_B^{so} = \frac{V_2}{\delta_B} \quad \pi_1^{so} = V_1 - \frac{V_2}{\delta_B}.$$

Proof of Lemma 13: From Lemma 6, $\pi_1^{so} = V_1 - \frac{V_2}{\delta_B}$. From Lemma 3, $\pi_1^{so} = V_1 - \max [V_2, \delta_B \pi_B^{bo}]$, so substituting the value of π_1^{so} derived from Lemma 6 yields $\frac{V_2}{\delta_B} = \max [V_2, \delta_B \pi_B^{bo}]$. Given that $\delta_B < 1$, it must be that $\delta_B \pi_B^{bo} = \max [V_2, \delta_B \pi_B^{bo}]$. Therefore, $\pi_B^{bo} = \frac{V_2}{\delta_B^2}$. By Lemmas 1 and 2, $\pi_1^{bo} = V_1 - \frac{V_2}{\delta_B^2}$ and $\pi_B^{so} = \frac{V_2}{\delta_B}$. These are the desired results. ■

Lemma 14 *If $V_2 = \delta_B (V_1 - \pi_1^{so})$, then $\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \leq V_2 \leq \delta_B^2 V_1$.*

Proof of Lemma 14: If $V_2 = \delta_B (V_1 - \pi_1^{so})$, then the stationary SPNE payoffs derived in Lemma 13 are $\pi_1^{so} = V_1 - \frac{V_2}{\delta_B}$ and $\pi_B^{bo} = \frac{V_2}{\delta_B^2}$. From the first condition, the restriction that $\pi_1^{so} \geq 0$ implies that the proposed solution's validity requires $V_2 \leq \delta_B V_1$. From the second condition, the restriction that $\pi_B^{bo} \leq V_1$ implies that the proposed solution's validity requires $V_2 \leq \delta_B^2 V_1$. The second constraint is tighter than is the first, so it is the relevant of the two constraints.

Another restriction is that $\pi_1^{bo} \leq \delta_1 \pi_1^{so}$. If not, then the buyer would be offering seller 1 strictly more than necessary to induce seller 1 to accept the buyer's offer. Given the values of the stationary SPNE derived in Lemma 13, the proposed solution's validity requires

$$V_1 - \frac{V_2}{\delta_B} \leq \delta_1 \left(V_1 - \frac{V_2}{\delta_B} \right) \iff \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \leq V_2.$$

Combining the two parameter restrictions implies that the proposed solution's validity requires $\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \leq V_2 \leq \delta_B^2 V_1$, which is the desired result. ■

Proof of Theorem 1: The proof demonstrates there are unique stationary SPNE payoffs for each set of possible values for V_2 .

If $V_2 \in \left[0, \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \right)$, then $V_2 \notin \left[\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1, V_1 \right]$. The contrapositives of Lemmas 10 and 12 imply it is not the case that $V_2 > \delta_B (V_1 - \pi_1^{so})$, while the contrapositive of Lemma 14 implies that it is not the case that $V_2 = \delta_B (V_1 - \pi_1^{so})$. Hence, it must be the case that $V_2 < \delta_B (V_1 - \pi_1^{so})$. Therefore, by Lemma 7 the payoffs must be as specified in Theorem 1.

If $V_2 \in \left[\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1, \delta_B^2 V_1 \right)$, then $V_2 \notin \left[0, \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \right)$ and $V_2 \notin (\delta_B^2 V_1, V_1]$. The contrapositive of Lemma 8 implies that it is not the case that $V_2 < \delta_B (V_1 - \pi_1^{so})$, while the contrapositives of Lemmas 10 and 12 imply it is not the case that $V_2 > \delta_B (V_1 - \pi_1^{so})$. Hence, it must be the case that $V_2 = \delta_B (V_1 - \pi_1^{so})$. Therefore, by Lemma 13 the payoffs must be as specified in Theorem 1.

If $V_2 \in (\delta_B^2 V_1, \delta_B V_1)$, then $V_2 \notin \left[0, \delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1 \right)$, $V_2 \notin \left[\delta_B^2 \left(\frac{1-\delta_1}{1-\delta_1\delta_B} \right) V_1, \delta_B^2 V_1 \right)$, and $V_2 \notin [\delta_B V_1, V_1]$. The contrapositive of Lemma 8 implies that it is not the case that $V_2 < \delta_B (V_1 - \pi_1^{so})$, the contrapositive of Lemma 14 implies that it is not the case that $V_2 = \delta_B (V_1 - \pi_1^{so})$, while the contrapositive of Lemma 10 implies that it is not the case that both $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$ hold. Hence, it must be the

case that $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$. Therefore, by Lemma 11 the payoffs must be as specified in Theorem 1.

If $V_2 \in [\delta_B V_1, V_1]$, then $V_2 \notin [0, \delta_B V_1]$. The contrapositive of Lemma 8 implies that it is not the case that $V_2 < \delta_B (V_1 - \pi_1^{so})$, the contrapositive of Lemma 14 implies that it is not the case that $V_2 = \delta_B (V_1 - \pi_1^{so})$, while the contrapositive of Lemma 12 implies that it is not the case that both $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 < \delta_B \pi_B^{bo}$ hold. Hence, it must be the case that $V_2 > \delta_B (V_1 - \pi_1^{so})$ and $V_2 \geq \delta_B \pi_B^{bo}$. Therefore, by Lemma 9 the payoffs must be as specified in Theorem 1. ■

Proof of Corollary 1: Recalling that the discount factor δ_k is defined as $\delta_k \equiv e^{-r_k \Delta}$, the buyer's payoffs from Theorem 1 can be written as

$$\begin{aligned} \pi_B^{bo} &= \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1 & \text{if } V_2 \in \left[0, e^{-2r_B \Delta} \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1 \right] \\ \pi_B^{bo} &= \frac{V_2}{e^{-2r_B \Delta}} & \text{if } V_2 \in \left[e^{-2r_B \Delta} \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1, e^{-2r_B \Delta} V_1 \right] \\ \pi_B^{bo} &= V_1 & \text{if } V_2 \in (e^{-2r_B \Delta} V_1, e^{-r_B \Delta} V_1) \\ \pi_B^{bo} &= V_1 & \text{if } V_2 \in [e^{-r_B \Delta} V_1, V_1] \end{aligned}$$

and

$$\begin{aligned} \pi_B^{so} &= e^{-r_B \Delta} \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1 & \text{if } V_2 \in \left[0, e^{-2r_B \Delta} \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1 \right] \\ \pi_B^{so} &= \frac{V_2}{e^{-r_B \Delta}} & \text{if } V_2 \in \left[e^{-2r_B \Delta} \left(\frac{1 - e^{-r_1 \Delta}}{1 - e^{-r_1 \Delta} e^{-r_B \Delta}} \right) V_1, e^{-2r_B \Delta} V_1 \right] \\ \pi_B^{so} &= e^{-r_B \Delta} V_1 & \text{if } V_2 \in (e^{-2r_B \Delta} V_1, e^{-r_B \Delta} V_1) \\ \pi_B^{so} &= V_2 & \text{if } V_2 \in [e^{-r_B \Delta} V_1, V_1]. \end{aligned}$$

By Lemmas 1 and 2 one can derive the associated values for π_1^{bo} and π_1^{so} . Taking the limit as $\Delta \rightarrow 0$ requires using L'Hopital's rule, which gives the desired result. ■

6.5 Proofs of Propositions 1 and 2

Proof of Proposition 1: The buyer strictly prefers multilateral negotiations if and only if its payoff from multilateral negotiations strictly exceeds its payoff from a first-price auction. That condition amounts to

$$\pi_B^{mln} > \pi_B^{fpa} \iff \max \left[V_2, \left(\frac{r_1}{r_1 + r_B} \right) V_1 \right] - C^{mln} > V_2.$$

The latter expression is equivalent to

$$C^{mln} < \max \left[0, \left(\frac{r_1}{r_1 + r_B} \right) V_1 - V_2 \right], \quad (1)$$

which is the desired result.

The buyer's relative preference for multilateral negotiations intensifies as $\pi_B^{mln} - \pi_B^{fpa}$ increases. That condition corresponds to relaxing (1), which is accomplished by decreasing C^{mln} , increasing V_1 , decreasing V_2 , or decreasing $\frac{r_B}{r_1}$. ■

Proof of Proposition 2: The buyer strictly prefers multilateral negotiations if and only if its expected payoff from multilateral negotiations strictly exceeds its expected payoff from a first-price auction. That

condition amounts to

$$\pi_B^{mln} > \pi_B^{fpa} \iff \frac{N-1 + \left(\frac{r_S}{r_S+r_B}\right)^N}{N+1} - C^{mln} > \frac{N-1}{N+1}.$$

The latter expression is equivalent to

$$C^{mln} < \frac{\left(\frac{r_S}{r_S+r_B}\right)^N}{N+1}, \quad (2)$$

which is the desired result.

The buyer's relative preference for multilateral negotiations intensifies as $\pi_B^{mln} - \pi_B^{fpa}$ increases. That condition corresponds to relaxing (2), which is accomplished by decreasing C^{mln} , decreasing N , or decreasing $\frac{r_B}{r_1}$. ■

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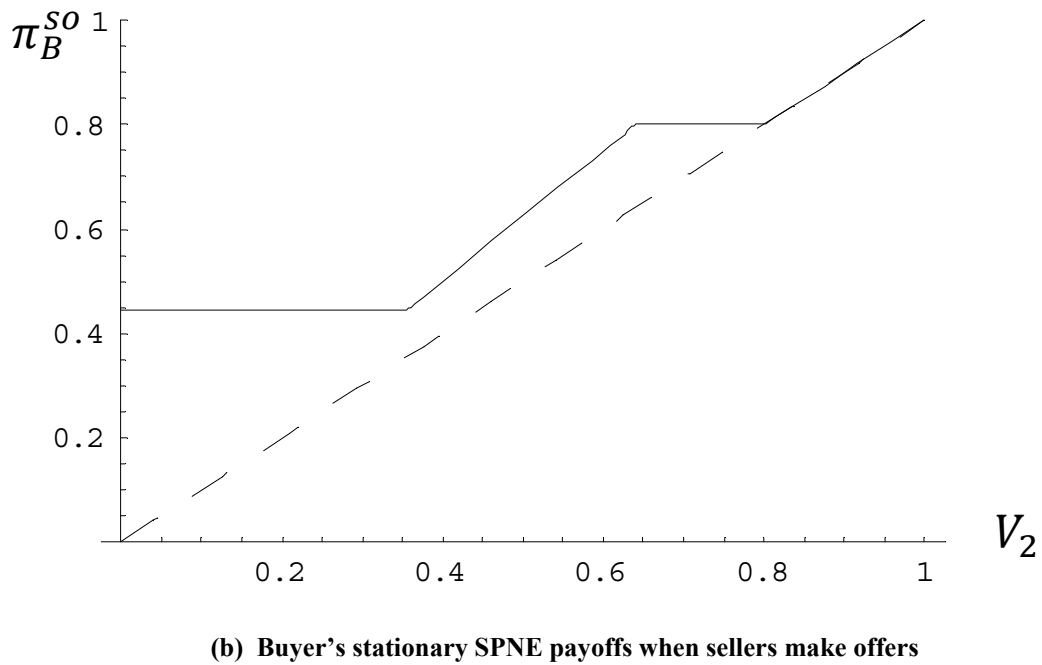
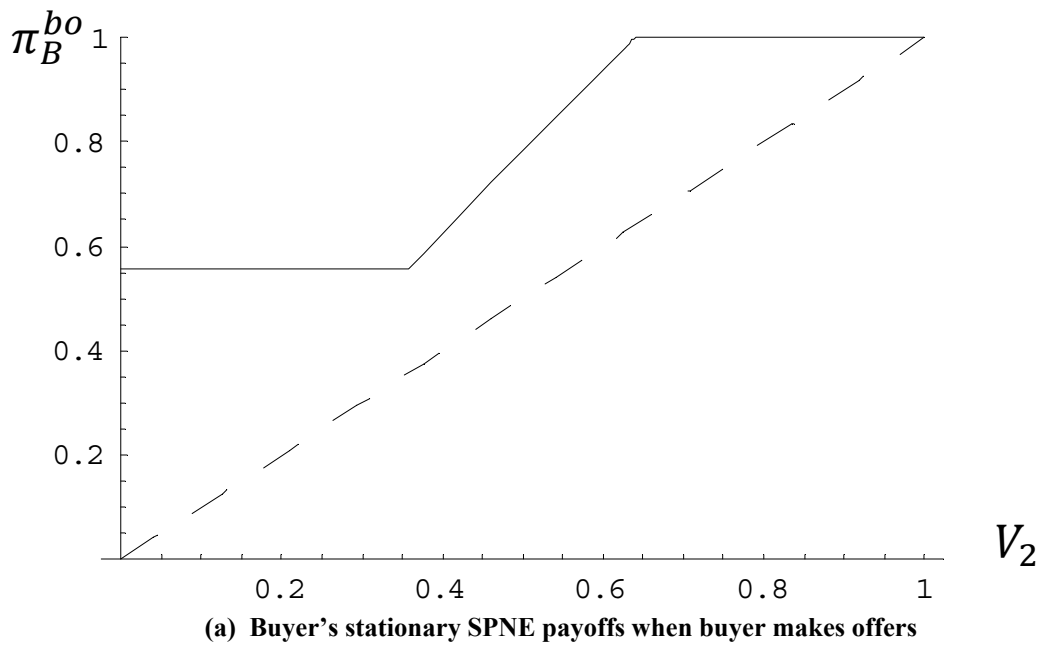
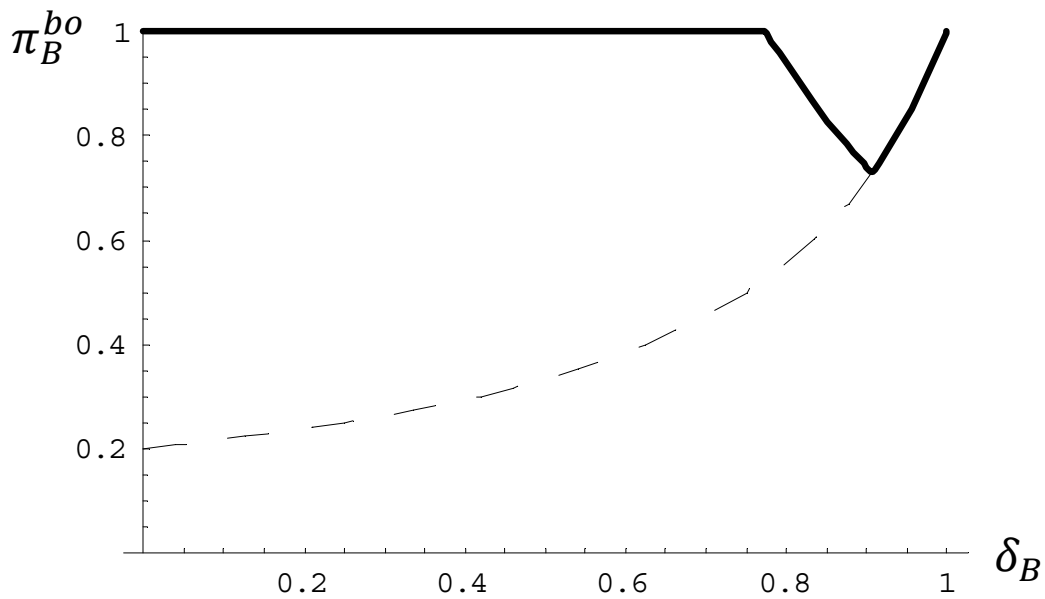
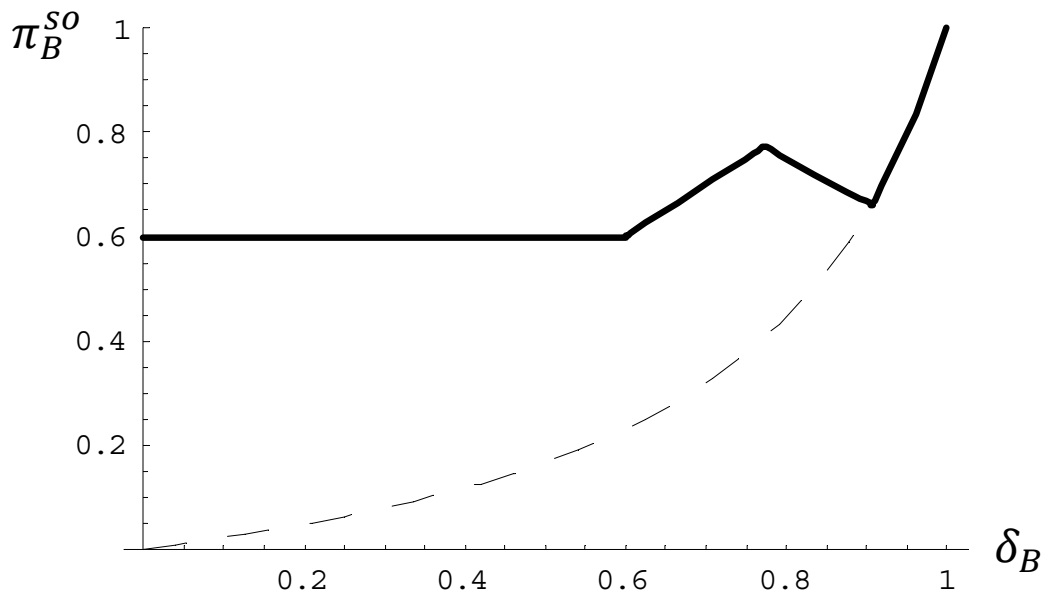


Figure 1. Buyer's Stationary Payoffs in Multilateral Negotiations, as a Function of V_2 . Panel (a) shows the buyer's SPNE payoffs as a function of V_2 , in subgames beginning with offers from the buyer. Panel (b) shows the buyer's SPNE payoffs as a function of V_2 , in subgames beginning with offers from the sellers. These figures are calculated using $V_1 = 1$, $\delta_1 = 0.8$, $\delta_B = 0.8$. Dashed line is 45°-line.



(a) Buyer's stationary SPNE payoffs when buyer makes offers



(b) Buyer's stationary SPNE payoffs when sellers make offers

Figure 2. Buyer's Stationary Payoffs in Multilateral Negotiations, as a Function of δ_B . Panel (a) shows the buyer's SPNE payoffs as a function of δ_B , in subgames beginning with offers from the buyer. Panel (b) shows the buyer's SPNE payoffs as a function of δ_B , in subgames beginning with offers from the sellers. These figures are calculated using $V_1 = 1$, $V_2 = 0.6$, $\delta_1 = 0.8$. Solid line is stationary SPNE payoff, and dashed line is SPNE payoff from bilateral negotiations.

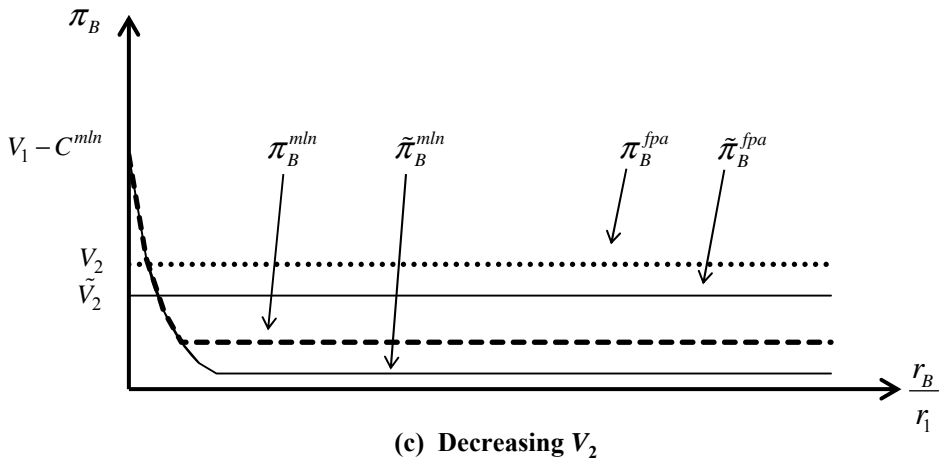
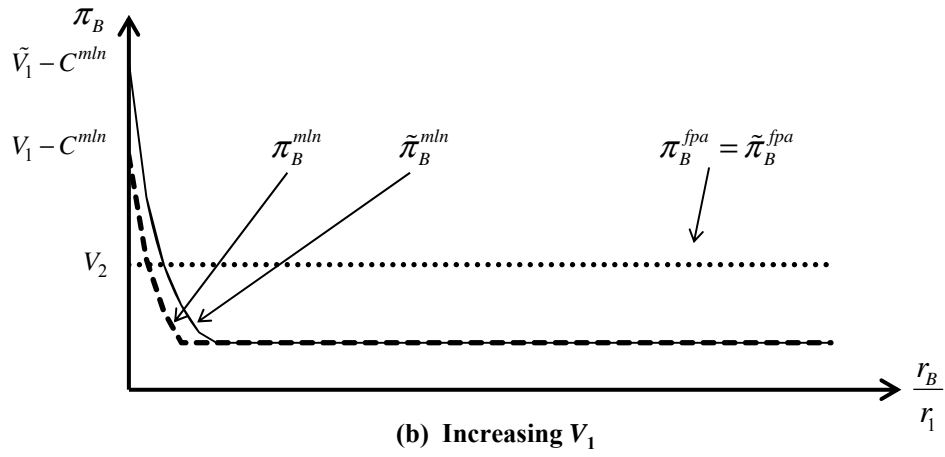
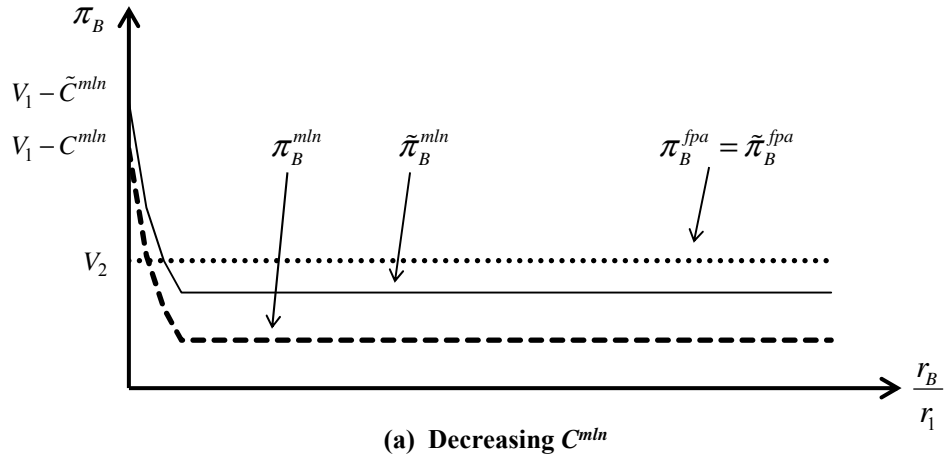
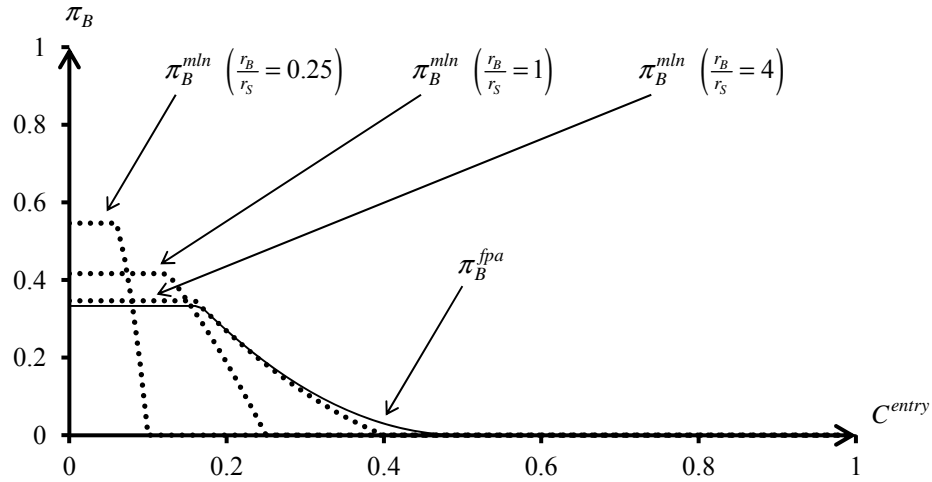
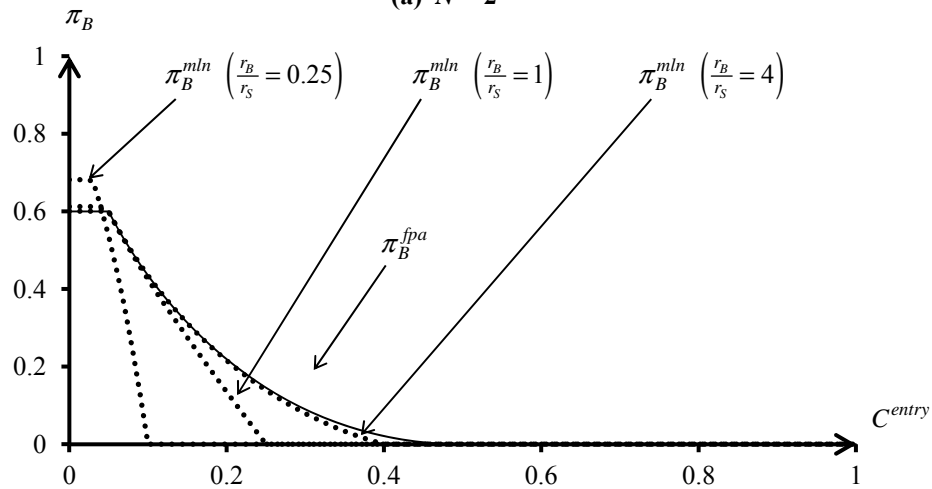


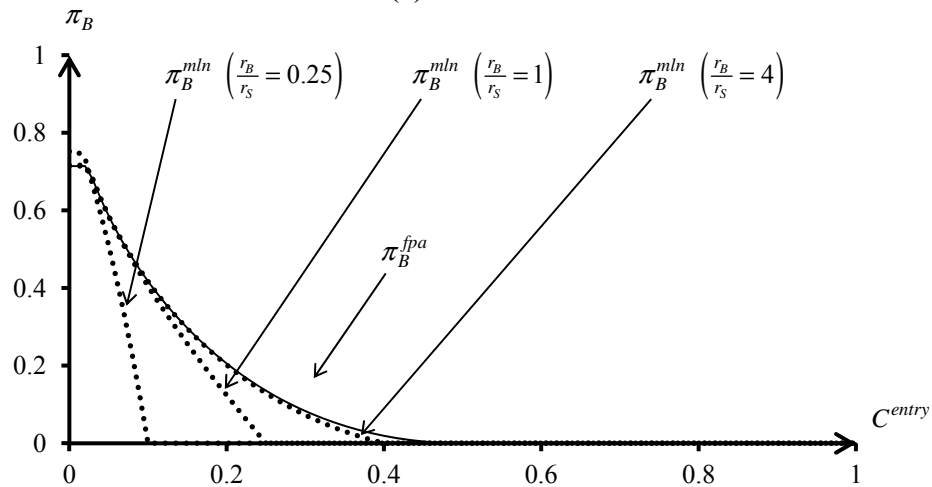
Figure 3. Comparative Statics of Buyer's Payoffs in Multilateral Negotiations and a First-Price Auction. Each panel shows the buyer's payoffs with initial parameters and with one changed parameter (changes denoted by a tilde). Panel (a) decreases C^{mln} , panel (b) increases V_1 , and panel (c) decreases V_2 . Each change intensifies the buyer's relative preference for multilateral negotiations over a first-price auction.



(a) $N = 2$

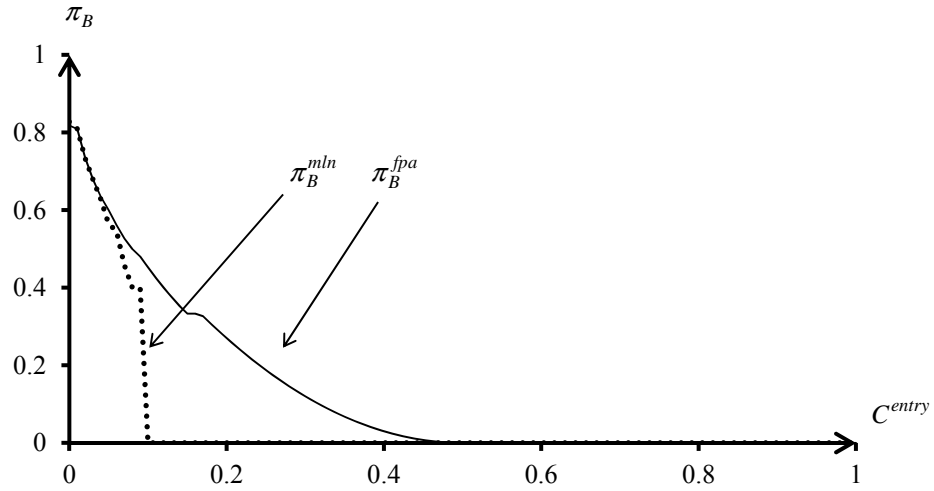


(b) $N = 4$

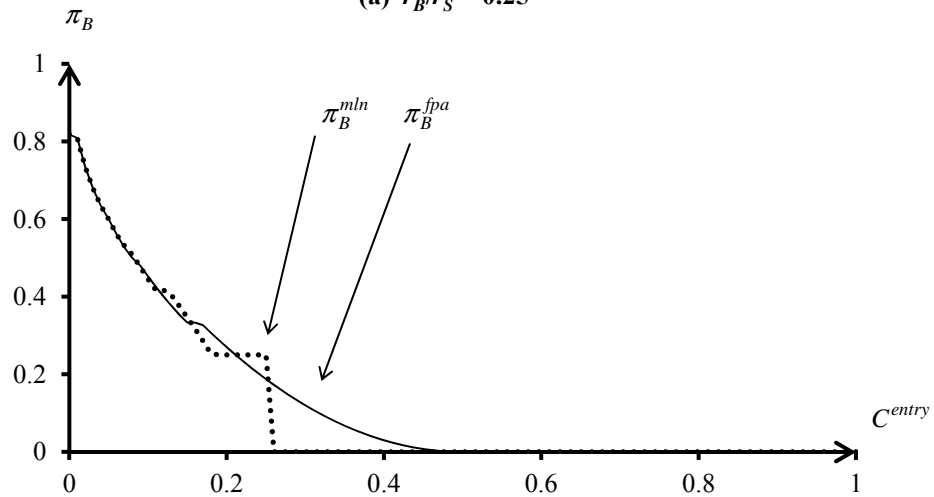


(c) $N = 6$

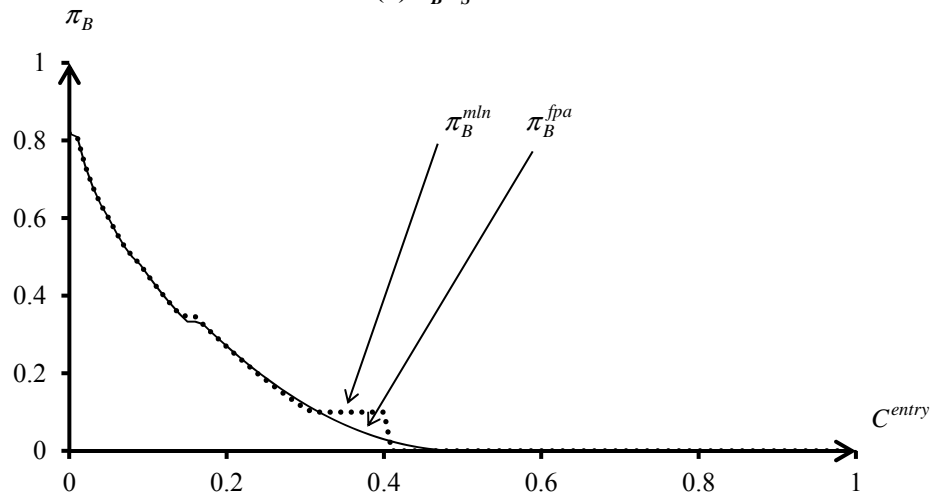
Figure 4. Buyer's Expected Payoffs in Multilateral Negotiations and a First-Price Auction, with Fixed Number of Sellers. Each panel shows the buyer's payoffs for a particular value of N , and with different ratios of discount rates r_B/r_S . $N = 2$ in panel (a), $N = 4$ in panel (b), and $N = 6$ in panel (c). Multilateral negotiation payoffs are dashed lines, first-price auction payoff is solid line.



(a) $r_B/r_S = 0.25$



(b) $r_B/r_S = 1$



(c) $r_B/r_S = 4$

Figure 5. Buyer's Expected Payoffs in Multilateral Negotiations and a First-Price Auction, with Optimal Number of Sellers. Each panel shows the buyer's payoffs for a particular ratio of discount factors, and with optimal number of invited sellers. $r_B/r_S = 0.25$ in panel (a), $r_B/r_S = 1$ in panel (b), and $r_B/r_S = 4$ in panel (c). Multilateral negotiation payoff is dashed line, first-price auction payoff is solid line.