Relinquinshing Power, Explotation and Political Unemployement in Democratic Organizations*

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Abstract

This paper focus on the dynamics of organizations and how they design its future according to the interest of their members. Agents are grouped into three classes, high, medium and low productivity. We analyze the evolution of organizations which take decisions by majority voting. We focus on the evolution of the political power and show that in some cases, rational agents who valuate the future may yield political power to other class. This is what we call the relinquish effect. We show that exploitation is possible in democratic societies and study its determinants. We also show that high productivity agents may be left in the cold because their entrance in an organization may threaten the dominance of other classes. We call this political unemployment.

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1. Introduction

The last two decades have seen a growing interest in the dynamic side of the theory of organizations. In particular, organizations evolve over time, changing in size, internal composition and distribution of output. The example that comes to our mind are University departments but our main ideas can be applied to other settings like cooperative firms or political representation.

As a starting point, one may think that when the future does not count much, a (rational) dominant class may take actions that in the future will endanger the political supremacy of this class. This corresponds to dictum by French king Louis XV "Apres de moi le deluge". But when the future counts, one would expect that no (rational) dominant political class will take actions that endanger their political supremacy. We may call this conjecture "The Iron Law of Political Power". According with this law, a dominant class will only endorse political supremacy to other class if a) it is myopic, b) it is violently deposed or c) when the threat of violent deposition is credible.[1] In this paper we present a three-class model in which this law does not hold.

We consider an organization that lasts for an infinite number of periods. We assume that organizations concentrate on their profit rather than on their preferences to hire workers. Agents outside the organization are called outsiders and those inside the organization are called insiders. These agents are distinguished by their productivity and can be of three types: H (high), M (medium) or L (low). We will refer to each group as a class. There is a pool of outsiders of all types that are potential entrants in the organization. In each period, the organization takes three decisions sequentially by majority voting. First, it decides how to share the output produced in this period. Second, it decides who joins the organization. For simplicity we will assume that the organization hires a new member only every period. The hired agent will participate on the voting of this rule and on the hiring of a new member from the next period on. Next, the agents who wish, leave the organization and join the pool of agents outside the organization. The leaving agents receive their reservation utility outside of the organization. Finally, production takes place and output is shared among insiders according to the rule that the organization has voted. A class has the political power -or is dominant- when a member of this class is a pivotal voter in the voting on the sharing rule.

We show that when the discount factor δ is close to one and the organization is capable of

[1] See Acemoglu and Robinson (2000) and Llavador and Oxoby (2005).

producing a surplus, if H is the dominant class at time 0, they will vote for meritocracy and they will hire H types only. But in any other case, M type agents become pivotal in the voting and, eventually they will vote egalitarianism and they will hire M and H agents in such a way that the stay pivotal. In particular if L agents are a majority at the starting time they will relinquish power to the M class. This is because, in the long run, the interest of the middle and low classes regarding distribution are aligned: they prefer an egalitarian distribution scheme. And hiring members of the middle class beefs up production and with an egalitarian policy this beefs up the payoffs of the low class. We call this the *Relinquish effect* and it happens any time that a dominant class relinquish power to another class that, for whatever reason, may be more efficient managing the economy, and this class will not alter neither the rules nor the outcome of the game. The rules may be voting or not. Thus, in imperial China under the Tang dynasty, the oligarchy relinquished a lot of their power into the Mandarin class which was more efficient than they were in managing the economy and had no plan to change neither the rules, nor the outcome of the game. Thus our paper aims to contributes to the understanding of how power is maintained (or lost) in organizations and how it translates into the distribution of resources.

In our model, initial insiders determine the long run performance of the organization. In particular organizations in which the high type is not dominant in the first period, will never achieve a majority of high type agents, even though the percentage of high type can be close to fifty percent. All these agents have to share their output with the other types so they are paid less than their productivity. We say that these agents are *Exploited*. And since there are high type agents that could have been hired we say that we have *Political Unemployment*, i.e. unemployment due to the political constraints inside the organization (in this case, majority voting).

Next, we consider finitely lived agents. In this scenario we see that the demographic structure at time zero plays an important role. In some cases, there is convergence to the equilibrium where meritocracy holds even if we start with a majority of low types.

This paper focus on the policies followed by rational classes and is written in the spirit of the Monumental "A General Theory of Exploitation and Class" (1982) in which John Roemer laid the foundations of a rigorous analysis of exploitation and class. Of course our methods are different -voting and dynamic game theory barely existed then - but we hope that our contribution opens new research avenues worth to be explored. [2]

[2] For a survey of elections in stationary environments see Duggan and Martinelli (2015)).

The remainder of the paper is organized as follows. Section 2 introduces the basic model. Section 3 presents our main results. Section 4 extends the basic model to finitely-lived agents. Section 5 concludes.

2. The Model

An organization is a productive facility. Some agents belong to the organization and are called insiders, and some do not belong to the organization and are called outsiders. Both the organization and all agents last for a countable infinite number of periods $\tau = (0, 1, ...t, ...)$.

Agents can be of three types: H, M and L. If an agent of type $T \in \{H, M, L\}$ is an insider, she produces a per period gross output of o_T. The per period reservation utility of an agent of type $T \in \{H, M, L\}$ is denoted by u_T. Let $x_T \equiv o_T - u_T$ be the surplus (or net output) that the organization obtains from an agent of type T. Assume that $x_H > x_M > x_L > 0$. Thus type H is the high productivity type, M is the medium productivity type, and L is the low productivity type.

At each period τ, the organization takes two decisions in turn. First, the insiders in period $\tau - 1$ decide how the total surplus in this period is going to be shared. Second, they decide who joins the organization and the outsiders decide if they accept the offer or not. Next, the agents who wish, leave the organization and receive their reservation utility. Finally, production takes place and output is shared among the new insiders. The timing of the decisions is motivated by the fact that the decision on the distributive rule must be made in advance of next hire, if not, insiders would not know the distributive effects of the hiring and outsiders would not know the exact offer made to them. Furthermore, if agents would leave the organization before voting this would be equivalent to consider an organization with a different set of agents. Thus, changes in population due to exit will have its effect both at the production moment and in the next period decision stage.

If there are n_T^τ insiders of type $T = H, M, L$ in the period τ (including the new hiring in this period), the surplus to be shared, denoted by X^τ, is

$$X^\tau \equiv n_H^\tau x_H + n_M^\tau x_M + n_L^\tau x_L.$$

Roberts (2015) and Razin, Sadkay and Suwankiriz (2015) present models in which the constituency is endogenous and show that a dynamic equilibrium exists. Dziuda and Loeper (2015) and Zapal (2015) present models of legislative bargaining.

All insiders at period τ are paid their reservation utility plus a share in the surplus.

At the beginning of period 0 the organization is populated by some insiders and has a rule (i.e. a constitution) that fixes how collective decisions are taken. In this paper we will concentrate in the case in which decisions are taken by majority voting (plurality).

In order to simplify the model we make the following assumptions.

Assumption A1. The pool of outsiders includes agents from the three types.

Assumption A2. The organization makes a single offer per period.

Assumption A3. Output is distributed by the following sharing rule

$$c_T^\tau = \theta^\tau \frac{X^\tau}{n_H^\tau + n_M^\tau + n_L^\tau} + (1-\theta^\tau)x_T + u_T,$$

where c_T^τ is the output allocated to an agent of type T in the period τ and $\theta^\tau \in [0,1]$. Given that aggregate surplus is positive all insiders receive a payoff larger than their reservation utility so any outsider who is invited to enter the organization accepts the offer and no insider wants to leave.

Notice that agents of type L (resp. H) would always prefer $\theta^\tau = 1$ (resp. $\theta^\tau = 0$). The preferences of agents of type M will depend on the group structure of the organization. If $x_M \leq X^\tau/(n_H^\tau + n_M^\tau + n_L^\tau)$, they will prefer $\theta^\tau = 1$, and if $x_M > X^\tau/(n_H^\tau + n_M^\tau + n_L^\tau)$ they will prefer $\theta^\tau = 0$. Thus, without loss of generality, we assume that voting takes place between $\theta^\tau = 0$ -a meritocratic rule- and $\theta^\tau = 1$ -an egalitarian rule.

Assumption A4. Whenever there is a tie, the highest productive agent in the organization decides. If an agent is indifferent between hiring agents from different types, he will vote for the agent from the highest type.

We remark that our tie breaking rules favour excellency.

Assumption A5. Agents vote sincerely.

Several comments are in order. A1 is made to discard problems associated with a rigid labor supply. A2 and A3 are made to simplify the voting. The deterministic tie-breaking rule in A4 is chosen to avoid lotteries that might complicate the history of the game. A5 is made in order to destroy equilibria in which all agents vote for the alternative which is at the bottom of their preferences. It means that each agent believes that his vote is pivotal. Without this assumption we may have very complicated vote patterns (that are highlighted in the paper by Barberá et alia, 2001). And we have a serious problem of multiplicity of equilibria.[3] Since the emphasis of our

[3] Mavridis and Serena (2015) offered an appealling solution to the problem of multiplicity of Nash equilibrium in

paper is on class interests we assume that each member of each class (types) behaves like if he were in command of this class.

At the beginning of each period τ, the history of the game can be summarized by a state, s^{τ}, defined as the number of insiders at $\tau-1$ because these are the voters in period τ. That is, $s^{\tau}=(n_H^{\tau-1},n_M^{\tau-1},n_L^{\tau-1})$. The state at $\tau+1$ depends on s^{τ} and the action taken at τ. Let S the set of all possible states. We only consider strategies that are state dependent. By our assumption on sincere voting, the strategies for all members of a class can be summarized in a strategy for a typical member of this class. Let $\sigma_T : S \rightarrow \{0,1\} \times \{M,L,H\}$ be a strategy for a member of type $T \in \{L,M,H\}$. Thus a strategy for a typical member of a type is a mapping that for each state yields the vote of this person on the sharing rule and on the hiring. Our equilibrium concept is Markov Perfect Equilibrium (MPE in what follows).

Given that the total net surplus is always positive, no insider wants to leave and no outsider will never reject an offer. This is why we do not model the strategy of an outsider and we do not consider the exit option.

3. Results

We analyze the influence of the ruling class on the evolution of the organization in terms both of political power and resource distribution.

We say that a type $T \in \{H,M,L\}$ is the dominant class at time τ if $n_T^{\tau-1} > n_J^{\tau-1}+n_K^{\tau-1}$, $J,K \in \{H,M,L\}\backslash\{T\}$. If T is the dominant class, the pivotal voter is an agent of type T.

Our main result is the following:

Proposition 1. *There exist $\bar{\delta}>0$ such that for all $\delta \geq \bar{\delta}$, $T \in \{H,M,L\}$ then:*
(i) In any MPE if H is the dominant class at period τ, then H is dominant at all subsequent periods and meritocracy will be the sharing rule.
(ii) In any MPE if M is the pivotal type at period τ, then M will be the pivotal type at any subsequent period and eventually, egalitarianism will be the sharing rule.
(iii) In any MPE if L is the dominant class at period τ, then eventually egalitarianism will be the sharing rule although they will eventually loose political power in favor of the medium class.

voting games when voting is costly.

We start by proving the first part of the proposition. For the other parts we need a series of auxiliary results that we present here. The proofs of these auxiliary results are given in the Appendix.

Proof of (i). A type H agent will always vote for a meritocratic sharing rule in any possible state. Furthermore, given A4, he will vote for hiring a high type agent. Thus, if H is the dominant class at τ, they will be the dominant class in any subsequent period. ■

Our first auxiliary result points out that, when there is no dominant class, for a sufficiently high δ, the pivotal voter is a medium type. When deciding on the sharing rule, a high type agent will always vote for a meritocratic sharing rule and a low type agent for an egalitarian rule independently on the hiring decision. Thus, when there is no dominant class the pivotal voter is a medium type agent. The following lemma provides conditions under which, with no dominant class, the medium type is also pivotal on the hiring decission.

Lemma 1. *There exist $\bar{\delta}_0 > 0.5$ such that for all $\delta \geq \bar{\delta}_0$ in any MPE, if at period τ there is no a dominant class, a type M agent is the pivotal voter.*

Lemma 1 formalizes the idea that the middle class enjoys an strategic advantage with respect to both "ends", low and high. And by exploiting the fact that one of the extremes will vote with the middle class, this class is pivotal in the voting game when the future is very important. And, again, when the future is very important, the middle class will target its votes on hiring to maintain this pivotality in the long run.

In the following Lemma we show that whenever $X^{\tau-1}/n^{\tau-1} < x_M < (X^{\tau-1} + x_H)/n^{\tau}$, there exist $\bar{\delta}_1$ such that for both, medium and low type, having meritocracy today and egalitarianism from tomorrow on and alternating the hiring between a medium type and a high type starting by a medium type, is better than having egalitarianism today and hiring a high type and meritocracy from tomorrow on. Let $V_E^1(\delta)$ the continuation payoff in the first case.

Lemma 2. *Let $X^{\tau-1}/n^{\tau-1} < x_M < (X^{\tau-1}+x_H)/n^{\tau}$. There exist $\bar{\delta}_1 > 0.5$ such that for all $\delta > \bar{\delta}_1$,*

$$x_M + \delta V_E^1(\delta) \quad > \quad (X^{\tau-1} + x_H)/n^{\tau} + \frac{\delta}{1-\delta} x_M, \ \ \textit{and} \tag{3.1}$$

$$x_L + \delta V_E^1(\delta) \quad > \quad (X^{\tau-1} + x_H)/n^{\tau} + \frac{\delta}{1-\delta} x_L. \tag{3.2}$$

In the following Lemma we show that whenever $x_M < X^{\tau-1}/n^{\tau-1} < (X^{\tau-1}+x_H)/n^\tau$, there exist $\bar{\delta}_2$ such that for both, medium and low type, having egalitarianism from now on and alternating the hiring between a medium type and a high type starting by a medium type, is better than egalitarianism today and hiring a high type and meritocracy from tomorrow on. Let $V_E^2(\delta)$ the continuation payoff in the first case.

Lemma 3. *Let $x_M < X^{\tau-1}/n^{\tau-1} < (X^{\tau-1} + x_H)/n^\tau$. There exist $\bar{\delta}_2$ such that for all $\delta > \bar{\delta}_2$,*

$$(X^{\tau-1} + x_M)/n^\tau + \delta V_E^2(\delta) \quad > \quad (X^{\tau-1} + x_H)/n^\tau + \frac{\delta}{1-\delta} x_M, \text{ and} \tag{3.3}$$

$$(X^{\tau-1} + x_M)/n^\tau + \delta V_E^2(\delta) \quad > \quad (X^{\tau-1} + x_H)/n^\tau + \frac{\delta}{1-\delta} x_L \tag{3.4}$$

We are now prepared to prove our main result.

Proof of Proposition 1.

(ii) First of all note that in all τ such that $n_H^{\tau-1} + 1 < n_M^{\tau-1} + n_L^{\tau-1}$, a medium type will always prefer to hire a high type because his pivotality on the sharing rule for next period is not at risk. The interesting periods are the ones at which $n_H^{\tau-1} + 1 = n_M^{\tau-1} + n_L^{\tau-1}$. In those periods, by voting on a high type the medium class will loose his pivotal vote from there on. However, let us see that for δ sufficiently high, the medium type will prefer to keep the power. Since the decision on the sharing rule can be reversed in the next period, only today's payoffs are relevant when voting for different rules. We distinguish three cases.

Case ii1. Suppose that $x_M \leq X^{\tau-1}/n^{\tau-1} < (X^{\tau-1} + x_H)/n^\tau$.

In this case the medium type will vote for egalitarianism independently on the hiring. But by voting for a high type, the medium class will have egalitarianism today and meritocracy for ever because high types will become the dominant class. However, if they vote for a medium type they will keep the power in the next period where again, there is no risk on losing power and a high type will be hired. In this case, they will have egalitarianism for ever. By Lemma 3 there exist $\bar{\delta}_2 > 0$ such that for all $\delta \geq \bar{\delta}_2$, alternating the hiring between medium and high type is better than hiring a high type.

Case ii2. Suppose that $(X^{\tau-1} + x_H)/n^\tau \leq x_M$.

In this case the medium type will vote for a meritocratic rule independently on the hiring. But by voting for a high type, the medium class will have meritocracy for ever because high types will become the dominant class. However, if they vote for a medium type they will keep the power in the next period where again, there is no risk on losing power and a high type will be hired. From period τ is better to alternate the hiring between medium and high starting by hiring a medium type. This will be profitable comparing with hiring a high type because eventually, at some period τ', $x_M < (X^{\tau-1} + x_H)/n^{\tau}$, and the medium type by keeping the power will get egalitarianism from there on. Case ii1 shows that this is better than getting meritocracy for ever.

Case ii3. Suppose that $X^{\tau-1}/n^{\tau-1} \leq x_M < (X^{\tau-1} + x_H)/n^{\tau}$.
First of all note that, if the medium voter is pivotal in this case is because $\delta \geq \bar{\delta}_0$ (by Lemma 1). But by voting for a high type, the medium class will have egalitarianism today and meritocracy for ever because high types will become the dominant class. However, if they vote for a medium type they will keep the power in the next period where again, there is no risk on losing power and a high type will be hired. In this last case, they will have meritocracy today but egalitarianism from tomorrow on. By Lemma 2 there exist $\bar{\delta}_1$ such that for all $\delta \geq \bar{\delta}_1$, keeping power is the best option.

Let $\bar{\delta}_M = \max\{\bar{\delta}_0, \bar{\delta}_1, \bar{\delta}_2\}$, by cases ii1, ii2, and ii3, for all $\delta \geq \bar{\delta}_M$, if M is the pivotal type at period τ, then M will be the pivotal type at any subsequent period and eventually, egalitarianism will be the sharing rule.

(iii) A type L will always vote for an egalitarian rule. Furthermore, whenever $n_L^{\tau-1} > n_M^{\tau-1} + n_H^{\tau-1} + 1$ they will vote for a high type and they will keep power in the next period. The interesting periods are the ones at which $n_L^{\tau-1} = n_M^{\tau-1} + n_H^{\tau-1} + 1$. In those periods, by voting on a high type the low class will cease to be the dominant class. If $n_M^{\tau-1} = 0$, the dominant class will become the high types, and if $n_M^{\tau-1} > 0$, the pivotal vote will be in hands of the medium class. We distinguish three cases.

Case iii1. Suppose that $n_M^{\tau-1} = 0$ and $x_M \leq (X^{\tau-1} + x_H)/n^{\tau}$.
By voting for a high type, the low types will loose power and they will get an egalitarian share today but a meritocratic one from tomorrow on. If they vote for a medium type, the pivotal vote tomorrow will be in hands of the medium class. Given that $x_M \leq (X^{\tau-1} + x_H)/n^{\tau}$ by Case ii1 the medium type will keep the pivotal vote from there on and they will get egalitarianism for ever. By Lemma 3 there exist $\bar{\delta}_2$ such that a low type prefers to hire a medium type rather than hiring a high type. And finally note that hiring a low type is dominated by hiring a medium type because in both cases egalitarianism will be obtained but the payoff by hiring a medium type is larger.

Case iii2. Suppose that $n_M^{\tau-1} = 0$ and $(X^{\tau-1} + x_H)/n^\tau \leq x_M$.

In this case the low type will prefer to hire a medium type than a high type. In both cases the low types will loose the political power. By hiring a high type they will have egalitarianism today and meritocracy for ever. By hiring a medium type they will have egalitarianism today, meritocracy for some periods and then egalitarianism for ever (because case ii2 will apply). A similar argument as the one in Lemma 3 can be applied here to show that there exist $\bar{\delta}_3$ such that for all $\delta \geq \bar{\delta}_3$, the low type prefers to hire a medium type than a high type. So either the low type hires a medium type and loose the political power or hires a low type and keeps it. In both cases, eventually egalitarianism will be the sharing rule.

Case iii3. Suppose that $n_M^{\tau-1} > 0$. In this case, by hiring a high type the low types will loose power in favor of the medium class which will keep the power for ever if $\delta \geq \bar{\delta}_M$. Clearly this option dominates the option of hiring a medium type. Because by doing that they are also relinquishing power to the medium type but the payoff today will be smaller. Finally, note that if $x_M \leq (X^{\tau-1} + x_H)/n^\tau$, egalitarianism will be the result from period τ on and this is better for the low type that hiring a new low type. If $x_M > (X^{\tau-1} + x_H)/n^\tau$ the low types will get egalitarianism today, meritocracy for some periods and, eventually egalitarianism again. If δ is sufficiently large, the loss in some periods due to meritocracy, will be compensated by a higher egalitarian payoff after those periods. If δ is not large enough, the low types will prefer to keep power. But also in this case the sharing rule will be meritocratic.

Summarizing, let $\bar{\delta} = \max\{\bar{\delta}_M, \bar{\delta}_3\}$. Then, for all $\delta \geq \bar{\delta}$, the proposition holds. ■

Proposition 1 says that when the future is very important, it is better for the high and the middle types to maintain political power that allows to select the sharing rule and the hiring rather than hiring an agent with larger productivity that in the short run might increase production but that may lead in the future to lose the ability to select sharing rules and hirings. In the case of the high type they would only hire high types (like first class departments only hire high class scholars). Middle types may be not the most numerous but they may be pivotal. And the low class will yield power to the middle class because they know that the middle class will be egalitarian.

Three consequences of Proposition 1 are:

1. *Exploitation*, defined as the existence of an agent whose payment is less than his productivity may arise in equilibrium in our democratic society in which voting -and not the possession of

capital- is the deciding method.

2. Agents with high productivity may be left idle because their hiring might jeopardize the dominance of the ruling class. We propose to call this *Political Unemployment*. Of course in order to have a complete theory of unemployment we should also model the supply side and to model how the market works. We have assumed that there is always supply of all types in the market so the short side of the market -the organization- is the determinant of employment. This model is akin to the Marxists theory of the "reserve army" in which some workers are left idle in order to maintain wages low. In our case, high types are not hired because they constitute a threat to the power of the other classes.

3. The low class *Relinquish Power* to the middle class. The middle class is more productive and eventually will vote for egalitarianism so if they are hired, there is more surplus to share.

Proposition 1 has implications about the long run composition of the organization. Let (m_H^t, m_M^t, m_L^t) be the proportion of insiders H, M and L respectively at time t. Let $(\tilde{m}_M, \tilde{m}_M, \tilde{m}_L) = \lim_{t\to\infty}(m_H^t, m_M^t, m_L^t)$.

Corollary 1. *For a $\bar{\delta}$ sufficiently close to* 1 *if the dominant class at $t = 0$ is H then $(\tilde{m}_M, \tilde{m}_M, \tilde{m}_L) = (1, 0, 0)$. In any other case $(\tilde{m}_M, \tilde{m}_M, \tilde{m}_L) = (0.5, 0.5, 0)$.*

The first part says that high quality organizations tend to maintain on time. The examples of Cambridge, Harvard, La Sorbonne and other educational institutions that have been outstanding from a long time ago come to our mind.[4] The second part says that institutions started by other class of agents will never achieve full excellency. In this sense our model highlights the importance of the founding fathers of an organization.

Next, we define the long run per capita output as $G = \lim_{t\to\infty}(m_H^t x_H + m_M x_M + m_L^t x_L)$. It is a measure of the efficiency of an organization. We have now the following:

Corollary 2. *For a $\bar{\delta}$ sufficiently close to* 1 *if the dominant class at $t = 0$ is H then $G = x_H$. In any other case $G = 0.5x_H + 0.5x_M$.*

[4] But there are models of dynamic organizations in which this conclusion does not hold. In Sobel (2001) standards of admission and the average quality of incumbents rise or fall without any bound (which is impossible here because we only have three types). In Corchón (2005) there is free entry and an organization populated by high types may be subject to the entry of many low types whose life is easier under the command of a high type. In our model there is no free entry.

We can define the instantaneous relative efficiency loss of G as $(x_H - G)/x_H$. In any inefficient MPE this loss is $(x_H - 0.5x_H - 0.5x_M)/x_H = 0.5(x_H - x_M)/x_H$. It ranges between zero and 50%.

To end this section, we note that Proposition 1 has some resemblance with the "Folk Theorem" in which when the weight of future payoffs is overwhelming, no deviation to an action that is immediately profitable is profitable in the long run. But our proof is not based on the classical device of maintaining a certain vector of payoffs by means of a punishment phase followed (or not) by a rewarding phase. We only use Markovian strategies and agents vote sincerely. Thus, there are neither a punishment nor a reward phase.

4. Finitely lived agents

Our assumption that agents live an infinite number of periods is meant to free the model from the "last day effect". In this section we explore the consequences of assuming that agents are finitely-lived. An agent in his last period inside the organization will be called retiring. In this new setting the state not only has to specify the number of each type of agents but who retires. We start with a simple observation: A retiring agent will always vote for hiring a high type.

This is because junior members of the organization work but do not vote. In an egalitarian society it is a dominant strategy for a retiring agent to vote for the H type regardless of what will happen tomorrow. If the society is meritocratic, then retiring agents are indifferent between any vote, given A4, they will vote for a high type.

Since the results below are meant to highlight possibilities that may arise under finitely-lived agents, we simplify the model here and assume two types only. We will also assume that, at most, one individual retires in each period. A state now includes the specification of the retiring agent.

We note first that, as in the previous section, if high types are the dominant class at some period, they will be the dominant class from there on.

Proposition 2. *In any MPE if H is the dominant class at period τ, then H is dominant at all subsequent periods and meritocracy will be the sharing rule.*

Proof. A type H agent will always vote for a meritocratic sharing rule in any possible state. Given A4 and the remark above, a high type agent, independently if he is a retiring agent or not, will vote for hiring a high type. Thus, if H is the dominant class at τ, they will be the dominant class in any subsequent period. ■

Thus, the existence of retiring agents will never jeopardize the predominance of the H type. However, this will not be the case for the low types. First note that, without retiring agents, an adaptation of Proposition 1 for two types will say that there exist $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta}$, if low types are the dominant class, they will be the dominant class for ever and the sharing rule will be egalitarian. In the following proposition we show that with retiring agents, low types will loose political power at some subgames.

Proposition 3. *Suppose that at τ, $n_H^{\tau-1} + 1 = n_L^{\tau-1}$ and the retiring agent is a low type. Then, for all $\delta > 0$, in any MPE low types will loose political power and meritocracy will be the sharing rule from $\tau + 1$ on.*

Proof. At τ, the political power is in hands of low types. Thus, egalitarianism will be the sharing rule. Given that high types and the retiring low type agents will vote for a high type, a high type will be hired and they will be the dominant class in next period. By Proposition 2, high types will be the dominant class from there on and the resulting sharing rule will be meritocratic. ■

From Proposition 3 we learn that demographics has a lot to say and determines entirely that, in some organizations, power will change hands even if all agents are non myopic.

Low types know that, at all subgames at which $n_H^{\tau-1} + 1 = n_L^{\tau-1}$ and the retiring agent is a low type, they will loose control. However, low types can be farsighted and keep control by using the strategy described below.

Vote always for an egalitarian sharing rule. As a retiring, vote for a high type. As non-retiring, if $n_H^{\tau-1} + 2 < n_L^{\tau-1}$ vote for a high type no matter who retires. If $n_H^{\tau-1} + 2 = n_L^{\tau-1}$ and the retiring agent is a high type, vote for a high type. If $n_H^{\tau-1} + 2 = n_L^{\tau-1}$ and the retiring agent is a low type, vote for a low type. And if $n_H^{\tau-1} + 1 \geq n_L^{\tau-1}$ they vote for a high type.

The following proposition shows that $\bar{\sigma} = (\bar{\sigma}_L, \bar{\sigma}_H)$, where $\bar{\sigma}_L$ is the strategy described above and $\bar{\sigma}_H$ is such that at all periods high types vote for meritocracy and for hiring a high type, is a MPE for a sufficiently high δ.

Proposition 4. *There is $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta}$, $\bar{\sigma} = (\bar{\sigma}_L, \bar{\sigma}_H)$ is a MPE. Furthermore, if $n_H^0 + 1 < n_L^0$, the low class will be the dominant class for ever.*

Proof. Note first that neither a high type nor a retiring low type have incentives to deviate from their strategy. Note also that under this strategy, if low types are the dominant class, they will not lose the political power and egalitarianism will be the sharing rule. At states where $n_H^{\tau-1}+2 < n_L^{\tau-1}$, is a dominant strategy for the low type to vote for a high type. Independently of who is the retiring agent, low types will not loose the political power and an egalitarian sharing rule gives larger payoffs with a high type. At states where $n_H^{\tau-1}+1 \geq n_L^{\tau-1}$, independently of who is retiring, a low type prefers to vote for a high type. Either because they are not the dominant class and nothing will change with their vote (A4 applies here), or they are the dominant class, that is $n_H^{\tau-1}+1 = n_L^{\tau-1}$, and the retiring agent is a high type (so hiring a high type will not change their political power), or $n_H^{\tau-1}+1 = n_L^{\tau-1}$ and the retiring agent is a low type and therefore $n_H^{\tau-1}+1$ agents will vote for a high type (so a high type will be selected independently of the vote of the non-retiring low types). Finally, let us see that there are no profitable deviations in states where $n_H^{\tau-1}+2 = n_L^{\tau-1}$. It is clear that no profitable deviation exist if the retiring agent is a high type. If the retiring agent is a low type and a non-retiring low type changes his vote and votes for a high type instead of a low type, a high type will be hired (a high type will receive the votes of the high types, the vote of the retiring low type and the vote of the deviant). Thus, this agent will receive a better payoff today (an egalitarian share with the surplus of a new high type), but the political power tomorrow will be in hands of the high types and will stay in their hands for ever. Let us see that there exist $\bar{\delta}$ such that this deviation is not profitable. By deviating, this agent will get

$$\frac{X^{\tau-1}-x_L+x_H}{n^\tau}+\frac{\delta}{1-\delta}x_L. \tag{4.1}$$

If he does not deviate from his strategy, in all subsequent periods $t \geq \tau$, $n_H^{t-1}+2 = n_L^{t-1}$. Furthermore, since a retiring low type (resp. high type) is replaced by a low type (resp. high type), $n^t = n^\tau$ for all $t \geq \tau$, and $X^{t-1} = X^{\tau-1}$. Then, the payoff associated to no deviation is:

$$\frac{X^{\tau-1}}{n^\tau}+\sum_{t>\tau}\delta^{t-\tau}(\frac{X^{\tau-1}}{n^\tau}) = \frac{1}{1-\delta}(\frac{X^{\tau-1}}{n^\tau}). \tag{4.2}$$

Thus, the deviation will not be profitable if

$$(\frac{X^{\tau-1}}{n^\tau}) \geq (1-\delta)\frac{X^{\tau-1}-x_L+x_H}{n^\tau}+\delta x_L. \tag{4.3}$$

Let $F(\delta) = (1-\delta)(X^{\tau-1}-x_L+x_H)/n^\tau+\delta x_L$. Note that $F(\delta)$ is decreasing in δ, $F(0) > X^{\tau-1}/n^\tau$ and $F(1) < X^{\tau-1}/n^\tau$. Thus, there exist $\bar{\delta}$ such that for all $\delta \geq \bar{\delta}$, the deviation is not profitable.

The second part of the Proposition follows directly from the fact that $\bar{\sigma}$ is a MPE and starting from a dominant position, in the equilibrium path the low types will not loose the political power. ■

5. Conclusions and extensions of the model

In this paper we have studied the evolution of resource allocation in a society in which decisions are taken by majority voting among three classes of citizens: highly (H), medium (M) and lowly (L) productive. We have shown that if agents are sufficiently patient, if H is dominant at time 0, they will vote for meritocracy and they will hire H types only. But in any other case, M type agents become pivotal in the voting and, eventually they will vote egalitarianism and they will hire M and H agents in such a way that the stay pivotal. In particular if L agents are a majority at the starting time they will relinquish power to the M class. As a result, some agents may be paid less than their productivity (exploitation) and highly productive agents may be left in the cold because their hiring will jeopardize political power of the dominant class (political unemployment). In short, voting in the workplace does not avoid the shortcomings of capitalistic economies. These shortcomings of voting -oppression of minorities or lack of efficiency- have been pointed out long time ago in the context of political systems.

The conclusions above were obtained in a particular model but it seems to us that they are general. Dominant classes may relinquish power to more productive classes as long as they do not jeopardize output distribution. And non dominant but very productive classes may be exploited by less productive classes which also will restrict the entry of potentially dangerous individuals into the organization. Our results point out the importance of the initial constituency. Thus a sometimes forgotten fact about the "Pilgrim Fathers" is their high level of education which might have pervaded the future of US. Thus William Brewster, John Harvard, Roger Williams, etc. were educated at Cambridge university. This initial constituency may help to explain why US converted so easily from an agricultural economy into a high tech one, a conversion that was not so successful in Australia and Argentina (in both countries the initial constituency was very different from the US one).

As for the possible extensions of the model, variations on the number of alternatives, the number of hired outsiders and the tie breaking rules are not likely to alter the qualitative conclusions of the model. The introduction of more types is likely to yield a similar relinquish from the less productive classes to the second more productive class. We think that the most promising (and difficult) extension is to consider competition among several organizations and to allow for the possibility of shortages of types in the market. We leave this for future research.

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6. Appendix

First, let us introduce some notation. Let $V_J(T)$ be the continuation payoff for a type J agent when hiring a type T agent. Note first that $V_L(T) \leq V_M(T) \leq V_H(T)$. This is because at each period, either the rule is meritocratic or egalitarian. If it is egalitarian everyone receives the same, if it is meritocratic the highest the type the highest the payoff that is obtained.

Proof of Lemma 1. Given A5, a high type agent will always vote for a meritocratic sharing rule and a low type agent for an egalitarian rule independently on the hiring decision. Thus, when there is no dominant class, when deciding on the sharing rule, the pivotal voter is a medium type agent. Let us see that for δ sufficiently high, the medium type is also pivotal when voting on the hiring. First note that, under A4 and A5, an agent will always vote for hiring another agent from his type or from a highest type. Thus, if a medium type prefers to hire a high type, a high type will be hired. Furthermore, given A4, and A5, in all τ such that $n_H^{\tau-1}+1 > n_M^{\tau-1}+n_L^{\tau-1}$, a medium type will always prefer to hire a high type because his pivotal vote on the sharing rule for next period is not at risk. If $n_H^{\tau-1}+1 = n_M^{\tau-1}+n_L^{\tau-1}$, there is a trade off because by hiring a high type, the medium class will loss its pivotal vote on the sharing rule in the next period in favor of the high type, and by part (i) in Proposition 1 we will get meritocracy from there on. To discuss the implications of the voting behavior we distinguish three cases.

(i) Suppose that at τ, $(X^{\tau-1}+x_H)/n^\tau \leq x_M$.

Since the pivotal voter on the sharing rule is the medium type, he will vote for a meritocratic sharing rule independently of the new hiring. Suppose that the medium type prefers to hire another medium type. Thus,

$$x_M + \delta V_M(M) > x_M + \delta V_M(H) = x_M + \frac{\delta}{1-\delta} x_M. \tag{6.1}$$

Which implies that some of the terms in $V_M(M)$ are the result of an egalitarian rule with a payoff greater than x_M. Given that $V_L(T) \geq (1/(1-\delta))x_L$ for all $T \in \{H, M, L\}$, $V_L(M) > (1/(1-\delta))x_L$. Thus, in this situation a low type agent will prefer also to hire a medium type. As a result, a medium type will be hired.

(ii) Suppose that at τ, $x_M \leq X^{\tau-1}/n^{\tau-1} < (X^{\tau-1}+x_H)/n^\tau$.

Since the pivotal voter on the sharing rule is the medium type, he will vote for an egalitarian sharing rule independently of hiring a high or a medium type. Suppose that the medium type prefers to hire another medium type. Thus,

$$\frac{X^{\tau-1}+x_M}{n^\tau}+\delta V_M(M) > \frac{X^{\tau-1}+x_H}{n^\tau}+\frac{\delta}{1-\delta}x_M. \tag{6.2}$$

Or equivalently

$$\frac{X^{\tau-1}+x_H}{n^\tau}-\frac{X^{\tau-1}+x_M}{n^\tau} < \delta(V_M(M)-\frac{1}{1-\delta}x_M). \tag{6.3}$$

Let us see that the low type also prefers to vote for a medium type. Note that $V_M(M)$ and $V_L(M)$ only differ in the periods where meritocracy is the resulting sharing rule (if any of these periods exist). Thus,

$$V_M(M)-V_L(M) < \frac{1}{1-\delta}(x_M-x_L). \tag{6.4}$$

Therefore, if (6.3) holds, then

$$\frac{X^{\tau-1}+x_H}{n^\tau}-\frac{X^{\tau-1}+x_M}{n^\tau} < \delta(V_L(M)-\frac{1}{1-\delta}x_L) \tag{6.5}$$

also holds and the low type will also prefer to hire a medium type.

(iii) Suppose that at τ, $X^{\tau-1}/n^{\tau-1} \leq x_M < (X^{\tau-1}+x_H)/n^\tau$.

Since the pivotal voter on the sharing rule is the medium type, he will vote for a meritocratic sharing rule if a medium type is hired and for an egalitarian one if a high type is hired. Suppose that the medium type prefers to hire another medium type. Thus,

$$x_M+\delta V_M(M) > \frac{X^{\tau-1}+x_H}{n^\tau}+\frac{\delta}{1-\delta}x_M. \tag{6.6}$$

Or equivalently

$$\frac{1-2\delta}{1-\delta}x_M+\delta V_M(M) > \frac{X^{\tau-1}+x_H}{n^\tau}. \tag{6.7}$$

Because in all periods where the pivotal vote of the medium type on the sharing rule is not at risk a new high type is hired, from $\tau+1$ on, the sharing rule will be egalitarian. This is because once we hire a high type, the average surplus is above x_M and from there on, no matter if a high or a medium type is hired, the average surplus will stay above x_M. Thus, $V_M(M)=V_L(M)$. Furthermore, let us see that the left hand side of (6.7) is increasing in δ. Let y_t denote the t-term in $V_M(M)$, recall that $y_t > x_M$ for all t. The derivative with respect to δ of the left hand side of (6.7) is

$$\frac{-1}{(1-\delta)^2}x_M+V_M(M)+\delta\frac{\partial V_M(M)}{\partial\delta}. \tag{6.8}$$

Given that $V_M(M) = \sum_{t=0}^{\infty} \delta^t y_t > \sum_{t=0}^{\infty} \delta^t x_M = (1/(1-\delta))x_M$, and

$$\frac{\partial V_M(M)}{\partial \delta} = \sum_{t=1}^{\infty} t\delta^{t-1} y_t > \sum_{t=1}^{\infty} t\delta^{t-1} x_M = (1/(1-\delta)^2)x_M, \tag{6.9}$$

then (6.8) is positive. Thus, the left hand side of (6.7) is increasing in δ. Therefore, if (6.7) holds for some δ, it holds for all $\delta' > \delta$. Finally note that for $\delta > 0.5$, whenever (6.7) holds,

$$\frac{1-2\delta}{1-\delta} x_L + \delta V_L(M) > \frac{X^{\tau-1} + x_H}{n^\tau} \tag{6.10}$$

also holds and thus low types will prefer also to hire a medium type.

Summarizing, there exist $\bar{\delta}_0 > 0.5$ such that for all $\delta \geq \bar{\delta}_0$ in any MPE, if at period τ there is no a dominant class, a type M agent is the pivotal voter. ■

Proof of Lemma 2. First of all note that $\delta V_E^1(\delta)$ is

$$\delta V_E^1(\delta) = \sum_{t=0}^{\infty} \delta^{2t+1} \left[\frac{X^{\tau-1} + (t+1)x_M + (t+1)x_H}{n^\tau + 2t + 1}\right] + \sum_{t=1}^{\infty} \delta^{2t} \left[\frac{X^{\tau-1} + (t+1)x_M + t x_H}{n^\tau + 2t}\right]. \tag{6.11}$$

The first term corresponds to all the periods where a high type is hired and the second term to the periods where a medium type is hired. First note that, since $\delta < 1$, all power series in $\delta V_E^1(\delta)$ are convergent. Note also that all terms in $\delta V_E^1(\delta)$ are larger than x_M. Let us see that there exist $\bar{\delta}_1 > 0.5$ such that the first three terms in $x_M + \delta V_E^1(\delta)$ are bigger than $(X^{\tau-1} + x_H)/n^\tau + \delta x_M + \delta^2 x_M$. Formally, let us see that there is $\bar{\delta}_1 > 0.5$ such that for all $\delta > \bar{\delta}_1$,

$$x_M + \delta\left(\frac{X^{\tau-1} + x_M + x_H}{n^\tau + 1}\right) + \delta^2\left(\frac{X^{\tau-1} + 2x_M + x_H}{n^\tau + 2}\right) > \frac{X^{\tau-1} + x_H}{n^\tau} + \delta x_M + \delta^2 x_M. \tag{6.12}$$

For $\delta = 0$, the left hand side of (6.12) is x_M which is smaller than $(X^{\tau-1} + x_H)/n^\tau$. As δ converges to 1, the left hand side converges to

$$x_M + \frac{X^{\tau-1} + x_M + x_H}{n^\tau + 1} + \frac{X^{\tau-1} + 2x_M + x_H}{n^\tau + 2}, \tag{6.13}$$

and the right hand side converges to

$$\frac{X^{\tau-1} + x_H}{n^\tau} + x_M + x_M. \tag{6.14}$$

Let us see that

$$\frac{X^{\tau-1} + x_M + x_H}{n^\tau + 1} + \frac{X^{\tau-1} + 2x_M + x_H}{n^\tau + 2} > \frac{X^{\tau-1} + x_H}{n^\tau} + x_M, \tag{6.15}$$

or equivalently,

$$\frac{X^{\tau-1}+2x_M+x_H}{n^\tau+2}-x_M > \frac{X^{\tau-1}+x_H}{n^\tau}-\frac{X^{\tau-1}+x_M+x_H}{n^\tau+1}, \tag{6.16}$$

$$\frac{X^{\tau-1}+x_H-n^\tau x_M}{n^\tau+2} > \frac{X^{\tau-1}+x_H-n^\tau x_M}{n^\tau(n^\tau+1)} \tag{6.17}$$

Since $n^\tau \geq 2$, $n^\tau(n^\tau+1)$ is larger that $(n^\tau+2)$, (6.15) holds.

Let

$$F(\delta)=(1-\delta-\delta^2)x_M+\delta(\frac{X^{\tau-1}+x_M+x_H}{n^\tau+1})+\delta^2(\frac{X^{\tau-1}+2x_M+x_H}{n^\tau+2}). \tag{6.18}$$

We have seen that $F(0)<(X^{\tau-1}+x_H)/n^\tau$, and that $\lim_{\delta\to 1}F(\delta)>(X^{\tau-1}+x_H)/n^\tau$, note also that since $(X^{\tau-1}+x_M+x_H)/(n^\tau+1)>x_M$, and $(X^{\tau-1}+2x_M+x_H)/(n^\tau+2)>x_M$, $F(\delta)$ is increasing in δ. Thus, we can find $\bar{\delta}_1>0.5$ such that for all $\delta>\bar{\delta}_1$, $F(\delta)>(X^{\tau-1}+x_H)/n^\tau$, as we wanted to proof.

Finally note that for $\delta>0.5$, if (3.1) holds, then (3.2) also holds. ■

Proof of Lemma 3. Note first that if (3.3) holds, given that $x_M>x_L$, (3.4) also holds. Also note that $\delta V_E^2(\delta)$ can be written as (6.11) in Lemma 2. As in Lemma 2, all terms in $\delta V_E^2(\delta)$ are larger than x_M. Let us see that there exist $\bar{\delta}_2$ such that the first three terms in $(X^{\tau-1}+x_M)/n^\tau+\delta V_E^2(\delta)$ are larger than $(X^{\tau-1}+x_H)/n^\tau+\delta x_M+\delta^2 x_M$. That is, there is $\bar{\delta}_2$ such that in three periods, we compensate the initial loss of getting $(X^{\tau-1}+x_M)/n^\tau$ compared with getting $(X^{\tau-1}+x_H)/n^\tau$. Formally, let us see that there is $\bar{\delta}_2$ such that for all $\delta>\bar{\delta}_2$,

$$(X^{\tau-1}+x_M)/n^\tau+\delta(\frac{X^{\tau-1}+x_M+x_H}{n^\tau+1})+\delta^2(\frac{X^{\tau-1}+2x_M+x_H}{n^\tau+2})>\frac{X^{\tau-1}+x_H}{n^\tau}+\delta x_M+\delta^2 x_M. \tag{6.19}$$

Given that $(X^{\tau-1}+x_M)/n^\tau>x_M$, we can apply the argument in Lemma 2 to ensure the existence of $\bar{\delta}_2$. ■