An Experimental Analysis of an Envy-Free Auction

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****Preliminary****

Abstract

Economic concerns for equity have motivated game theorists to study envy-free mechanisms. Though not incentive compatible, these mechanisms implement in Nash equilibria efficient allocations at which no agent prefers the consumption of any other agent to their own. In experimental allocation decisions between two players, an envy-free first-price auction achieves similar efficiency and far greater no-envy than ultimatum bargaining. Both unsophisticated subject bidding and coordination failure are responsible for the departure from Nash equilibrium behavior in the envy-free auction, as bidding strategies vary greatly among subjects. Quantal response equilibrium and level-k models can explain most of this subjects bidding behavior.

JEL classification: D63, C72, C91

Keywords: experimental economics; no-envy; mechanism design; behavioral game theory.

1 Introduction

This paper experimentally evaluates an "envy-free" first-price-auction-type mechanism, which we refer to simply as a first-price auction, for the allocation of a social endowment of indivisible goods when monetary compensation is possible. In an experimental setting this first-price auction achieves similar efficiency and greater no-envy than ultimatum bargaining. We document

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how experimental outcomes differ from the Nash equilibrium prediction for the first-price auction. Then, we identify subjects unsophisticated bidding as an important source for these deviations. Finally, we derive policy implications from these results. In particular, we identify the non-strategic equivalence of envy-free mechanism when subjects are boundedly rational, which calls for the design of an optimal envy-free mechanism.

An envy-free allocation is that in which no agent prefers the allotment of any other agent to her own. To some extent these allocations provide all agents with equal opportunity to benefit from resources. They are desirable outcomes in situations in which all agents have equal rights over the resources but their preferences may differ (Foley, 1967; Varian, 1974; see Thomson, 2006 for a survey).[1] In our model, each envy-free allocation is efficient (Svensson, 1983).

We are interested in envy-free mechanisms for the allocation of indivisible goods and money. We have in mind situations like the allocation of the rooms and the division of the rent among house-mates who collectively lease a house. Since rooms may differ, then agents' preferences over bundles of rooms and rent may differ as well. It is known that under general conditions on preferences there are envy-free allocations for each such a situation (Alkan et al., 1991; see Velez (2011a) for a domain restriction that guarantees the existence of envy-free allocations in which each agent contributes a non-negative amount to rent).[2] Moreover, a rich family of mechanism doubly implement, in Nash and strong Nash equilibria, the correspondence that selects all envy-free allocations (Tadenuma and Thomson, 1995a; Āzacis, 2008; Beviá, 2010; Velez, 2011b; Fujinaka and Wakayama, 2011). Our first-price auction belongs to this family.

We test the extent to which these theoretical predictions are obtained in the laboratory. That is, each agent's ability to manipulate our first-price auction is limited by the others and as a result, only envy-free allocations ensue. Additionally, we test an alternative mechanism, ultimatum bargaining, which provides a non-envy-free benchmark. To our knowledge no previous experiments have analyzed the performance of envy-free mechanisms.[3]

[1] Let us emphasize that equal rights across agents precludes situations in which agents differentially contributed to the production of the resources to divide.

[2] Existence of envy-free allocations holds for domains that include other regarding preferences Velez (2011b).

[3] Herreiner and Puppe (2009) investigate whether envy-free allocations are outcomes

We obtain experimental results from two sessions, one using our first-price auction and the other using ultimatum bargaining. From a large group of subjects we randomly divided subjects into pairs and had them use each mechanism with five distinct "valuations" for ten periods each (groups were formed each period). Our results suggest that the ultimatum bargaining is slightly superior in efficient allocations realized, but the first-price auction achieves a much greater number of envy-free allocations and higher earnings per round for subjects.

That being said, the performance of the first-price auction differs from the theoretical predictions—envy-free allocations are achieved less than half the time, and efficient outcomes occur 72 percent of all outcomes. We examine two possible explanations for this disparity. Since the auction mechanism takes simultaneous bids, players may not coordinate on a Nash equilibria. Alternatively, players due to bounded rationality, may submit non-Nash bids or try to exploit those who do. Our evidence suggests that we cannot rule out any of these effects as the cause for the deviations from equilibrium. However, a special configuration of valuations for which coordination is trivial, allows us to conclude that players' unsophistication is an important issue. We use both the quantal response equilibrium (McKelvey and Palfrey, 1995, 1998) and a variation of the level-k model applied to auctions (similar to Crawford and Iriberri, 2007) to characterize unsophisticated subject bidding.

Our results have consequences for the design of an optimal envy-free mechanism, i.e., a mechanism that maximizes the probability to achieve an envy-free allocation. We observe that given the non-Nash bidding of subjects, the number of envy-free and efficient outcomes of an envy-free mechanism may differ from one mechanism to the other. This is in contrast to the outcome equivalence under the Nash equilibrium prediction. Our preliminary analysis indicates, that a greater number of envy-free outcomes could be achieved with an auction other than our first-price auction. Section 6 summarizes these ideas and provides suggestions for further research.

The remainder of the paper is organized as follows. Section 2 introduces

from an "infinite proposals" bargaining with time limit. They document that this mechanism achieve few envy-free allocations. The main difference with our study is that the Nash equilibrium (and subgame perfect Nash equilibrium) outcomes of this infinite bargaining procedure are generally non-envy-free.

our model and the mechanisms we study. Section 3 presents our experimental design. Section 4 describes alternatives to rationalize behavior in our experiments. Section 5 presents our data and analysis. Section 6 discusses our results. Section 7 concludes.

2 Model

In this section we introduce our model and the mechanisms the we experimentally study.

2.1 Environment

There are two agents $N \equiv \{1, 2\}$ who are collectively endowed with two objects $\{A, B\}$. Each agent consumes an object and an amount of money. Consumptions of money should add up to zero. An allocation is a list of two bundles $z \equiv (t, \sigma)$ where $t \equiv (t_A, t_B)$ is a pair of transfers of money that add up to zero and σ is a bijection from N to $\{A, B\}$. The agent who receives object A transfers t_A to the agent who receives object B. Symmetrically, the agent who receives object B transfers t_B to the agent who receives object A. Thus, $t_A + t_B = 0$. In different words, the amount of money consumed by the agent who receives object A is $-t_A$ and the amount of money consumed by the agent who receives object B is $-t_B$. Agent 1's consumption at z is $z_1 \equiv (t_{\sigma(1)}, \sigma(1))$. Analogously, agent 2's consumption at z is $z_2 \equiv (t_{\sigma(2)}, \sigma(2))$. The set of allocations is Z. Agents' true preferences are quasi-linear

$$u_1^0(z) \equiv -t_{\sigma(1)} + v_1^0(A)1_{\sigma(1)=A} + v_1^0(B)1_{\sigma(1)=B},$$

$$u_2^0(z) \equiv -t_{\sigma(2)} + v_2^0(A)1_{\sigma(2)=A} + v_2^0(B)1_{\sigma(2)=B}.$$

In order to reduce the dimension of the strategy space, we assume that[4]

$$v_1^0(A) = v_2^0(A) = 100.$$

[4]This assumption is without of loss of generality. Since each agent has to receive one object, then the possibility of receiving money and no object is precluded. Thus, $v_1^0(A) = v_1^0(B) = 100$ is a normalization.

We assume that this information, except the values of $v_1^0(B)$ and $v_2^0(B)$, is known to the mechanism designer.

A mechanism is a pair $\langle M_1 \times M_2, S\rangle$ in which M_1 and M_2 are message spaces and $S : M_1 \times M_2 \to Z$ is an outcome function. The game induced by $\langle M_1 \times M_2, S\rangle$ and the true preference profile u^0 is $\langle M_1 \times M_2, S, u^0\rangle$. The set of Nash equilibrium and pure-strategy Nash equilibrium outcomes of this game are $\mathcal{O}\langle M_1 \times M_2, S, u^0\rangle$ and $\mathcal{O}_p\langle M_1 \times M_2, S, u^0\rangle$, respectively.

A solution is a function that associates with each preference profile an allocation in Z. Since each agent's preferences are determined by the agent's valuation of object B, then a solution, generically denoted by S, can be seen as a function

$$\begin{array}{rccc} S: & \mathbb{R} \times \mathbb{R} & \to & Z \\ & v \equiv (v_1(B), v_2(B)) & \mapsto & S(v). \end{array}$$

A solution S induces a direct revelation mechanism in which each agent's strategy space is her preference space and the outcome function is the solution itself, i.e., $\langle \mathbb{R} \times \mathbb{R}, S\rangle$.

2.2 Properties

We consider two properties of allocations. The first, no-envy, requires that no agent prefer the consumption of the other agent to her own (Foley, 1967; Varian, 1974). Formally, $z \in Z$ is envy-free for u if $u_1(z_1) \geq u_1(z_2)$ and $u_2(z_2) \geq u_2(z_1)$. The set of envy-free allocations for u is $F(u)$. The second property, efficiency, is defined as usual. An allocation $z \in Z$ is efficient for u if there is no other $z' \in Z$ that is weakly preferred to z by both agents and strictly preferred to z by at least one agent. Since there is at least as many agents as objects in our model, then each envy-free allocation is efficient (Svensson, 1983).

We now characterize the set of envy-free allocations. We assume without loss of generality that $v_1(B) \leq v_2(B)$. There are two cases.

Case 1: $v_1(B) < v_2(B)$. Let $z \equiv (t, \sigma) \in F(u)$. Since each envy-free allocation is efficient, then $\sigma(1) = A$ and $\sigma(2) = B$. Since $u_1(z_1) \geq u_1(z_2)$, then $-t_A + v_1(A) \geq -t_B + v_1(B)$. Since, $t_A + t_B = 0$, then

$$t_B \geq \frac{v_1(B) - v_1(A)}{2}.$$

Since $u_2(z_2) \geq u_2(z_1)$, then $-t_B + v_2(B) \geq -t_A + v_2(A)$. Moreover, since $t_A + t_B = 0$, then

$$t_B \leq \frac{v_2(B) - v_2(A)}{2}.$$

Under our normalization $v_1(A) = v_2(A) = 100$, these restrictions become

$$\frac{v_1(B) - 100}{2} \leq t_B \leq \frac{v_2(B) - 100}{2}. \quad (1)$$

Conversely, one can easily see that if $\sigma(1) = A$, $\sigma(2) = B$, and t satisfies (1), then $z \equiv (t, \sigma) \in F(u)$.

The difference between the maximal and minimal amount transferred at an envy-free allocation, intuitively measures the size of the set of these allocations (Tadenuma and Thomson, 1995a). In our model this difference, which we refer to as the equity surplus, is exactly

$$ES(u) \equiv \frac{v_2(B) - v_1(B)}{2}.$$

Case 2: $v_1(B) = v_2(B)$. One can easily see that $z \equiv (t, \sigma) \in F(u)$ if and only if $t_B = \frac{v_2(B) - v_2(A)}{2}$. Thus, when agents have identical preferences, the amount of money received by the agent who receives object B in a envy-free allocation is uniquely determined. Of course, the amount of money received by the agent who receives object A, $t_A = -t_B$, is uniquely determined too. Let us remark that in this case the equity surplus is zero and no restriction is imposed in the allocation of objects.

Fig. 1 provides a geometric interpretation of the equity surplus and the set of envy-free allocations for all possible valuations configurations.

It follows from our characterization that a solution that selects envy-free allocations is defined by: (i) the agents' consumptions of money when $v_1(B) \neq v_2(B)$ (allocation of objects is unique in Case 1), and (ii) the allocation of objects when $v_1(B) = v_2(B)$ (consumptions of money are unique in Case 2).

2.3 Mechanisms

We are interested in mechanisms that implement envy-free allocations. In this section we introduce two mechanisms. The fist is a first-price-auction-type mechanism for which Nash equilibrium predicts it achieves our goal.

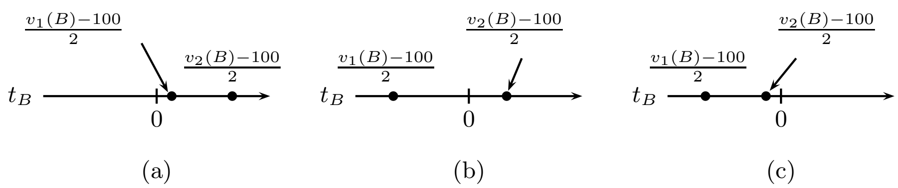

Figure 1: Equity surplus and non contestable allocations: Panels (a), (b), and (c) display the possible transfers for the agent who receives object B in an envy-free allocation when: (a) $100 \leq v_1(B) \leq v_2(B)$, (b) $v_1(B) \leq 100 \leq v_2(B)$, and (c) $v_1(B) \leq v_2(B) \leq 100$. The maximal and minimal transfers of money at an envy-free allocation for the agent that consumes object B are $\frac{v_2(B)-100}{2}$ and $\frac{v_1(B)-100}{2}$, respectively. The equity surplus is the difference between these two amounts (the distance between the points displayed on axis t_B).

The second is an ultimatum-bargaining mechanisms for which the subgame-perfect Nash equilibrium predicts the opposite.

2.3.1 First-price auction

Our first mechanism is a first-price-auction-type mechanism in which agents report bids (possibly negative) for object B. Bid b is interpreted as the amount of money that they are willing to transfer to (or receive from) the other agent in order to receive object B. Then an agent with the highest bid receives object B and pays her bid, say b (thus her consumption of money is $-b$). The other player receives object A and b. In case of a tie, an agent with highest true valuation of object B receives object B and pays her bid.

Recall that without loss of generality we assume that $v_1^0(B) < v_2^0(B)$. Formally, this mechanism is $\langle \mathbb{R} \times \mathbb{R}, \mathcal{A} \rangle$ where given reports $b \equiv (b_1, b_2)$, $\mathcal{A}$ recommends $\mathcal{A}(v) \equiv (t, \theta)$ where $t \equiv (t_A, t_B)$ and $\theta^v : N \to \{A, B\}$ are given by:

$$t_A \equiv -\max\{b_1, b_2\} \quad \text{and} \quad t_B \equiv \max\{b_1, b_2\},$$

and

$$\begin{array}{llll} \theta(1) \equiv A & \text{and} & \theta(2) \equiv B & \text{if} \quad b_1 < b_2, \\ \theta(1) \equiv B & \text{and} & \theta(2) \equiv A & \text{if} \quad b_1 > b_2, \\ \theta(1) \equiv A & \text{and} & \theta(2) \equiv B & \text{if} \quad b_1 = b_2. \end{array} \tag{2}$$

Let us remark that the mechanism selects an efficient allocation for true preferences when agents reports are identical. True preferences are available to the mechanism designer in the lab environment. If this information were not available, one would have to enlarge the agents' strategy space in order to define a solution with efficient Nash equilibrium outcome correspondence (see Tadenuma and Thomson, 1995b, Beviá, 2010, Velez and Thomson, 2009, and Velez, 2011a).[5]

Our auction mechanism is strategically equivalent to the one induced by the solution that selects the envy-free allocation that assigns the maximal surplus to the agent who receives A. Formally, there is a bijection between agent i's bid, b_i, and her possible valuations, $v_i(B) \equiv 100 + 2b_i$. Thus, one can think of an agent's bid as the report of her preferences. Given report $(v_1(B), v_2(B))$, $\mathcal{A}$ selects the envy-free allocation at which the agent with highest valuation for B transfers the other agent $\frac{\max\{v_1(B),v_2(B)\}-100}{2}$.

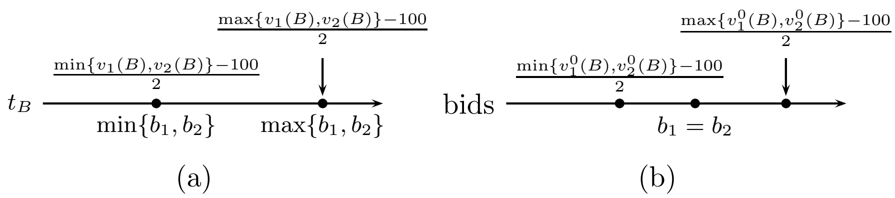

Figure 2: First-price-auction-type mechanism: (a) given bids (b_1, b_2) this mechanism selects an envy-free allocation at which the agent who receives object B makes the maximal transfer among all envy free allocations for valuations $(100 + 2b_1, 100 + 2b_2)$; (b) in each the pure-strategy Nash equilibria of each game induced by the mechanism both agents bid equal amounts in between the maximal and minimal transfers, t_B, in an envy-free allocation.

It is well known that the pure-strategy Nash equilibrium correspondence of the mechanism induced by any selection from the envy-free set is itself envy-free (Tadenuma and Thomson, 1995a,b; Beviá, 2010; Velez, 2011a; Fujinaka and Wakayama, 2011). Thus, all equilibrium allocations of our first-price-auction-type mechanism are envy-free. One can easily see that for each envy-free allocation for the true preferences, $z \equiv (t, \theta) \in F(u^0)$, there is an equilibrium with outcome z in which each agent bids t_B. Thus, the pure-strategy Nash equilibrium correspondence of our first-price-auction-type mechanism is exactly the envy-free correspondence. That is, $\mathcal{O}_p\langle\mathbb{R} \times \mathbb{R}, \mathcal{A}, u^0\rangle = F(u^0)$.

[5] For instance, one could require agents also report the object they request. If Case 2 is reached, then this information determines the allocation of objects.

2.3.2 Ultimatum bargaining

Finally, we consider an ultimatum mechanism in which one agent,chosen at random, say agent 1, is selected to propose an allocation z such that $u_2^0(z_2) \geq 0$. Then agent 2 accepts or rejects the proposal. They both receive no object and no transfer of money if the proposal is rejected. One can easily see that at each subgame-perfect equilibrium outcome of this mechanism, agent 1 proposes an efficient allocation z^* such that $u_2^0(z_2^*) \leq 1$ and agent 2 accepts the proposal (for the true valuations considered in our experiments, these allocations are not envy-free).[6]

The following table summarizes the theoretical properties of first-price-auction-type and ultimatum-type mechanisms.

Mechanism	Efficient	Envy-free
First price auction	+	+
Ultimatum	+	−

Table 1: Theoretical properties of mechanism.

3 Experimental Design

This experiment implemented the theoretical environment described in section 2.1. Subjects in groups of two chose how to allocate two indivisible items with possible transfer payments. In all possible allocations, each subject would receive *exactly* one item.[7].

All experiments were held at the Experimental Research Labratory (ERL) in the Economics Department at Texas A&M University. Subjects sat at computer terminals and made their decisions on software programmed in the z-tree language (Fischbacher, 2007). Dividers were used to make sure

[6] A proposal such that $u_2^0(z_2^*) = 1$ is strictly preferred by agent 2 to rejecting it. A proposal where $u_2^0(z_2^*) = 0$, could also be subgame perfect depending on the assumptions made about agent 2's actions when she is indifferent. There are a great number of Nash equilibria for this mechanism that are not subgame perfect. Any strategy that player 1 offers could be a Nash equilibrium, provided player 2 would reject all proposals that offer her a lower share than that strategy.

[7] In the case of ultimatum bargaining if a proposal was rejected each subject would receive nothing. However, all ultimatum proposals must have each subject receiving *exactly* one item

anonymity of subjects was preserved. Subjects were 50 Texas A&M undergraduates from a variety of majors, twenty-six subjects took part in the ultimatum bargaining session on October 21, 2010; twenty-four subjects took part in the envy-free first-price auction session on October 22, 2010.

Each period, subjects would receive points for acquiring either item, equivalent to their valuation of that item. Thus, the values for each item were induced valuations.[8] They would also gain or lose points included in any transfer payments imposed by the mechanism. Subject values for each item were common knowledge to both subjects. Exactly as in our theoretical environment (section 2.1), all valuations had identical subject values for item A at 100 units, but most had different valuations for item B. Table 2 provides a sample valuation.).

Player	Value of Item A	Value of Item B
Player 1	100	120
Player 2	100	80

Table 2: A sample valuation: In this valuation a player values item A at 100 and item B at 120. As in our theoretical environment (Section 2.1, the other player whom she is paired has the same valuation for item A, but values item B at only 80. Each player has an equal chance of receiving the high value on item B for any period. Valuations are common knowledge to both players.

Table 2 provides an example valuation, the third valuation used in the experiments. In this case, both subjects have the same valuation for item A, but their valuations for item B are different: player 1 has the high valuation for item B (120 points), and player 2 has the low valuation for item B (80 points). There were five total valuations used in the experiment. Each valuation was used for 10 consecutive periods, for a total of 50 periods. For any period, for each grouping of subjects, one subject would be randomly assigned the high valuation on item B, the other would receive the low valuation. Thus a subject's valuation could change for any period, but the valuation structure (the two subject valuations for item B) would remain the same for ten periods. For each of the five valuations, subjects always valued item A at 100. In order of appearance, the pairing of subject valuations for item B were $(40, 80)$, $(120, 160)$, $(40, 120)$, $(160, 160)$, and $(0, 40)$.

[8]Alternatively, one could say each item had a redemption value for each subject.

To avoid incentives associated with repeated play, subjects were randomly re-assigned to each other at the beginning of each period. Subjects were instructed that they would be randomly rematched each period, but no identifying information (e.g., subject number) was disclosed to a subject about their match in any round. Each period would begin with each subject seeing the valuation for the period

3.1 Envy-free Auction Procedures

In the auction session, after observing the valuation for the period, subjects would simultaneously submit their bids for item B. The subject with the higher bid receives the item B, and the subject with the lower bid receives item A. In the case of equal bids, the subject with the higher valuation of item B receives item A.[9] The subject who receives item B, then pays the subject who receives item A the full amount of her bid. In this way, the auction mechanism is a first-price auction. However, there is no restriction that any bid must be positive. Bids were only restricted so that no bid could be lower than the opposite of twice the value of item A (-200, always) and no bid could be higher the twice the maximum value of item B (varies by valuation, i.e, 160, 320, 240, 320, and 80 for each valuation, respectively).

After submitting a bid, each subject was allowed to submit a possible value for the other players bid. The experimental software would then display the outcome (i.e., who gets which item, what amount is transferred for each player, each players earnings for that period) that would occur with those two bids as well as a table that showed all possibilities that could happen if the other player's bid were below, equal to, or above the subject's bid (see Figure 3). After a subject viewed these possibilities, she could choose to confirm her bid, or choose an alternate bid. If she chose an alternate bid, the process would repeat again. The process would end when a subject confirmed her bid.

After both subjects submitted their bids, they would be asked to guess what they believed the other subject bid. If they guessed correctly they would receive a small bonus of 5 points. The value of this bonus was deliberately chosen to be small, so that subjects would not alter their bidding

[9] In the case of the fourth valuation where both subjects value item B at 160, one subject was randomly selected to win ties, this was known before bids were submitted (i.e., ex-ante).

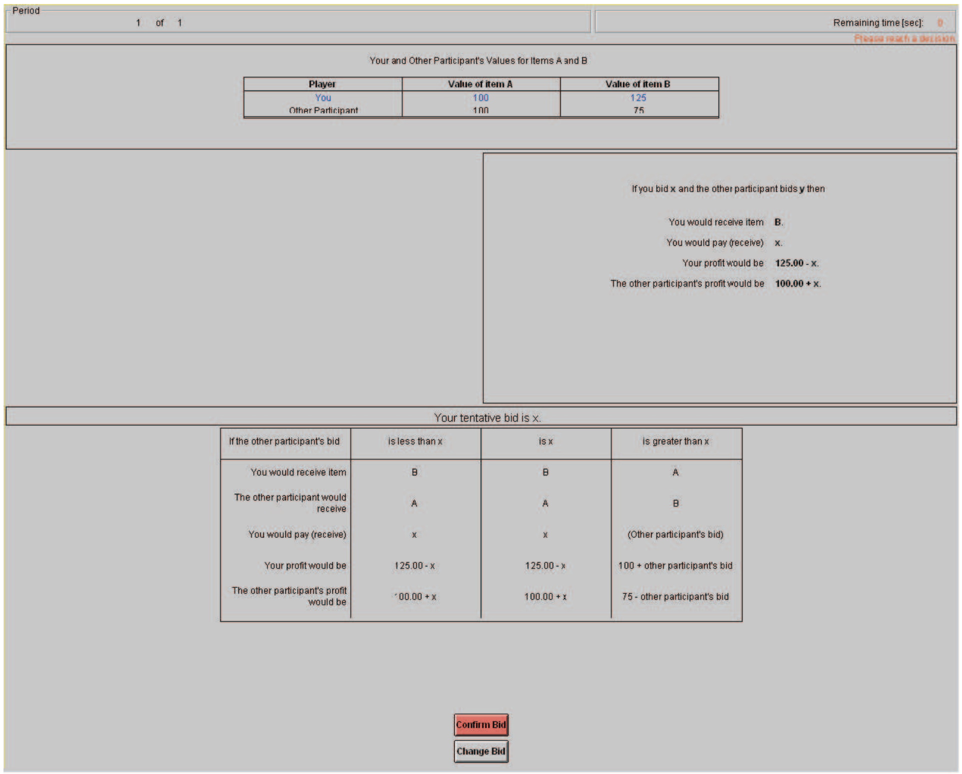

Figure 3: Confirmation screen for subjects: Subjects have the option to review their bid and the possible outcomes associated with it after submitting their initial bid.

strategy to receive the bonus. After both subjects submitted their guesses they would see the outcome of their bidding. They would learn what the other player bid, which items they both received, the transfer payment between them, their earnings, and their partners' points earned. Subjects would learn if they received a bonus for guessing the other player's bid correctly, but would not learn if the other player had received the bonus for guessing their bid correctly. After this information was disclosed, a new period would begin. This process would continue for 50 periods.

3.2 Ultimatum Bargaining Procedures

In the ultimatum bargaining session, at the beginning of each period, one subject would be randomly selected to be the proposer. That subject would choose who would receive each item, and if any transfer payments should be paid from one subject to the other. Transfer payments were limited so that no subject could receive negative earnings for each period (so, for example in Table 2, player 1 cannot propose player 2 receive item A and pay player 1 a transfer payment of 101 points, because that would result in player 1 receiving negative earnings).

After one subject had made a proposal, she would see a table of possible outcomes similar to figure 3 that would display the two possible outcomes (i.e., who gets which item, what amount is transferred for each player, each players earnings for that period) when the other subject accepted or rejected her proposal. Note that in the case of rejection, the outcome would be that each subject receives no items and no points for the period. The subject would have the opportunity to confirm or try another proposal. If she chose to try another proposal, the process would repeat until she confirmed a proposal. Once a proposal was confirmed, the other subject would view the proposal. The display would show him two outcomes—what would happen if he chose to accept or reject the other subject's proposal. The subject then would have the opportunity to accept or reject the proposal. After that decision was made, both subjects would view the outcome of the period. They would see what the first player had proposed, whether that proposal was accepted or rejected, the items and transfers received by each subject (if applicable) and the points earned of each subject for the period. After this information was disclosed, a new period would begin. This process would continue for 50 periods.

3.3 End of Experiment Procedures

Once the 50 periods were complete. Subjects would complete four surveys about their personality and preferences regarding fairness (see Appendix, Section 2). Three of the surveys were commonly used psychological surveys, the Barrat Impulsivity Scale (Patton et al., 1995), the Zuckermann Sensation Seeking-Scale (Zuckerman, 1994), and a five-factor personality assessment (John et al., 2008). The other survey asked subjects about their opinions of the other players who they had been matched, the mechanism used, and their general feelings of what fairness means. They were also given the opportunity to provide a tip up to $5, that would be doubled and divided among all other subjects.[10] Finally they were told they would play one more period at a valuation 10 times greater than before. All subjects then voted between the ultimatum and auction mechanisms. After all subjects completed the vote, the winning mechanism was implemented for the final period.[11] Since only one mechanism was used per experimental session a brief description was provided of the other mechanism. As carefully explained to the subjects a majority vote was required to change to the new mechanism, meaning the status-quo won all ties.[12]

After the final period, point values were totaled and converted to cash at the rate of 400 points=$1.00, rounded up to the nearest dollar. Subject earnings ranged from $14.70 to $33.80 ($25.44 average earnings) $14.20 to $33.80 ($26.40 average) for the ultimatum and auction sessions, respectively.

4 Applications of Nash Equilibrium, Quantal Response Equilibrium, Level-k with Quantal Response Models to our Auction Mechanism

Three different models may characterize the possible bidding strategies and outcomes for the subjects with the auction mechanism. We summarize these models in this section and discuss their possible issues in predicting subject behavior with the auction mechanism.

4.1 Nash Equilibrium

As explained in section 2.3.1, the auction mechanism implements all envy free allocation in Nash Equilibria. These equilibria exist on a continuum defined by equation 1 where the points transferred by the player who acquires item B, fall between $\left(\frac{v_1(B)-100}{2}, \frac{v_2(B)-100}{2}\right)$. Table 3 displays the range of equilibria, defined by the above interval, for each of the five valuations in the experiment.

[10] Surveys and tips were not found to vary by mechanism nor were they correlated with subject behavior.

[11] The results of this last period are not included in the data analysis.

[12] The status-quo won in both sessions though by a smaller margin in the ultimatum session

	lowest val. item B	highest val. item B	lowest possible Nash equil. bid	higest possible Nash equil. bid	lowest possible bound-rational bid	highest possible bound-rational bid
val 1	40	80	-30	-10	-60	0
val 2	120	160	10	30	0	60
val 3	40	120	-30	10	-60	20
val 4	160	160	30	30	0	60
val 5	0	40	-50	-30	-100	0

Table 3: Characterization of Nash Equilibria with Auction Mechanism: For the five valuations different continua of equilibria exist. Note that the symmetric fourth valuation only has one equilibrium where each player bids 30. The region of "boundedly rational bids" are defined in section 4.2.

4.2 Quantal Response Equilibrium

Developed in McKelvey and Palfrey (1995, 1998), the quantal response equilibrium (QRE) model uses the concept of noisy best response to allow equilibria where players will not play the action that yields the highest expected value with full probability. We may define such model in this context. Let each player i have a set of possible bidding strategies S_i and utility $u_i(z(s_i, s_{-i}))$ where $s_i \in S_i$ is her strategy and $s_{-i} \in S_{-i}$ represents the other player's strategies. Using the logit specification with parameter λ, if $\pi_j(s, \lambda)$ denotes the probability player j plays strategy $s \in S_i$. Then the player's noisy best response is to play strategy s_i with probability defined as

$$\begin{aligned}
\pi_i(s_i, \lambda) &= \frac{\exp\left[\lambda(E_{s_{-i}\in S_i}\left[u_i(z(s_i, s_{-i}))\right]\right]}{\sum\limits_{s\in S_i}\exp\left[\lambda E_{s_{-i}\in S_i}\left[z(u_i(s_i, s_{-i}))\right]\right]} \\
&= \frac{\exp\left[\lambda \sum\limits_{s_{-i}\in S_i} \pi_{-i}(s_{-i}, \lambda) u_i(z(s_i, s_{-i}))\right]}{\sum\limits_{s\in S_i}\exp\left[\lambda \sum\limits_{s_{-i}\in S_i} \pi_{-i}(s_{-i}, \lambda) u_i(z(s_i, s_{-i}))\right]} \qquad (3)
\end{aligned}$$

For any λ there exists a fixed point where equation 3 holds for both players. That is, all players are noisily best responding to each other. As $\lambda \to 0$ the system approaches each player uniformly randomizing over each action, as

$\lambda \to \infty$ the system approaches the Nash equilibrium as each player is best responding to each other.

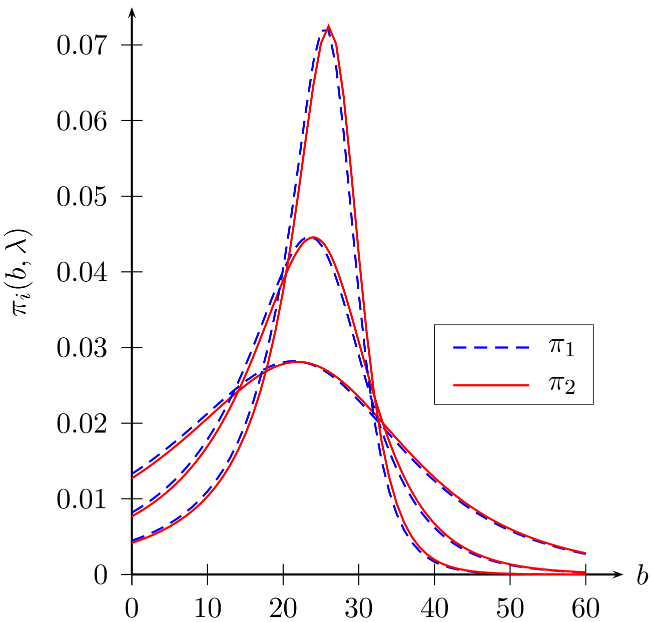

Figure 4: QRE first-price-auction-type mechanism: valuations $v_1^0(B) = v_2^0(B) = 160$ with support $[0, 60]$ and $\lambda = 0.0645,\ 0.15,\ 0.30$. Subject 2 is assigned object B with probability one in case of a tie.

In parameterizing our models in later sections, we make assumptions about the set of feasible bids, S_i. Although subjects may enter bids from the opposite of twice the value of item A (-200), to twice the highest valuation of item B (varies by valuation, i.e., 160, 320, 240, 320, 80 for valuations 1–5), we restrict our models over a range of bids we define as "boundedly rational".

Definition 1. *Consider 2 players bidding for items A and B. For each player define the following set for each player:*

$$\beta_i = \{b : min(v_i(B) - v_i(A), 0) \leq b \leq max(v_i(B) - v_i(A), 0)\}$$

Any bid, b' that falls in the union of each player's set, $b' \in \beta_i \bigcup \beta_j$ is said to be boundedly rational.

Basically boundedly rational bidding restricts players to bid non-negative amounts on item B if they prefer B to A and non-positive amounts for item B if they prefer A to B. The one exception is best responding to the other player may require one to violate the rule (e.g., a player prefers item B to A, but bids -10 because she believes the other player will bid -11). To allow this type of bidding as boundedly rational, we take the union of the two β_i sets created from each players preferences. Table 3 provides the ranges of boundedly rational bids for each of the five valuations. Note that all Nash equilibrium bids are boundedly rational.

Assumption 1. *In both parameterizations of QRE and level-k with quantal response we restrict the set of feasible bids, S_i to those bids which are boundedly-rational.*

4.3 Level-k with Quantal Response

While not an equilibrium model, level-k models (Stahl and Wilson, 1994; Nagel, 1995; Stahl and Wilson, 1995; Costa-Gomes et al., 2001; Costa-Gomes and Crawford, 2006) allow agents in a population to have heterogeneous levels of sophistication. Starting from level-0, each agent best responds to the previous level, with higher levels achieving more sophistication and ultimately converging to equilibrium. We apply a similar version of Crawford and Iriberri's (2007) continuous, level-k model to the auction mechanism. Though we apply the model to a discrete choice setting as subjects can only bid integer amounts. Their model also features a quantal response parameter, allowing noisy-best-response across subjects. To begin any level-k model, assumptions must be made about the level-0 types. We assume that each level-0 plays all feasible strategies with equal probability. That is,

$$\pi_i(s_i, \lambda, 0) = 1/\left|\left|S_i\right|\right|.$$

All level-k types then noisily best respond to level-$k-1$.

$$\pi_i(s_i, \lambda, k) = \frac{\exp\left[\lambda \sum\limits_{s_{-i} \in S_i} \pi_{-i}(s_{-i}, \lambda, k-1) u_i(s_i, s_{-i})\right]}{\sum\limits_{s \in S_i} \exp\left[\lambda \sum\limits_{s_{-i} \in S_i} \pi_{-i}(s_{-i}, \lambda, k-1) u_i(s_i, s_{-i})\right]} \tag{4}$$

Mechanism	Auction	Ultimatum
efficient outcomes	437	500*
(percent)[a]	(0.728)	(0.769)
envy-free outcomes[b]	283	234*
(percent)	(0.472)	(0.360)
efficient and envy-free outcomes[c]	283	134***
(percent)	(0.472)	(0.206)
average profit (in points)	99.917	88.644***
(standard error)[d]	(1.111)	(1.731)
percent of maximum possible profit	0.935	0.829***
(standard error)[a]	(0.005)	(0.014)

* significant difference at the 10% level.
** significant difference at the 5% level.
*** significant difference at the 1% level.

Table 4: Outcomes of Auction and Ultimatum Mechanisms: Efficient, envy-free, and profit values for each period in the auction and ultimatum experimental sessions.
a. There are 600 and 650 observations of subject-pairs for the auction and ultimatum mechanism, respectively.
b. The 100 rejections (0 earnings for each player) are included in envy-free outcomes for the ultimatum mechanism.
c. Only accepted, envy-free proposals would be both efficient, envy-free allocations in the ultimatum mechanism (12 such proposals were rejected). All envy-free allocations are also efficient with the auction mechanism.

Note that as $k \to \infty$, if this model converges it will converge to a quantal response equilibrium of parameter λ.[13]

5 Data and analysis

5.1 Outcomes

Both the auction mechanism and ultimatum bargaining sessions consisted of 50 periods. With 24 subjects and 26 subjects there are 1200 and 1300 subject choices in the auction and ultimatum sessions, respectively. Since each subject was paired each round, this gives 600 and 650 unique outcomes in each session.

Table 4 characterizes each of these outcomes for each session with the auction and ultimatum mechanisms. Subjects using the ultimatum mecha-

[13]At higher values of λ (above 0.5) the model does not converge. Instead it cycles between strategies. These high values were not encountered when characterizing subject behavior.

nism realized efficient outcomes at a higher frequency than those using the auction mechanism (77 vs. 73 percent), albeit at a moderately significant level (p-value≈ 0.095). Subjects using the auction mechanism achieved significantly more envy-free outcomes, more profit per period, and a higher percentage of the maximum profit possible than ultimatum bargaining (all p-values< 0.01).

It should be noted that table 4 shows that envy-free proposals were rejected at a much lower rate than proposals with envy. Note c states that only 12 of 146 such proposals were rejected (92 percent acceptance) as opposed to 100 of the 504 proposals (80 percent acceptance). However, we cannot conclude that envy-free proposals are less likely to be rejected since they are highly correlated with the amount the receiver acquires in this setting. Thus it cannot be ascertained whether the receiver just accepts higher offers and does not care about no-envy, or if she accepts offers with no envy at higher rates.

The differences in the mechanisms likely explain the differing results in Table 4. The ultimatum mechanism is a sequential mechanism and involves less complex strategy than the simultaneous auction mechanism. Thus both coordination failures and improper bidding in the auction mechanism may reduce the efficiency of that mechanism relative to the ultimatum mechanism (which of these two factors was responsible will be discussed later in this section). However, the auction mechanism implements envy-free allocations as Nash equilibria, which explains its higher level of envy-free allocations achieved. Since the lowest combined earnings in the ultimatum mechanism occur at rejection—where both players receive nothing (an outcome that occurs 15 percent of the time)—as opposed to at an inefficient outcome in the auction mechanism, it achieves much fewer outcomes that both are efficient and envy-free, as well as lower earnings for both players. The equal-opportunity property of the auction mechanism also leads to the greater number of envy-free allocations.

Given that subjects using the auction mechanism achieved envy-free allocations less than half the time, and achieved efficiency less often than the ultimatum mechanism, it is natural to investigate the causes of such performance. One possibility is that subjects bid at Nash levels, but did not coordinate on the same Nash equilibrium, leading to inefficient outcomes with envy. Another possibility subjects did not bid at Nash levels, caus-

valuation (low val., high val.)	mean bid (sandard error)	Nash bids (percent)	boundedly rational bids (percent)[a]
val. 1[b]	−28.183	100	164
(40,80)	(2.395)	(0.417)	(0.683)
val. 2	3.704	114	143
(120,160)	(1.594)	(0.475)	(0.596)
val. 3	−13.158	200	220
(40,120)	(1.765)	(0.833)	(0.917)
val. 4	19.721	49	216
(160,160)	(0.894)	(0.204)	(0.900)
val. 5	−50.142	140	198
(0,40)	(2.491)	(0.583)	(0.825)
Totals	−13.162	603	941
	(1.108)	(0.503)	(0.784)

Table 5: Subject Bidding by Type and Valuation: Proportion of bids that are Nash or boundedly rational in each valuation.
a. See Table 3 for ranges of each type of bid by valuation.
b. Each valuation contains 240 bids.

ing the auction mechanism to achieve outcomes that were neither efficient nor envy-free. One reason subjects might bid this way is that they do not understand the mechanism fully, and do not make boundedly-rational bids (see section 4.2 for the definition of "boundedly rational" in this context).

Tables 5 and 6 explore these questions further. Table 5 counts the number of Nash, boundedly rational, and other bids for each of the five valuations. In all valuations, the mean bid is significantly lower than the lowest Nash equilibrium bid.[14] On average, subjects bid a Nash Equilibrium bid only half the time. Bidding generally varies by valuation: valuation 4 which has only one value corresponding to a Nash bid, has the lowest number of Nash bids; valuation 3 which has a large Nash Equilibrium bid range, a range which also contains 0 (the modal bid, and likely focal point) has the highest number of Nash bids. Subjects generally make boundedly rational bids (roughly 80 percent of the time). The figure greatly increases over the last three valuations, indicating that subjects may be learning how to bid better with experience.

Table 6 breaks down the efficient and envy-free outcomes by paired bid types. The most noticeable result is that envy-free allocations do not occur

[14]This relationship is generally true for median bids (except in valuations 3 and 5) indicating that a few outliers are not driving this result.

event	frequency (percent)	efficient allocations within event (percent)	envy-free allocations within event (percent)
2 Nash bids	162 (0.270)	126 (0.778)	126 (0.778)
1 Nash bid and 1 non-Nash boundedly rational bid	128 (0.213)	77 (0.602)	59 (0.461)
1 Nash bid and 1 non-bound. rational bid	151 (0.252)	111 (0.735)	98 (0.605)
2 non-Nash, boundedly rational bids	66 (0.110)	63 (0.955)	0 (0.000)
1 non-Nash, bound. rational bid and 1 non-bound. rational bid	55 (0.092)	40 (0.727)	0 (0.000)
2 non-boundedly rational bids	38 (0.063)	20 (0.526)	0 (0.000)
Totals	600	437 (0.728)	283 (0.472)

Table 6: Outcome by Type of Bids: The 600 outcomes of the auction mechanism divided by bid type. See section 4.1 and4.2 for classification of Nash and boundedly rational bids. All Nash bids are boundedly rational, therefore all bids can be divided into the three categories with no overlap.

when both players fail to makes a Nash bid. This suggests that the lack of Nash bidding (only about half of all bids) is largely responsible for the low number of envy-free outcomes. When both players do Nash bid, they achieve efficient, envy-free outcomes roughly 77.8 percent of the time, a great increase in envy-free allocations. However, this number shows that even when subjects bid ideally, coordination failure present in simultaneous bidding does reduce the number of efficient outcomes by about 20 percent. With the exception of 2 non-Nash, boundedly rational bids–where efficiency is achieved in an astonishing 96 percent of outcomes–all other pairings of bids have lower frequency of efficient outcomes being realized, suggesting that lack of Nash bidding is also responsible for the reduction in efficient outcomes of the auction mechanism.

It should be noted that selection does greatly influence the numbers in table 6. Valuation 4 contained the fewest number of Nash bids (49 of 240) and consisted entirely of efficient outcomes since all outcomes would turn out efficient regardless of bids. Of the 63 efficient allocations that occurred with 2 non-Nash, boundedly rational bids, 55 were in valuation 4. This means excluding valuation 4, only 8 of 11 cases of 2 non-Nash, boundedly rational bids were efficient. So we must be careful interpreting numbers in the table. In the next section we will parameterize subjects by their biding strategy, and again look at outcomes by bidding types to determine the effects of more sophisticated bidding by types (see table 9).

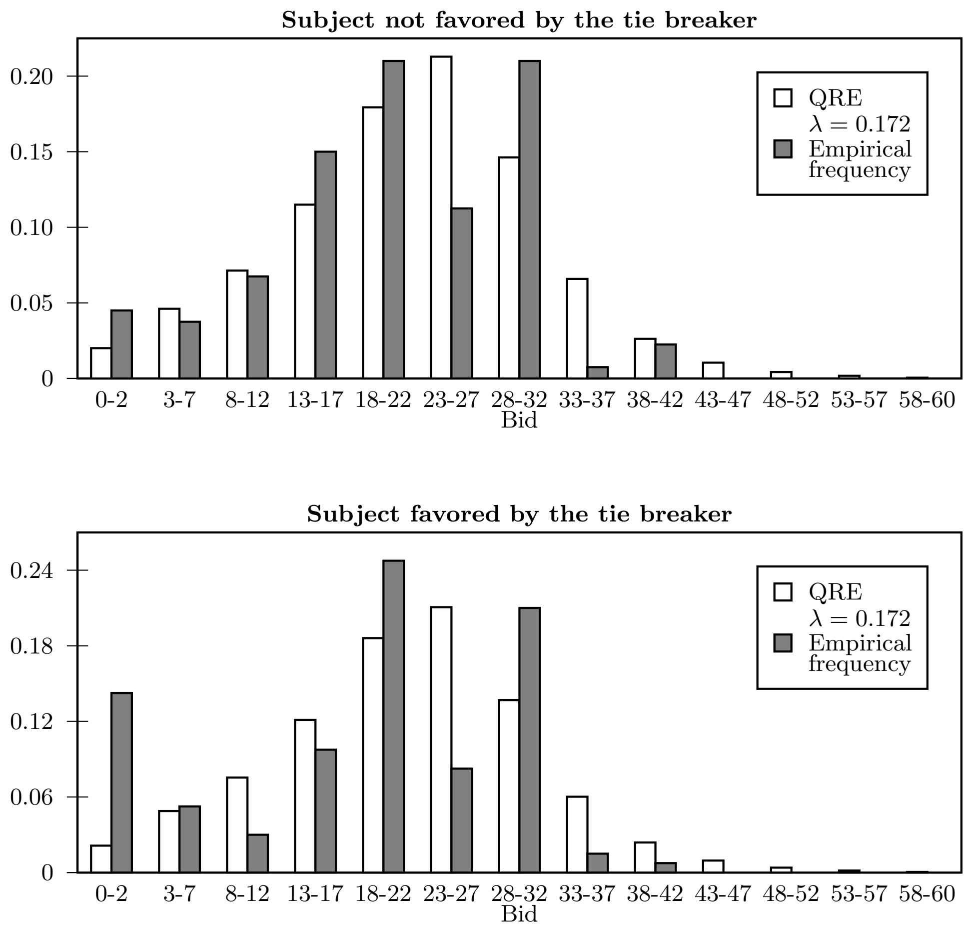

Figure 5: Estimated QRE vs. empirical frequencies: valuations $v_1^0(B) = v_2^0(B) = 160$ with support $[0, 60]$ and $\lambda = 0.172$.

5.2 Classifying subject types

In this section we will characterize subject bidding behavior using the QRE model (McKelvey and Palfrey, 1995, 1998) and level-k model applied to auctions (similar to Crawford and Iriberri, 2007).

When analyzing bids, we will restrict our bids to integer values that fit our definition of boundedly rational (see section 4.2). This restriction gives us a finite number of bids in our feasible bid set, S_i. We may assume that in any period, player 2 has the high valuation on item B (or wins the tie-breaker in the case of valuation 4). Then we will characterize S_i for each player as $S_i = (s_i 1 \ldots s_i N)^T$ where s_{i1} and s_{iN} are the lower and upper boundaries of boundedly rational bids for player i (see table 3 for such ranges for each valuation). The using the QRE model, there exists a vector of probabilities, $\vec{\pi_i}(S_i, \lambda) = (\vec{\pi_{i1}}(\lambda) \ldots \vec{\pi}_{iN}(\lambda))^T$ for any λ where $\vec{\pi}_i(S_i, \lambda)$ is the probability of $s_i \in S_i$ being played by player i at a quantal response equilibrium with parameter λ. Let Xij be a $10 \times N$ matrix that contains subject i's bids for valuation j. Specifically, matrix Xij's elements are of the form

$$x_{ij_{mn}} = \begin{cases} 1 & : \textit{ if subject } i \textit{ bid } b_m \textit{ in the } n'th \textit{ period of valuation } j \\ 0 & : \textit{ otherwise} \end{cases}$$

Further, define h_{ij} as a 1×10 column vector that indicates (by a 1) whether subject i in valuation j had the high or low value on item B for each of the 10 periods in that valuation. Then we may construct the log likelihood function for any λ of each subject bidding the way they did over the five valuations.

$$L(\lambda) = \sum_{i=1}^{I} \sum_{j=1}^{5} \left[X_{ij} \log\left[\vec{\pi}(S_2, \lambda)\right] h_{ij} + X_{ij} \log\left[\vec{\pi}(S_1, \lambda)\right] (1 - h_{ij}) \right] \qquad (5)$$

Table 7 provides maximum likelihood values of λ for each valuation and over all valuations. The overall maximum likelihood value is $\lambda \approx 0.040$.

One reason the QRE estimates appear so smooth relative to the spikes observed i the data is that they are unable to classify subjects by types. We alleviate this limitation by using a level-k model with quantal response to

Valuations	best-fitting λ (standard error)	log likelihood	boundedly rational bids (percent)
val. 1	0.012***	-747.33	164
(40, 80)	(6.23E-05)		(0.683)
val. 2	0.092***	-674.348	143
(120, 160)	(5.76E-05)		(0.596)
val. 3	0.018***	-980.382	220
(40, 120)	(4.50E-05)		(0.917)
val. 4	0.172***	-850.963	216
(160, 160)	(2.19E-05)		(0.900)
val. 5	0.027***	-1018.43	198
(0, 40)	(4.99E-05)		(0.825)
aggregate	0.040***	-4368.81	941
estimation	(6.23E-05)		(0.784)

* significant difference at the 10% level.
** significant difference at the 5% level.
*** significant difference at the 1% level.

Table 7: Maximum Likelihood λ by Valuation: Best-fitting λ in the QRE model by observations grouped by subject valuations 1–5. Each valuation featured a total possible 240 bids. The value $\lambda \approx 0.040$ is the maximum likelihood value when observations taken from all valuations are pooled. All values were estimated over the boundedly rational ranges.

classify subjects by type. To create a likelihood function we redefine our probability vector $\vec{\pi}_i(S_i, \lambda, k)$ where k is the level of steps of thinking and λ is the quantal response parameter. For every lambda, we calculate a best fitting k for each subject, then we calculate the log likelihood as before.

$$L(\lambda) = \sum_{i=1}^{I} \min_k \sum_{j=1}^{5} \left[X_{ij} \log\left[\vec{\pi}(S_2, \lambda, k)\right] h_{ij} + X_{ij} \log\left[\vec{\pi}(S_1, \lambda, k)\right](1 - h_{ij})\right] \tag{6}$$

The value of $\lambda \approx 0.060$ minimizes equation 6, with most subjects estimated to have $k > 0$, indicating they are doing at least one step of iterative thinking. The frequencies of levels of thinking are given in table 8. Appendix table A.1 provides more detail in this estimation process, including a breakdown of maximum likelihood estimates by valuation.

The type classifications of the level-k model allow us to return to the question we investigated in table 6, but in a way that escapes the selection issues from that table. Since all types are randomly paired with each other,

level of steps of thinking in level-k model	number of subjects	percent of subjects
0	5	0.208333
1	10	0.416667
2	2	0.083333
3	5	0.208333
4	1	0.041667
5	1	0.041667

Table 8: Subject Distribution by Level-k: Frequency of maximum likelihood level-k values for all subjects. The maximum likelihood quantal response parameter is $\lambda \approx 0.06$. Subject observations across all valuations are pooled. All values were estimated over the boundedly rational ranges. No subject had an estimated level-k value above 5.

we may observe how outcomes depend on the types of players involved in a way that does not depend on valuation. We will classify high types as those that do two or more steps of thinking, and low types as those that do zero or one steps. Table 9 provides a breakdown of efficient and envy-free outcomes

event	frequency (percent)	efficient allocations within event (percent)	envy-free allocations within event (percent)
2 high level types	79 (0.132)	65 (0.823)	44 (0.557)
1 high level type, 1 low level type	292 (0.487	205 (0.702)	133 (0.455)
2 low level types	229 (0.382)	167 (0.729)	106 (0.463)
Totals	600	437 (0.728)	283 (0.472)

Table 9: Outcomes by Subject Sophistication: Efficient and envy-free allocations separated by type-interaction. Subjects with best fitting level-k values 2 or higher are classified as high types; the others are classified with low types.

by the interaction of types. Two high types are more likely to produce efficient and envy-free allocations than two low types. But it appears that one low type and one high type do worse than any other pairing in both categories. The results are only suggestive, we may only say that 2 high-level types have significantly more efficient and envy-free outcomes at the 0.10 level. Similar to table 6, the best types still fail to achieve efficient outcomes roughly 20 percent of the time, suggesting this may be an upper bound on efficiency caused by coordination failure.

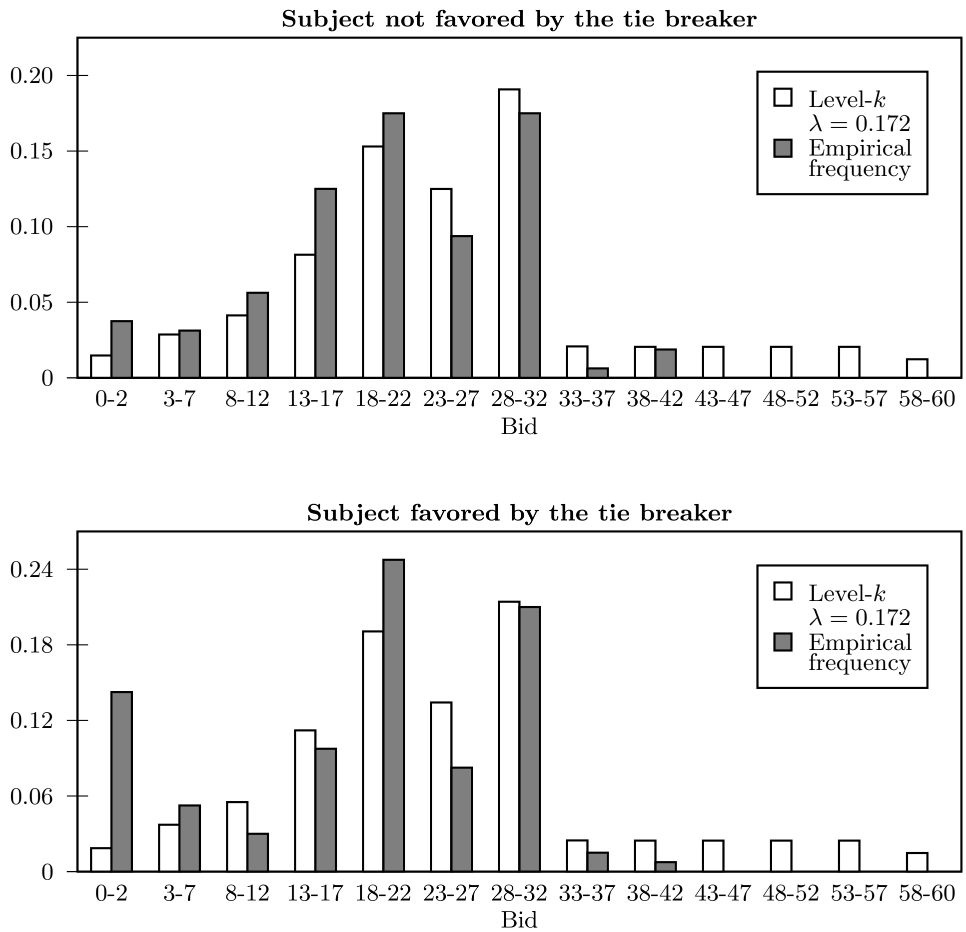

Figure 6: Estimated Level-k with Quantal Response Model vs. empirical frequencies: valuations $v_1^0(B) = v_2^0(B) = 160$ with support $[0, 60]$ and $\lambda = 0.172$.

6 Discussion

In this section we introduce a family of mechanisms that generalize our first-price-auction-type mechanism. Then we analyze the possibility of selecting, out of this family, the mechanism that maximizes the probability of obtaining a envy-free allocation in a QRE for the mechanism.

6.1 α-auction

Let $\alpha \in [0,1]$. The α-auction is the mechanism in which first agents report bids (possibly negative) for object B. Then an agent with the highest bid receives object B and pays the α convex combination between the maximum and the minimum bid, i.e., $\alpha \max\{b_1, b_2\} + (1-\alpha)\min\{b_1, b_2\}$. The other player receives object A and the transfer of money from the agent who receives object B. In case of a tie, an agent with highest true valuation of object B receives object B and pays her bid.

Pure strategy Nash equilibria and pure strategy Nash equilibrium outcomes coincide for all α-auctions. Recall that we assume without loss of generality that $v_1^0(B) \leq v_2^0(B)$. A profile (b_1, b_2) is a Nash equilibrium of the α-auction if and only if $b_1 = b_2$ and the common bid is in the interval

$$\left[\frac{v_1^0(B) - 100}{2}, \frac{v_2^0(B) - 100}{2}\right].$$

The family of α-auctions generalizes our first-price-auction-type mechanism ($\alpha = 1$). Additionally, this family contains the second-price-auction-type mechanism ($\alpha = 0$) and the average-price-auction-type mechanism ($\alpha = \frac{1}{2}$).

6.2 Maximizing no-envy

We have documented that in an experimental setting, 1-auction (i.e., first-price) outcomes deviate from the Nash equilibrium prediction. Moreover, these deviations are explained to some extent by players unsophisticated bidding. We now perform a simulation exercise in which we assume that agents exhibit the same sort of unsophisticated bidding in α-auctions, and highlight some open questions and avenues for future research. For simplicity in the presentation, we concentrate in our QRE explanation of agents' behavior.

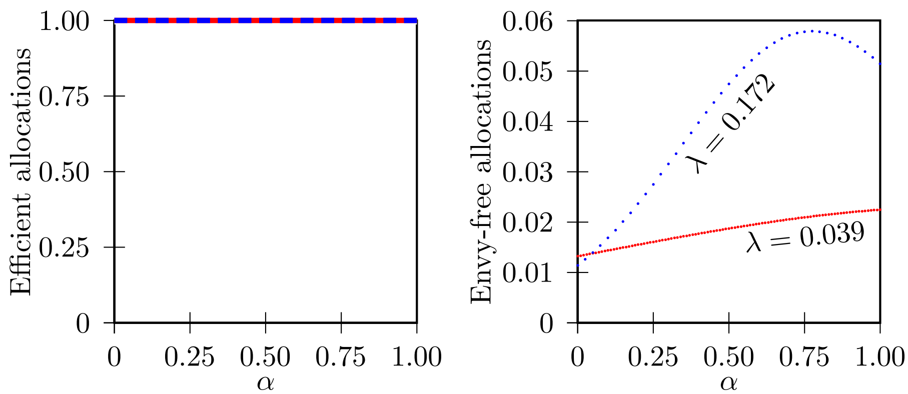

Figure 7: Probability of obtaining an efficient and envy-free allocation in a QRE of the α-auction: valuations $v_1^0(B) = v_2^0(B) = 160$ with support $[0, 60]$ and $\lambda = 0.039,\ 0.172$.

Figure 7 shows the probability with which efficient and envy-free allocations are realized in a QRE equilibrium for each α-auction with $\alpha \in [0, 1]$ when $v_1^0(B) = v_2^0(B) = 160$, $\lambda = 0.039$, and $\lambda = 0.172$ (these values correspond to valuation 4 in the experiment). Since each allocation for these valuations is efficient, there is no trade off between no-envy and efficiency. Generically, the probability that an envy-free allocation is realized in a QRE equilibrium in the α-auction is greater when $\lambda = 0.172$ than when $\lambda = 0.039$. This is expected since as λ converges to infinity, QRE equilibria converge to Nash equilibria.[15] A more surprising finding is that for a fixed sophistication index λ, the probability to attain an envy-free allocation significatively varies with α. For $\lambda = 0.172$, the difference between the optimal envy-free α-auction and the 0-auction (second-price) is close to 5 percent. More striking differences hold among α-auctions when valuations are $v_1^0(B) = 40$ and $v_2^0(B) = 80$ and $\lambda = 0.039$, for example. Figure 8 shows the performance, both in terms of efficiency and no-envy in this case. The 0-auction (second-price) achieves higher efficiency than each other α-auction. However, this gain in efficiency is at most 0.0025. By contrast, the optimal envy-free auction achieves no-envy with 0.10 more probability than the 0-auction.

[15]Technically, the limit of QRE equilibria in our model are mixed-strategy Nash equilibria; for valuation $v_1^0(B) = v_2^0(B) = 160$, it is easy to prove that all these equilibria are envy-free; it is an open question to characterize mixed-strategy equilibria for arbitrary valuations in our model.

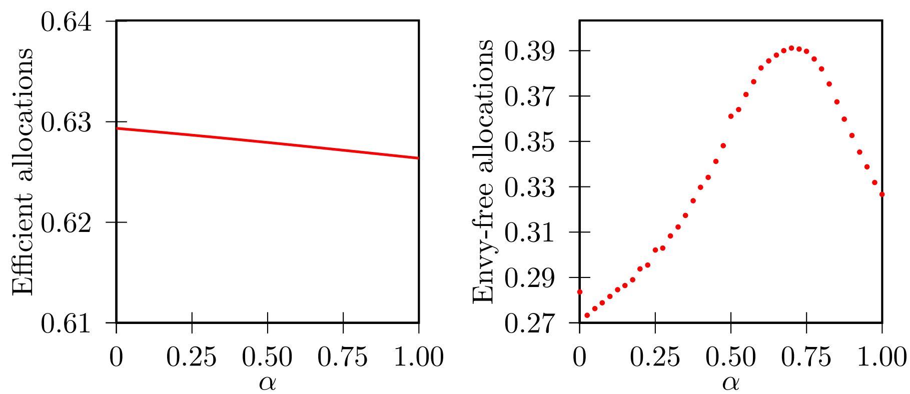

Figure 8: QRE first-price-auction-type mechanism: valuations $v_1^0(B) = 40$ and $v_2^0(B) = 80$ with support $[-60, 0]$ and $\lambda = 0.039$.

The simulation exercises in Figures and 7 and 8 illuminate us about the effect of players unsophisticated bidding in the possibility to achieve envy-free allocations. Even though all α-auctions are Nash equilibrium outcome equivalent, in experimental settings our results suggest some auctions may perform better than the others. It is an open question to better calibrate agents' behavior in α-auctions for $\alpha \neq 1$. A relevant issue here is that in our simulation we assumed the support of each QRE to be uniform across auctions. A more refined theory can be constructed if one better calibrates the relation of these supports and α (numerical calculations indicate that the probability with which envy-free allocations are achieves in the QRE of an α-auction depend on the choice of strategies support).

7 Conclusions

This paper characterizes the equilibria of an envy-free, first-price, auction mechanism and compares its outcomes experimentally to those achieved through ultimatum bargaining. Both mechanisms achieve relatively equal numbers of efficient outcomes—ultimatum bargaining is slightly higher—but the auction mechanism achieves significantly more envy-free allocations and higher earnings for subjects. Nonetheless, these envy-free outcomes are achieved less than half the time. Dividing outcomes by bid type and subject sophistication, our results suggest that with sophisticated bidding subjects may achieve efficient outcomes 80 percent and envy-free outcomes 55–75 percent. Coordination failure likely causes the remaining discrepancy.

Since subject bids are not fully sophisticated, our results are dependent on the first-price auction mechanism. Our estimates suggest that a first-price auction mechanism is not ideal in this setting—an average of the two bids could achieve a higher proportion of envy-free outcomes. Future research will need to confirm this prediction.

References

Alkan, A., Demange, G., Gale, D., 1991. Fair allocation of indivisible goods and criteria of justice. Econometrica 59, 1023–1039.

Āzacis, H., 2008. Double implementation in a market for indivisible goods with a price constraint. Games Econ Behavior 62, 140–154.

Beviá, C., 2010. Manipulation games in economies with indivisible goods. International Journal of Game Theory 39 (1), 209–222.

Costa-Gomes, M., Crawford, V. P., Broseta, B., 2001. Cognition and behavior in normal-form games: An experimental study. Econometrica 69 (5), 1193–1235.

Costa-Gomes, M. A., Crawford, V. P., 2006. Cognition and behavior in two-person guessing games: An experimental study. The American Economic Review 96 (5), 1737–1768.

Crawford, V. P., Iriberri, N., 2007. Level-k auctions: Can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions? Econometrica 75 (6), 1721–1770.

Fischbacher, U., 2007. Z-tree: Zurich toolbox for ready-made economic experiments. Experimental Economics 10 (2), 171–178.

Foley, D., 1967. Resource allocation and the public sector. Yale Economic Essays 7, 45–98.

Fujinaka, Y., Wakayama, T., 2011. Maximal manipulation in fair allocation, Mimeo, Tokio Institure of Technology.

Herreiner, D. K., Puppe, C. D., 2009. Envy freeness in experimental fair division problems. Theory and Decision 67, 65–100.

John, O. P., Naumann, L. P., Soto, C. J., 2008. Paradigm shift to the integrative big-five trait taxonomy: History, measurement, and conceptual issues. In: John, O. P., Robins, R. W., Pervin, L. A. (Eds.), Handbook of Personality: Theory and Research. Guilford Press, New York, NY, pp. 114–158.

McKelvey, R. D., Palfrey, T. R., 1995. Quantal response equilibria for normal form games. Games and Economic Behavior 10 (1), 6–38.

McKelvey, R. D., Palfrey, T. R., 1998. Quantal response equilibria for extensive form games. Experimental Economics 1 (1), 9–41.

Nagel, R., 1995. Unraveling in guessing games: An experimental study. American Economic Review 85 (5), 1313–1326.

Patton, J. H., Stanford, M. S., Barratt, E. S., 1995. Factor structure of the barratt impulsiveness scale. Journal Of Clinical Psychology 51 (6), 768–774.

Stahl, D. O., Wilson, P. W., 1994. Experimental evidence on players' models of other players. Journal of Economic Behavior and Organization 25 (3), 309–327.

Stahl, D. O., Wilson, P. W., 1995. On players' models of other players: Theory and experimental evidence. Games and Economic Behavior 10 (1), 218–254.

Svensson, L.-G., 1983. Large indivisibles: an analysis with respect to price equilibrium and fairness. Econometrica 51, 939–954.

Tadenuma, K., Thomson, W., 1995a. Games of fair division. Games Econ Behavior 9, 191–204.

Tadenuma, K., Thomson, W., 1995b. Refinements of the no-envy solution in economies with indivisible goods. Theory and Decision 39, 189–206.

Thomson, W., 2006. Fair allocation rules, Unpublished Manuscript, University of Rochester.

Varian, H., 1974. Equity, envy, and efficiency. J Econ Theory 9, 63–91.

Velez, R., 2011a. Are incentives against economic justice? J Econ Theory 146, 326–345.

Velez, R., 2011b. Fairness and externalities, Mimeo.

Velez, R., Thomson, W., 2009. Let them cheat!, RCER Working paper #552 http://rcer.econ.rochester.edu/RCERPAPERS/rcer_552.pdf.

Zuckerman, M., 1994. Behavioral Expressions and Biosocial Bases of Sensation Seeking. Cambridge, University Press, Cambridge.

A Appendix

Appendix table A.1 provides a breakdown of maximum likelihood estimates by valuation in the level-k model with quantal response.

val. (l.v., h.v.)	best-fit. λ (st.err.)	log l-hood	prop. level 0 (st.err.)	prop. level 1 (st.err.)	prop. level 2 (st.err.)	prop. level 3 (st.err.)	prop. level 4 (st.err.)	prop. level 5 or above (st.err.)
val. 1	0.083***	-733.631	0.542***	0.208***	0.083*	0.125***	0.042**	0.000
(40, 80)	(0.007)		(0.059)	(0.032)	(0.043)	(0.042)	(0.019)	(0.000)
val. 2	0.188***	-650.579	0.167***	0.417***	0.083***	0.000	0.000	0.333***
(120, 160)	(0.005)		(0.009)	(0.042)	(0.013)	(0.011)	(0.004)	(0.037)
val. 3	0.108***	-959.436	0.667***	0.250***	0.000	0.042***	0.000	0.042
(40, 120)	(0.010)		(0.073)	(0.051)	(0.063)	(0.007)	(0.019)	(0.025)
val. 4	1.791***	-757.297	0.333***	0.125***	0.000	0.000	0.000	0.542***
(160, 160)	(0.053)		(0.008)	(0.010)	(0.013)	(0.011)	(0.016)	(0.031)
val. 5	0.072***	-993.97	0.333***	0.083***	0.000	0.542***	0.042***	0.000
(0, 40)	(0.007)		(0.039)	(0.008)	(0.021)	(0.039)	(0.004)	(0.021)
aggregate	0.060***	-4338.88	0.208**	0.417***	0.083	0.208***	0.042	0.042***
estimation	(0.006)		(0.079)	(0.043)	(0.074)	(0.038)	(0.025)	(0.002)

* Significant at the 10% level

** Significant at the 5% level

*** Significant at the 1% level

Table A.1: Level-k with quantal response maximum likelihood estimates for λ with level-k selected for each subject. All estimates are taken from the range of boundedly rational bids.