

Sequentially Pricing Multiple Products: Theory and Experiments*

January 2009

John Aloysius
Information Systems Dept.
University of Arkansas

Cary Deck
Economics Dept.
University of Arkansas

Amy Farmer
Economics Dept.
University of Arkansas

Technological advances enable sellers to price discriminate based upon a customer's revealed purchasing intentions. E-tailers can track items in "shopping carts" and RFID tags enable retailers to do the same in bricks and mortar stores. In order to leverage this information, it is important to understand how this new visibility impacts pricing and market outcomes. We examine the theoretical implications of sequential pricing of multiple products. Specifically, we focus on independently-valued goods, goods that have values which are positively or negatively correlated, and goods with super-additive or sub-additive values. The results indicate that sequential pricing increases profit relative to simultaneous pricing for substitute goods. When sellers can condition the second good's price on the buyer's decision to purchase the first good, sequential pricing increases profits relative to mixed bundling when the goods are highly positively correlated. We also use experiments to examine sequential pricing in competitive markets where a segment of customers comparison shop. The behavioral results indicate that conditional pricing does not lower social welfare or harm consumers when customers observe prices sequentially. Further, the ability to price discriminate does not change the price of the initially offered good, but changes the price of the subsequently offered good.

Key Words: Sequential Pricing, Price Discrimination, E-commerce, Market Experiments

* The authors wish to thank Scott Burton, Yongmin Chen, Robert Letzler, Donald Lichtenstein, John List, Bart Wilson and participants at the FTC Conference on Microeconomics, the Southern Economic Association Annual Meetings, and the IFREE Workshop in Experimental Economics for helpful comments. Financial support from the Center for Retailing Excellence and the Information Technology Research Institute, both of the Sam M. Walton College of Business, is gratefully acknowledged.

1. Introduction

Imagine walking into a large department store and searching for a new outfit. The buyer observes a variety of shirts with posted prices, selects one, and then moves on to an area containing pants. The price of each pair of pants was set before the buyer selected a shirt and therefore the seller was dealing with a simultaneous pricing problem. One way sellers have attempted to exploit information on the underlying distribution of preferences among goods is by selling a collection of items in a bundle; however the prices are still set *ex-ante*. For example, the department store could sell an outfit rather than pants and shirts. The term pure bundling refers to offering only the bundle, whereas mixed bundling refers to offering both the individual items and the bundle simultaneously. Only offering the items separately is termed pure components pricing.

Now imagine that the seller is able to identify which shirt the customer selected (or even simply picked up for a moment) before setting the prices of the pants. The shirt selection reveals information about the buyer's tastes and preferences, thereby enabling the seller to better estimate the buyer's willingness to purchase any specific pair of pants. The seller could then effectively raise the price for items that are more likely to be purchased, perhaps by offering a smaller coupon for coordinating pants than for other pairs. Recent advances in technology have enabled exactly this type of monitoring of shoppers, so that pricing decisions for multiple products can be sequential rather than simultaneous.

Consider the situation where two people, High (H) and Low (L), each value two products A and B . Suppose, $V_A^H = V_B^H = 100$ and $V_A^L = V_B^L = 20$ and the value of consuming the bundle containing both products is $V_A^i + V_B^i$ for $i = H, L$. For simplicity, assume that the marginal cost of each product is 0. Under pure components, the maximum attainable profit is \$200, reaped by selling both products at a price of \$100 to the high valued buyer, H . Under pure bundling, the maximum profit is again \$200 and generated by selling the bundle to person H for \$200. Even with mixed bundling the firm can only earn \$200, all from person H , who will again purchase both goods. The inability of mixed bundling to increase profits is due to the fact that no discount can be offered for the bundle that would result in L buying the bundle while H bought the items separately. However, under sequential pricing with discrimination, the seller can obtain a profit of \$220. Without loss of generality, assuming that the decision to purchase A is made first, the seller can set the price of good A at \$100. H will purchase A and L will not. The seller can

then set the good B price at \$100 for those who purchased A and set the good B price at \$20 for those who did not. In this case both H and L would buy good B but at different prices.¹

The preceding illustration motivates the current research – what is the optimal pricing strategy for sellers who can monitor a customer’s initial purchase decision? The problem of conditional sequential pricing is one of first being able to identify the customer’s action and then exploiting this knowledge. This is straightforward in online markets where buyers place items in electronic shopping carts and cookies allow a shopper to be tracked. For example, when purchasing airline tickets through an online service, shoppers are often shown ads for hotels and attractions near their destination. It is easy to imagine that a different set of ads and promotional offers is displayed to those who have booked a Saturday night stay (or purchased a child’s ticket). Can the same level of monitoring occur in physical stores? Yes. Currently, RFID (radio frequency identification) technology is being used to track which products buyers in bricks and mortar stores have in their physical shopping carts. This technology is being employed primarily for theft detection, but other applications are being explored by industry and academia (Cromhout et al. 2008). Retailers, such as the Dillard’s department store among others, have introduced item level tagging in pilot stores and are planning expansion of the program. Sam’s Club, Walmart’s retail warehouse club division, expanding on its previous pallet level tagging mandate, recently introduced an item level tagging mandate for its suppliers, requiring that they tag all items shipped to 22 distribution centers by 2010 (Weier 2008). The chain is poised to unveil a new RFID enabled customer checkout system that will considerably reduce transactions costs and improve inventory control. In physical stores current practices are such that all buyers observe the same quoted

¹ This example highlights two important assumptions of the paper. The first assumption is that the prices can be presented in some order. In bricks and mortar retail, the flow of people through the store is carefully arranged. Grocery stores consistently have patrons enter through the fresh produce and leave impulse items for the checkout stand. In online markets shoppers have to actively negotiate websites to observe prices and the seller can track this history. For our purposes it does not matter if the presentation order is exogenous or endogenous as long as the seller can identify the order. The second important assumption is that buyers are myopic. The theoretically common fully rational buyer would form consistent beliefs regarding the prices of the second good and behave accordingly. However, there is evidence that actual buyers are only boundedly rational and thus some question about the appropriateness of modeling buyers as being fully rational. To the degree that we are worried about behavior in the naturally occurring world, our assumption is reasonable. Another main use of the technology that enables sequential pricing is making recommendations, which may be for products with which the buyer is otherwise unfamiliar and thus unlikely to have formed a price expectation beforehand.

prices; however, the use of new smart shopping carts can transfer the price display from the shelf to the shopper's cart allowing each shopper to receive a unique price or coupon.² Alternatively, individualized coupons can be sent to each shopper's phone. According to a December 17, 2008 article in the *New York Times* by Bob Tedeschi, this technology is already being employed by companies such as Cellfire and 8Coupons to offer coupons from retailers such as Sears and Kroger's.

Sellers also need to know how the buyer's value for an item is related to the buyer's value for other items. For this purpose, sellers have access to vast databases that can be mined to determine underlying relationships in buyer values across goods. Sellers routinely record the contents of every shopping basket sold. If purchases are made with credit or debit cards or some other form of identification such as a frequent buyer cards, a customer's shopping history within and across retailers can be compiled. Techniques like collaborative filtering and content filtering enable websites such as Amazon.com to provide recommendations to specific customers for other products based upon the information in such databases. Ansari et al. (2000) point out that among other sources, a customer's preferences or choices is information that can be used to make recommendations to customers. Knowing that one person is more likely than someone else to enjoy an item also suggests that the person is willing to pay more.³

This research considers two ways in which a buyer's value for two products might be related – the degree of complementarity and the degree of correlation between the valuations of the two goods. Two products which have greater utility to a customer when they are consumed together are complements; the values of the two items are superadditive in that the bundle is worth more than the sum of its parts. Similarly two products are substitutes if the value of consuming both is less than the sum of values from consuming the single items; that is the values are subadditive. Notice that the

² United States Patent 5729697 is for just such a device. The carts are also touted as way to monitor the health content of a shopper's purchases. Even without such technology, buyers may still pay different prices depending on the coupons they have, some of which could have been mailed specifically to them.

³ Conditional sequential pricing may be classified as a form of third degree price discrimination (Pigou 2006). Ayres (2007) gives many examples of firms that currently engage in such practices, ranging from the most visible examples of supermarkets to those less visible such as Harrah's casinos. Harrah's records real-time data on players winning or losing, and in combination with demographic information, uses this information to offer complementary promotional benefits to players who lose more than a critical threshold amount. In this way they avert these players leaving with a negative experience from their visit to the casino.

complements/substitutes relationship is distinct from the correlation between the values of the two goods. Two books on a related topic can be substitutes or complements depending on the overlap in their content, but people who dislike the topic will likely have a low value for both, while people who like the topic will likely have a high value for both.

To our knowledge, the literature is silent on how using the same information that enables sellers to make recommendations could be used to set sequential prices. However, once item level RFID or some similar technology emerges into the mainstream, retailers will likely seek opportunities to further utilize the technology to gain benefits other than the efficient management of inventories. Among these possibilities is the opportunity to leverage the same insight into planned purchases that will be available in stores as is already available online. When this happens in the near future, the benefits from optimal deployment of sequential pricing practices will have potential impact on a scale far beyond that imaginable even a few years ago. The theoretical results presented in this paper suggest that sequential pricing with discrimination outperforms mixed bundling when the goods have highly positively correlated values. Sequential pricing even without discrimination is more profitable than simultaneous pricing of pure components when the goods are close substitutes.

The rest of this paper is organized as follows: Section 2 reviews research on pricing relevant to the current research framework. Section 3 outlines an analytical framework that models sequential pricing decisions faced by a monopolist. Section 4 considers the problem faced by firms which compete for buyers of the initial good. The theoretical model is described and experiments are presented to demonstrate how human decision makers perform when making sequential pricing decisions and the likely market outcomes. In Section 5 we summarize the results.

2. Literature Review

Research on sequential pricing and exploiting the underlying relationship among the goods is surprisingly sparse given the rapid proliferation of technologies that enable retailers to gather information on likely purchases that could be used to set prices on those candidate goods in order to maximize expected profit. Mulhern and Leone (1991) review multi-product pricing and develop a framework for retail pricing and promotion policies. Using empirical data, they estimate the influence of regular and promotional prices on sales of substitute and complementary goods, and thus demonstrate the effectiveness of price promotions as a means of exploiting interdependencies in demand among retail products.

Instead, the literature on selling multiple products has been primarily focused on bundling by monopolists.⁴ Bundling has been shown to be an effective price discrimination tool even when the consumer's willingness to pay for each good is independent of the value of the other good and the value of the bundle is the sum of the values of the components (Adams and Yellen 1976). Customers with a high degree of asymmetry in product valuations will buy an individual product that they favor, while customers with more symmetric valuations will buy the bundle.⁵ Venkatesh and Kamakura (2003) present an analytical model of contingent valuations and find that the degree of complementarity or substitutability in conjunction with marginal cost levels determines whether products should be sold as pure components, pure bundles, or mixed bundles. They also find that typically, complements and substitutes should be priced higher than independently valued products. Nettesine, Savin, and Xiao (2006) present a stochastic dynamic program for analyzing the selection of complementary products.

There are some studies of dynamic pricing of goods, although these are confined to single goods and do not consider cross category effects on other goods. Cope (2006) presents dynamic strategies for maximizing revenue in internet retail by actively learning customers' demand responses to price. Zhang and Krishnamurthi (2004) provide a decision support system of micro-level promotions in an internet shopping environment, that provides recommendations as to when, how much, and to whom to give price promotions. The system derives the optimal price promotion for each household, on each shopping trip by taking into account the time-varying pattern of purchase behavior and the impact of the promotion on future purchases.

There are several examples of sellers using customer behavior to infer preferences, and using that information either to drive revenues or for customer relationship management. Montgomery et al. (2004) show how clickstream data about the sequence of pages or path navigated by web buyers can be used to infer users' goals and future path. There is a literature on behavior-based price discrimination

⁴ There are a few studies of price bundling in competitive markets. McAfee, McMillan, and Whinston (1989) extend their monopoly results to a duopoly and show that independent pricing can never be a Nash equilibrium when the reservation prices for the single goods are independent. Chen (1997) analyzes a situation in which firms compete in a duopoly for a single product and the firms also produce other products under conditions of perfect competition. Bundling as a product differentiation device proves to be an equilibrium strategy for one or both of the firms. Aloysius and Deck (2008) report behavioral experiments where firms engage in bundling while competing for informed customers and maintaining monopoly power over uninformed customers. Their results indicate that sellers tend to be overly competitive, using bundling as a competitive weapon rather than as a tool for price discrimination.

⁵ Schmalensee (1984) finds similar results in a model with continuous (bivariate normal) valuations. McAfee, McMillan and Whinston (1989) provide conditions under which such bundle pricing is optimal. Hanson and Martin (1990) show how to compute optimal bundle prices using a mixed integer linear program.

(for a survey see Fudenberg and Villas-Boas 2006) in which firms use information about consumers' previous purchases to offer different prices and/or products to consumers with different purchase histories. Empirical data shows that even the information contained in observing one historic purchase occasion by a customer boosts net target couponing revenue by 50% (Rossi et al. 1996). Better ability to predict preferences has been shown to potentially reduce price competition (Chen et al. 2001). Acquisti and Varian (2005) show analytically that it is optimal to price so as to distinguish between high-value and low-value customers. There is empirical evidence that competing firms have been able to price discriminate profitably by charging different prices across consumer segments (Basenko et al 2003). Moon and Russell (2008) develop a product recommendation model based on the principle that customer preference similarity stemming from prior purchase behavior is a key element in predicting current purchase. These studies exploit customer revealed preference for a good in order to set prices for future purchases of that good. The current research extends the issues explored in previous research to study how buyer revealed preferences inferred from initial purchase decisions of one good, can be used to set optimal prices for purchases of other goods. One advantage of this new mode of target marketing is that information on revealed preference can be used in the same online or in-store visit. Furthermore, it is not dependent on identifying customers in order to access their buying history.

3. Sequential Pricing in Monopoly Conditions

3.1 Monopolist's Problem

We begin with the simple assumption of a monopolist facing a pricing decision in two sequentially ordered markets. The products may be substitutes, complements, or neither. However, it is assumed that the consumer's value for one item is distributed independently of the other. In other words, the value of the product purchased second may depend on whether the initial good was purchased, but the values of the two goods separately are not correlated. Of course, the optimal pricing strategy depends upon whether the monopolist can use information regarding the consumer's decision in the market for the first good when setting the price for the second good. We first consider the case in which the monopolist cannot use such information and then follow that with an analysis of the case in which the monopolist is able to price discriminate in the market for the second good.

Before considering these two cases, it is useful to note that sequential pricing in the absence of price discrimination is substantively different from simultaneous pricing. The sequential pricing problem is one in which the monopolist recognizes the impact of the price of good A on the purchase of good B.

By recognizing the behavioral response at the second stage (good B decision) to the outcome in the first stage (good A decision), the monopolist will consider those results in expectation when pricing good A. Consumer behavior differs in that the sequential problem does not provide the consumer with full information when making a choice. Rather, the consumer in this model is assumed to be myopic, choosing to purchase A solely on its price relative to value and then a choice regarding B will occur subsequently. Given that consumer behavior is entirely different due to the timing effects, the pricing strategy also is entirely different. As such, it is important to fully model the case of sequential pricing absent price discrimination.

We begin with the general market set-up and then consider the each case in turn. Assume a market exists for two products denoted A and B , and a consumer has a choice first to buy A followed by a choice to buy B in sequence. Let the consumer's value for A be distributed $V_A \sim f_A(V_A)$. Then the consumer will buy A iff $P_A \leq V_A$. Similarly, the consumer's independent value for B follows the distribution $V_B \sim f_B(V_B)$. Following Venkatesh and Kamakura (2003), a consumer's joint value from purchasing both A and B is denoted $V_{AB} = (1 + \theta)(V_A + V_B)$, where θ represents complementarity if $\theta > 0$, and substitutability if $\theta < 0$. A consumer who chooses not to purchase A will buy B iff $P_B \leq V_B$. However, if A was purchased, then the joint value becomes relevant and the consumer will buy B iff $P_B < (1 + \theta)(V_A + V_B) - V_A$ which can be rewritten as $V_B > \frac{P_B - \theta V_A}{1 + \theta}$. A consumer will purchase only one unit of either item, and the monopolist produces each item at constant marginal costs of C_A and C_B respectively.

Given this framework, now consider a monopolist's problem in setting prices.

Case 1: Monopoly Pricing without Price Discrimination

When the monopolist sets P_B , it is as if there is no information concerning the decision to buy A . Rather, the monopolist will know the probability that A will have been chosen, conditional on the price of A and the distribution of preferences. Similarly, the monopolist sets the price of A knowing the probability A will be bought and therefore, how that probability will affect the subsequent purchase of B . Let us consider these stages in reverse. In other words, conditional on a price of A and the corresponding probability that A was purchased, how then should the monopolist price B ? And then, given that optimal response to a price of A , how should the monopolist price A in the first place?

Stage 2: Price of B

The monopolist will maximize expected profit with respect to the price of B conditional on a price of A and the distributions over preferences. Profits are $P_B - C_B$ if a sale is made and 0 otherwise. The monopolist maximizes equation (1).

$$\begin{aligned} \max_{P_B} E\Pi(P_B|P_A) = & (P_B - C_B) \int_{P_B}^{\infty} \int_0^{P_A} f_A(V_A) f_B(V_B) dV_A dV_B + \\ & (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f_A(V_A) f_B(V_B) dV_B dV_A \end{aligned} \quad (1)$$

The first term is profit from those who buy B but not A and the second term is profit from those who buy both B and A . Differentiating (1) with respect to P_B to determine the first order condition and solving for $P_B^* = f(P_A)$ gives an optimal response function based upon the choice in the first market.

Stage 1: Price of A

Given P_B^* , the monopolist must choose the optimal P_A for stage 1 by maximizing expected profit for the sum of both stages, recognizing that the choice of P_B depends on P_A . In stage 1, the monopolist maximizes equation (2).

$$\begin{aligned} \max_{P_A} E\Pi(P_A) = & (P_A - C_A) \int_{P_A}^{\infty} \int_0^{\frac{P_B^* - \theta V_A}{1 + \theta}} f_A(V_A) f_B(V_B) dV_B dV_A \\ & + (P_B^* - C_B) \int_{P_B^*}^{\infty} \int_0^{P_A} f_A(V_A) f_B(V_B) dV_A dV_B \\ & + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B^* - \theta V_A}{1 + \theta}}^{\infty} f_A(V_A) f_B(V_B) dV_B dV_A \end{aligned} \quad (2)$$

where the first term is profit from those who buy A only, the second term is profit from those who buy B only, and the third term is the profit from those who buy both. The general solution is derived by finding the first order condition of (2) with respect to P_A , solving this first order condition for P_A^* and then calculating P_B^* .

This exercise is intractable in general, so we now consider the case of a uniform distribution for consumer preferences. Specifically, let $f_A(V_A) \sim U[0,100]$ and $f_B(V_B) \sim U[0,100]$. In this case (1) can be rewritten as

$$\max_{P_B} E\Pi(P_B|P_A) = (P_B - C_B) \int_{P_B}^{100} \int_0^{P_A} \frac{1}{100^2} dV_A dV_B + (P_B - C_B) \int_{P_A}^{100} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{100} \frac{1}{100^2} dV_B dV_A \quad (1')$$

After integrating and simplifying (1') the problem becomes

$$\max_{P_B} E\Pi(P_B|P_A) = \frac{(P_B - C_B)}{100^2} [100^2 - P_A P_B + \frac{P_B}{1+\theta} (P_A - 100) + \frac{\theta}{1+\theta} (\frac{100^2 - P_A^2}{2})] \quad (1')$$

Differentiating (1') with respect to P_B and simplifying the first order condition yields

$$P_B^* = \frac{C_B(P_A\theta - 100) - \frac{100^2}{2}(2+3\theta) + \frac{\theta P_A^2}{2}}{-2(P_A\theta + 100)} \quad (3)$$

Performing the same exercise using uniform distribution, (2) can be rewritten as

$$\begin{aligned} \max_{P_A} E\Pi(P_A) = & (P_A - C_A) \int_{P_A}^{100} \int_0^{\frac{P_B^* - \theta V_A}{1+\theta}} \frac{1}{100^2} dV_B dV_A + \\ & (P_B^* - C_B) \int_{P_B^*}^{100} \int_0^{P_A} \frac{1}{100^2} dV_A dV_B + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{100} \int_{\frac{P_B^* - \theta V_A}{1+\theta}}^{100} \frac{1}{100^2} dV_B dV_A \end{aligned} \quad (2')$$

Integrating (2') and simplifying yields (2'')

$$\max_{P_A} E\Pi(P_A) = \frac{(P_A - C_A)}{100^2} \left[100^2 - P_A P_B^* + \frac{P_A P_B^* - 100 P_B^*}{1+\theta} + \frac{\theta}{2(1+\theta)} (100^2 - P_A^2) \right] \quad (2'')$$

where P_B^* is found in equation (3). Solving the first order condition for (2'') we need to differentiate with respect to P_A given that P_B is a function of P_A . The solution is quite cumbersome, but can be found using Mathematica.⁶ We do note that when $C_A = C_B = \theta = 0$ we can find $P_A^* = 50$, $P_B^* = 50$ which is precisely the optimal monopoly price in the two independent markets taken separately.

Case 2: Monopoly Pricing with Price Discrimination

In this case the monopolist will know when setting the price of B whether the consumer has purchased A or not. Formally, the decision is to choose either $P_B|(q_A=0)$ or $P_B|(q_A=1)$ where $q_A=0$ if A was not purchased and $q_A=1$ otherwise. In other words, the monopolist selects a state contingent price for B.

Stage 2: Price of B if $q_A=0$, i.e. $V_A < P_A$

Since the buyer's values for A and B are independent and since A is not purchased, the monopolist's problem is to maximize

$$E\Pi(P_B|(q_A = 0)) = (P_B - C_B) \int_{P_B}^{\infty} f_B(V) dV_A \quad (4)$$

Taking the first order condition of (4) and solving yields $P_B^*|(q_A=0)$.

⁶ The equation is available upon request from the authors.

For the uniform distribution example (4) simplifies to (4').

$$(P_B - C_B) \int_{P_B}^{100} \frac{1}{100} dV_B = \left(\frac{P_B - C_B}{100} \right) (100 - P_B) \quad (4')$$

Maximizing (4') with respect to P_B and solving yields $P_B^* | (q_A = 0) = \frac{100 + C_B}{2}$. When $\theta=0$ and $C_B=0$ we get the standard monopoly solution of $P_B^*=50$.

Stage 2: Price of B if $q_A=1$, i.e. $V_A \geq P_A$

In this case the monopolist considers the joint valuation of both products when pricing B. In other words, $V_{AB} = (1 + \theta)(V_B + V_A)$. Thus, the marginal value of B = $(1 + \theta)(V_B + V_A) - V_A$ and the consumer will buy B iff $\frac{P_B - \theta V_A}{1 + \theta} \leq V_B$.

Given this information, the monopolist chooses to maximize equation (5).

$$\max_{P_B} E\Pi(P_B | (q_A = 1)) = (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_B) f(V_A | V_A \geq P_A) dV_B dV_A \quad (5)$$

Taking the first order condition of (5) and solving yields $P_B^* | (q_A=1)$.

Under the assumption of the uniform distribution this can be rewritten as (5').

$$\begin{aligned} \max_{P_B} E\Pi(P_B | (q_A = 1)) &= (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_B) f(V_A | V_A \geq P_A) dV_B dV_A \\ &= \frac{P_B - C_B}{100} \left(100 - \frac{P_B - \theta(100 - P_A)/2}{1 + \theta} \right) \end{aligned} \quad (5')$$

Taking the first order condition of (5') and solving for the price of B yields

$P_B^* = \frac{100(1 + \theta) + \theta(100 - P_A)/2 + C_B}{2}$. Note that once again when $\theta=0$ and $C_B=0$ we get the standard monopoly solution of $P_B^*=50$.

Stage 1: The Price of A

We now need to solve for P_A given what will occur in stage 2. Specifically we need to know $E\Pi$ when $q_A=0$ or $q_A=1$. Plugging the solution for $P_B^* | (q_A=0)$ and $P_B^* | (q_A=1)$ into (4) and (5) respectively gives the expected profit in each state. The monopolist will maximize total expected profit over both stages,

knowing both the probability that A will be purchased at a given price and the resulting expected profits in stage 2 based on the follow-up price of B.

In general this problem is not tractable, but again we can set it up for the uniform case and find the solution. Recall that $P_B^*(q_A = 0) = \frac{100+C_B}{2}$. Computing the resulting profit at the second stage yields $E\Pi_B(q_A = 0) = (P_B - C_B) \int_{\frac{100+C_B}{2}}^{100} \frac{1}{100} dV_B$ which can be simplified to $25 - \frac{C_B}{2} + \frac{C_B^2}{4(100)}$.

When $q_A=1$, $P_B^*(q_A = 1) = \frac{100(1+\theta)+\theta(100-P_A)/2+C_B}{2}$ which yields a corresponding second stage expected profit of $\Pi_B(q_A = 1) = (P_B - C_B) \int_{P_A}^{100} \int_{\frac{P_B-\theta V_A}{1+\theta}}^{100} \frac{1}{100} \frac{1}{100-P_A} dV_B dV_A$. This can be simplified to $E\Pi_B(q_A = 1) = \frac{1}{200} (100(1 + \theta) + \theta(100 - P_A)/2 - C_B) \left(50 + \frac{\theta(100-P_A)/2-C_B}{(1+\theta)} \right)$

Therefore, for the uniform case we have that the first stage profit as a function of P_A can be written as

$$\begin{aligned} E\Pi(P_A) &= \left(\int_0^{P_A} \frac{1}{100} dV_A \right) \left(25 - \frac{C_B}{2} - \frac{C_B^2}{400} \right) + \left(\int_{P_A}^{100} \frac{1}{100} dV_A \right) \left\{ P_A - C_A + \frac{1}{200} \left[100(1 + \theta) + \frac{\theta(100-P_A)}{2} - C_B \right] \right\} \\ &= \frac{1}{100} \left\{ P_A \left(25 - \frac{C_B}{2} - \frac{C_B^2}{400} \right) + \frac{(100 - P_A)}{100} \left(P_A - C_A + \frac{1}{200} \left[\dots \right] \frac{[\dots]}{2(1 + \theta)} \right) \right\} \end{aligned}$$

The first order condition is that

$$25 - \frac{C_B}{2} - \frac{C_B^2}{400} + 100 - 2P_A + C_A - \frac{1}{200} \left[100(1 + \theta) + \frac{\theta(100-P_A)}{2} - C_B \right] \left(50 + \frac{\theta(100-P_A)/2-C_B}{2(1+\theta)} \right) = 0.$$

$$\text{Solving for } P_A \text{ yields } P_A^* = \frac{50 - C_B + \frac{C_B^2}{200} + 100 + C_A - \frac{1}{100} [100(1+\theta) + \theta(100-P_A)/2 - C_B] \left(50 + \frac{\theta(100-P_A)/2-C_B}{2(1+\theta)} \right)}{2}.$$

As above, when $\theta=0$, $C_B=0$ and $C_A=0$ we find the standard monopoly price $P_A^*=50$.

3.2 Impact of the ability to conditionally price

In the special case where $\theta=0$ and the values are independently distributed, the purchase of good A has no direct effect on the buyer's value for good B (the goods are neither complements nor substitutes when $\theta=0$) and knowing that $V_A \geq P_A$ provides no information to the seller as to the likely values of V_B . Therefore, P_B should be the same for everyone even if the seller could set conditional prices and this will be the same price that would be charged if the seller could not discriminate. With $C_A=0$ and $C_B=0$, the price is 50 for all three buyer types.

In general these three prices and the resulting profits will differ if the goods are complements or substitutes. Given the complexity in the solutions above, in Table 1 we provide a numerical comparison of sequential pricing with and without the ability to price discriminate for goods with independent values where the additivity in bundle values varies from $\theta = -0.5$ to $\theta = 1.0$. The above analysis does not examine the situation where the underlying values for the two goods are not independent. While there are many distributions that one could use for such analysis, the choice is arbitrary unless one has information about a specific set of naturally occurring product markets, which are unlikely to follow a normal, uniform, or any other mathematically nice distribution. Therefore, we offer a numerical comparison for a series of distributions where the correlation varies from $\rho = -1$ to $\rho = 1$. Specifically, the distributions used for this comparison are created by removing the opposing corners from the square domain $[0,100] \times [0,100]$. These distributions are easy to describe in a manner similar to the independent values case, thus making them appropriate for behavioral exploration in the laboratory, which is the focus of next two sections of the paper.

*****Insert Table 1 here*****

As one would expect, the ability to price discriminate increases profitability for the monopolist. Further, if the seller were forced to charge a single price for good B, this price would lie somewhere between the two prices that would be charged if discrimination were possible, a result similar to standard third degree price discrimination. The profit increase becomes more pronounced as θ or ρ becomes more distant from 0. When the values are not correlated, those buyers who did not purchase good A observe the same price for good B that the monopolist would have charged if the markets had been treated separately (as in pure components). For those who did buy good A, the price for B will be lower (higher) with sequential pricing than it would have been if the monopolist treated the goods separately when the goods are substitutes (complements). Finally, we note that sequential pricing without price discrimination is symmetric with respect to correlation. That is, the set of prices and the resulting profits are based upon $|\rho|$.

3.3 Comparison of sequential pricing with simultaneous pricing

Venkatesh and Kamakura (2003) explore the optimal bundle prices for a similar framework again under the assumption of a uniform distribution with independently distributed values. The complexity of both models makes a direct theoretical comparison difficult. However, it is reasonable to ask if sequential price discrimination outperforms traditional mixed bundling. To explore this, we again offer

a numerical comparison. For completeness, we also include the optimal prices and profits for a monopolist that charges a single simultaneous price for each good (i.e. pure components). The numerical results are presented in Table 2.

The information presented in Tables 1 and 2 indicate that sequential pricing with discrimination can outperform mixed bundling when the goods are very close substitutes ($\theta = -0.4, -0.5$). This result is somewhat intuitive in that when the goods are close substitutes, mixed bundling essentially gives the second product away. Sequential pricing allows the seller to exploit more fully those buyers who have a high value for both goods by charging them a (still very small but) larger marginal price for the second good. Sequential pricing with discrimination can also outperform mixed bundling when the values of the goods are highly positively correlated ($\rho = 0.75, 0.85, 1.0$). This is the same pattern observed in the example discussed in the introduction. The comparison of sequential pricing without discrimination and pure components, the two cases where sellers set a single price for each good, is also revealing. While the two practices lead to identical outcomes when the values of the goods are correlated, when the goods are substitutes ($\theta < 0$), sequential pricing without discrimination outperforms pure components pricing. It is also interesting to note that the benefits of mixed bundling relative to pure components seem to be greatest the closer θ is to 0 or ρ is to -1.

*****Insert Table 2 here*****

4. Sequential Pricing in Competitive Markets with Informed and Uninformed Buyers

The above analysis focuses on the problem faced by a monopolist. However, most of the examples described in the paper occur in more competitive markets. Competitive markets create a tension between using prices to set up future price discrimination and attracting customers in the first place. To examine this setting, we incorporate competition à la Varian's (1980) model of sales. In this model uninformed shoppers account for a fraction α of the market and the remaining $1-\alpha$ are informed. Informed buyers observe the price of good A offered by the n sellers in the market. These buyers may learn of the price via online shopping comparison websites or by reading advertisements in the newspaper. If the lowest offered price is at or below the informed buyer's reservation value, then the buyer will visit the low price seller, make the purchase and then observe the price of good B and otherwise the buyer will not visit any seller and will make no purchase. In this sense, good B is a new or

unknown product or an impulse type item.⁷ The uninformed buyers visit only one seller, observe that seller's price of good A, make a purchase decision, and then observe the possibly conditioned price of B. These buyers can be thought of as being loyal to the seller, having a preference for a particular seller, an unawareness of competitors, or travel costs that prohibit visiting other sellers. It is assumed that an equal fraction of uninformed buyers visit each seller. Therefore each seller acts as a monopolist to α/n of the market.

Again, a seller's problem differs if the price of good B can be conditioned on the decision to purchase good A. The appropriate solution concept is a symmetric mixed strategy Nash equilibrium. First, one can note that a seller always has the option to set monopoly prices and fully exploit the fraction of uninformed customers who visit. The profit from this action is referred to as the security profit since the seller can unilaterally guarantee itself this amount. We also note that there is no pure strategy equilibrium in this game as a firm would always prefer to lower one of its prices by some $\epsilon > 0$ and capture the entire market or simply receive the security profit. Since any pure strategy in the mixed strategy equilibrium must generate the same expected payoff, one can use then use the security payoff to implicitly define the mixing distribution. See Deck and Wilson (2006) and Aloysius and Deck (2008) for details. While the approach is intuitive, the implementation is not practical given the complexity of the problem. Therefore, we turn directly to a series of laboratory experiments for exploring the likely market outcomes of sequential pricing with and without price discrimination.⁸

4.1 Experimental Design

To explore the impact of sequential pricing in competitive markets, a total of 24 laboratory sessions were conducted. The sessions include 4 replicates of each treatment in a 3x2 design. The first factor of the design is the relationship between two experimental goods, A and B. In the "independent values" condition, the buyer's values for the two goods are independent and the value of the bundle made from purchasing both goods is the sum of the values of the separate items (i.e. $\theta=0$). Specifically,

⁷ An alternative approach is to assign the informed buyer who chooses not to purchase A to some store randomly and then observe the B price there. However, it is not clear why the buyer would choose to go to the store or website if they know that they are not going to purchase the item of interest.

⁸ Both Deck and Wilson (2006) and Aloysius and Deck (2008) look at pricing strategies in competitive markets using controlled laboratory methods and find that observed behavior is not consistent with the limited theoretical predictions.

$V_A \sim U[0,100]$ and $V_B \sim U[0,100]$ in the independent values case. For the “complements” condition, the single item values are the same as in the previous case but the value from buying both items is $1.3(V_A + V_B)$, that is $\theta = 0.3$. The third condition involves positively “correlated values,” where V_A and V_B are jointly distributed according to the density function $f(V_A, V_B) = \begin{cases} 1/7651 & \text{if } |V_A - V_B| \\ 0 & \text{else} \end{cases}$ for V_A, V_B integers $\in [0,100]$. For this distribution the correlation is $\rho = 0.5$. Bundle values are additive in the correlated values condition (i.e. $\theta = 0$). The distributions used in the laboratory are the same as those presented in the numerical comparisons in Tables 1 and 2.⁹ The second factor in the experimental design was the ability or inability to price discriminate by conditioning the price of good B on the buyer’s decision to purchase good A.

Subjects were recruited from undergraduate courses at a state university. While many of the participants are in the business school, students from other disciplines participated as well. The students were recruited directly from classes and through the laboratory’s subject database. Some of the subjects had prior experience in experiments; however, none had previously participated in any related experiments. Each laboratory session lasted 90 minutes, including approximately 30 minutes for the self paced written directions and the completion of a comprehension handout.¹⁰ After completing the handout, responses were checked by an experimenter and any remaining questions were answered. Once all of the participants were ready, the actual experiment began.

Sessions lasted for 750 paid market periods, including 250 periods that served as practice to allow the subjects to become familiar with the interface and competitive pressures in these markets.¹¹ During the experiment, subjects had an onscreen tool that would identify which potential uninformed customers would buy each good and the expected profit based upon a subject specified pricing strategy. This tool is shown in Figure 1. Value combinations that would lead to purchases of good A only were

⁹ The specific choices are not ones where sequential pricing are expected to outperform bundling as the main purpose of this paper is not to compare the two practices, but rather to explore the implications of sequential pricing as a novel strategy. The chosen distributions are not extreme and were selected with a eye towards the ease with which they could be explained to subjects.

¹⁰ Copies of the directions and the handout are available from the authors upon request.

¹¹ Subjects did not know the total number of periods in the experiment nor were the practice periods singled out for the subjects. Subjects did know the total length of the session, but typically the experiments finished before the allotted time had expired.

shaded in yellow while value combinations that would lead to the purchase of good B only were in blue. Value combinations such that the person would buy both A and B were shaded green. Combinations for which the person would not buy anything are white and areas shaded black could not occur given the distribution of values. A subject could click on an area in the diagram on the right and the diagram on the left would focus on the specified area revealing the actual buyer values in that region. For simplicity, the marginal cost of producing both types of goods was $C_A = C_B = 0$ and therefore profit and revenue are identical.

Each session involved $n = 4$ seller subjects.¹² Sellers could adjust their prices at any point and received feedback about the prices charged and profit earned by each rival after every period. In each period a single buyer would enter the market and make a purchase decision based upon their randomly determined values for goods A and B. Since buyers demand a single unit of each good, there is no incentive for them to not truthfully reveal their willingness to buy. Therefore, the buyer role was automated, a common practice in posted offer market experiments where demand withholding is not a critical element of the design (see Davis and Holt 1993).

*****Insert Figure 1 here*****

Loyal (or uninformed) customers accounted for $\alpha = 80\%$ of the market so each seller served as a monopolist to $\alpha/n=20\%$ of the market. Comparison (or informed) shoppers accounted for $1 - \alpha = 20\%$ of the market. If multiple sellers set the same price, a comparison shopper would randomly select one low price seller from whom to purchase.

At the conclusion of the experiment, subjects were paid based upon their earned profit at the rate \$400 in profit = US \$1. The average salient payment was approximately \$18.00. Participants also received a fixed payment of \$7.50 for arriving on time and participating in the study. Subjects were paid in private and were dismissed from the experiment once they had collected their money. Multiple sessions occurred concurrently in the laboratory. This prevented subjects from being able to identify which other participants were sellers in the same market. It also controlled for idiosyncrasies from one time in the laboratory to the next.

4.2 *Experimental Results*

¹² The total number of subjects was thus 4 sellers per session \times 4 sessions per treatment \times 6 treatments = 96 total subjects.

The analyzed data consist of 48,000 market pricing decisions.¹³ Of course, observations from the same subject or even from the same session are not independent. Therefore, linear mixed effects models are utilized to control for the repeated measures present in the data at the period level. Standard non-parametric tests are utilized for comparisons at the session level since each session is independent.

The results are presented separately for each of the three market conditions (independently valued goods, complementary goods, and positively correlated goods). For each market a series of five results are presented.

1. The impact of the ability to sequentially price discriminate on the price of good A.
2. The impact of the ability to sequentially price discriminate on the price of good B.
3. The impact that comparison shopping has on the price for good A.
4. The impact that comparison shopping has on the price for good B.
5. The welfare implications (buyer surplus and seller profit) of the ability to engage in price discrimination.

4.2.1 *The Case of Independent Values*

Figure 2 shows the distribution of prices for good A (left panel) and good B (right panel). Not surprisingly, good A prices tend to be much lower than good B prices, as sellers are competing for customers via good A prices. Notice further that there is evidence for a leader pricing strategy, as a significant proportion of sellers (regardless of whether or not they could price discriminate) priced at or close to marginal cost (Hess and Gerstner 1987).

*****Insert Figure 2 here*****

Impact on price of good A: The ability to sequentially price discriminate leads to nominally but not significantly greater prices.

Support: The top portion of Table 3 provides the econometric support based upon the linear mixed effects model estimation. The conclusion is supported by the lack of significance on α_1 .

¹³ 500 periods per subject \times 96 subjects = 48,000 market decision periods. The last 500 periods of each session are used to control for learning effects.

The average price of good A is only 18 when sellers cannot base the price of good B on the good A purchase decision. This amount nominally increases to $21.6 = 18 + 3.6$ when sellers do have the ability to set conditional prices for good B. In both cases, competition has forced the price of good A to be low and in fact it is not uncommon to see sellers setting a price of 0 for good A in both treatments.

With independent values, sellers gain no information about the consumer's valuation of good B based upon the decision to purchase good A. Therefore, one would expect there to be no difference between the price for someone who did buy A and the price for someone who did not. Further, these prices should equal that observed when sellers cannot discriminate. Given the parameters used in the experiments, the optimal price of good B should be 50 for all buyers in independent values conditions.

*****Insert Table 3 here*****

Impact on the price of Good B: As expected, sellers do not set different prices for buyers based upon the decision to purchase good A. The ability to engage in sequential price discrimination leads to nominally, but not significantly, higher prices for good B.

Support: The lower section of Table 3 provides the econometric support. The lack of a difference for those who did and those who did not buy good A is based upon a test of the hypotheses that $\beta_1 = \beta_2$, which cannot be rejected at standard levels. That the ability to set prices sequentially does not impact the price of good B is evidenced by the lack of significance on β_1 and β_2 . One must reject the hypothesis that sellers set the optimal price for good B when sellers cannot price discriminate (i.e. $\beta_0 = 50$). However, one would not reject the hypothesis that the average price is 50 for either segment when sellers can price discriminate at the 5% significance level (i.e. $\beta_0 + \beta_1 = 50$ and $\beta_0 + \beta_2 = 50$). Of course, the prediction for good B price is for every seller to charge 50 in every period, which does not occur as evidenced by the right hand panel of Figure 2.

The next two findings evaluate the impact that comparison shopping has on consumers. Since the best price for good A that comparison shoppers observe is the minimum of four prices, whereas uninformed buyers observe a single price drawn from the same distribution, comparison shoppers must have weakly lower prices. The amount by which comparison shopping lowers the expected price for good A relative to what an uninformed buyer would pay is a function of the variance within a market period.

Ultimately, there is considerable variation within a market resulting in large gains from comparison shopping.¹⁴

Impact of Comparison Shopping on Price for Good A: Comparison shopping lowers the price of good A by 66% when sellers cannot price discriminate and 50% when they can.

Support: Table 4 provides the estimation results for linear mixed effects models for the minimum good A price. Comparison shoppers observed an average price of 6.1 without price discrimination and 11.7 with it. The results in Table 3 identify the typical price paid by uninformed consumers.

Unlike good A purchases for which comparison shoppers consider the lowest of four prices, comparison shoppers only observe the good B price for the seller who offered the lowest good A price. These consumers could fare worse than typical uninformed customers if sellers who set the lowest price for good A systematically charge higher prices for good B.

*****Insert Table 4 here*****

Impact of Comparison Shopping on Price for Good B: Sellers who set the lowest prices for good A do not charge substantially different prices for good B as compared to other sellers.

Support: Table 4 also provides the estimation result for the linear mixed effect model for the price of Good B charged by sellers who set the lowest price for good A. This is comparable to results in Table 3 for the typical uninformed consumer. The seller with the lowest price for good A charged 32.0 on average for good B, which is not statistically different from the 33.2 that the average seller charged.

The final result in this subsection focuses on the welfare implications of the ability to price discriminate. Buyer (consumer) surplus is the difference between the buyer's value for the item and the price actually paid. Seller profit is the difference between the price received and the item's cost (here normalized to $C_A = C_B = 0$). Efficiency is the percentage of possible gains from trade (consumer surplus + seller profit) that are actually realized. In this market maximum efficiency is obtained when every buyer purchases both goods. While this will occur if the price of both goods is 0, the sellers would earn no

¹⁴ As the percentage of the customers who comparison shop changes, the distribution used by sellers to set prices will also change. Therefore one cannot predict that the same levels or treatment effects will hold under different search frequencies. See Deck and Wilson (2006) for a discussion of how changes in the percentage of informed customers can change seller behavior in a similar market experiment.

profit at this price. Ultimately, these markets were highly efficient, averaging 84% without the ability to price discriminate and 79% with sequential pricing, an insignificant difference. Further, the ability to sequentially price discriminate did not change buyer surplus or seller profit as made explicit in the following finding.

Welfare Implications of Sequential Price Discrimination: Sellers' ability to sequentially price discriminate does not change the welfare outcomes in these markets.

Support: Figure 3 plots buyer surplus and seller profit for each session in the two independent values conditions. Using the session average as the unit of measure, one cannot reject the null hypothesis of no treatment effect on buyer surplus, seller profit, or efficiency based upon the Wilcoxon Rank Sum test.

*****Insert Figure 3 here*****

4.2.2 *The Case of Complementary Goods*

The analysis of complementary goods closely parallels the analysis of the independent values case. Figure 4 shows the distribution of prices for each good by condition. Evident from the figure is that the price of good A tends to be higher when sellers can sequentially price discriminate, although this difference is not significant. Also evident is the result that buyers who purchased good A paid significantly more for good B than those who did not buy good A when sellers could price discriminate. This effect is consistent with the model since purchasing good A increases the marginal value of good B. Interestingly, the distribution of the price of good B when sellers cannot conditionally price is similar to the distribution of prices for customers who could be identified as having purchased good A. That is, rather than the practice of price discrimination leading to higher prices for the high valued buyers, it actually leads to a price discount for the low value segment. These results are formalized in the next two findings:

*****Insert Figure 4 here*****

Impact on price of good A: The ability to sequentially price discriminate leads to nominally but not significantly greater prices. To a greater degree than even the baseline case there is evidence for a leader pricing strategy as (independent of the ability to price discriminate), close to 40% of sellers priced at or very close to marginal cost.

Support: The top portion of Table 5 provides the econometric support based upon the linear mixed effects model estimation ($\alpha_1=0$).

*****Insert Table 5 here*****

Impact on the Price of Good B: As expected, sellers set greater prices for buyers who purchased good A than those that did not when sellers could set conditional prices. When unable to discriminate, sellers set prices similar to those set for buyers who could be identified as having purchased good A.

Support: The lower portion of Table 5 provides the econometric support. The difference in price for those who did and those who did not purchase good A when sellers could price discriminate is based upon a test of the null hypotheses that $H_0: \beta_2 - \beta_1 = 0$ against the one sided alternative that $\beta_2 - \beta_1 > 0$. The second claim is supported by the lack of significance for β_1 .

As in the case of independent values, comparison shopping leads to a dramatic reduction in the price that a buyer considers given the within period variation in prices. Also, sellers who set the lowest price for good A do not charge substantially different prices for good B as compared to the other sellers in the market.

Impact of Comparison Shopping on Price for Good A: Comparison shopping lowers the price of good A by 62% when sellers cannot price discriminate and 50% when they can.

Support: Table 6 provides the estimation results for linear mixed effects models for the minimum good A price. Comparison shoppers observed an average price of 5.4 without price discrimination and 9.1 with it. The results in Table 5 identify the typical price paid by uninformed consumers.

*****Insert Table 6 here*****

Impact of Comparison Shopping on Price for Good B: Sellers who set the lowest prices for good A do not charge substantially different prices for good B as compared to other sellers.

Support: Table 6 also provides the estimation results for the linear mixed effect model for the price of good B charged by sellers who set the lowest price for good A. The seller with the lowest price for good A on average charged 33.8 for those shoppers identified as not having bought Good A and 53.4 for other shoppers. These are comparable to results in Table 5 for the typical uninformed consumer where the typical shopper who could be identified as having not bought A was charged 34.1 and other shoppers were charged 51.8.

The analysis now turns to the welfare implications of the ability to price discriminate with conditional pricing. Ultimately, the practice does not significantly impact buyer surplus, seller profit, or efficiency. This result seems surprising in light of the fact that the practice of sequential pricing leads to lower good B prices for some buyers. However, the good A prices are so low due to the competition that the percentage of buyers who do not purchase good A is relatively small and the surplus lost when these buyers do not purchase good A is necessarily small since they have low valuation for good A.

Welfare Implications of Sequential Price Discrimination: Sellers' ability to sequentially price items does not change the welfare outcomes in these markets.

Support: Figure 5 plots buyer surplus and seller profit for each session in the two complementary goods conditions.¹⁵ Using the session average as the unit of measure, one cannot reject the null hypothesis of no treatment effect on buyer surplus, seller profit, or efficiency based upon the Wilcoxon Rank Sum test.

*****Insert Figure 5 here*****

4.2.3 The Case of Correlated Values

The analysis of this subsection closely parallels the two previous subsections. Figure 6 plots the distribution of prices in the correlated values treatments.

Impact on Price of Good A: The ability to sequentially price discriminate has no impact on the average price of good A. However in this case as well we note a large proportion of sellers following a leader pricing strategy.

Support: The top portion of Table 7 provides the econometric support based upon the linear mixed effects model estimation ($\alpha_1 = 0$).

*****Insert Figure 6 here*****

*****Insert Table 7 here*****

Impact on the Price of Good B: The average posted price when sellers cannot discriminate based upon who purchased good A was statistically the same as the average price posted to buyers who could be

¹⁵ For making comparisons across value conditions it is important to keep in mind that the total surplus possible is greater with complementary goods than with either of the other conditions. Hence it is possible for both buyer surplus and seller profits to be higher in this condition than in the other two, which is in fact what occurred.

identified as having purchased good A. Buyers who could be identified as not having purchased good A were quoted a (marginally) significant lower price for good A than those buyers whose decision could not be identified by a seller, as predicted. However, contrary to the predictions, there is no statistical difference between the prices paid by those who had and those who had not bought good A when sellers could discriminate.

Support: The lower portion of Table 7 provides the econometric support. The claim that the price quoted to buyers in the absence of conditional pricing is the same as the price quoted to buyers who purchased good A when sellers could discriminate is supported by the lack of significance for β_2 . The marginal price break for those identified as not purchasing good A relative to the no price discrimination case is evidenced by the marginal significance of β_1 in a one sided test with $H_a: \beta_1 < 0$. The result that there is no difference between the price for those who did and those who did not purchase good A when sellers could discriminate is based upon a test of $\beta_1 = \beta_2$.

As in the two previous cases, comparison shopping leads to substantially lower prices for good A due to the high within period price variation and that the sellers charging the lowest price do not charge different prices for good as compared to their rivals.

Impact of Comparison Shopping on Price for Good A: Comparison shopping lowers the price of good A by about 44% when sellers cannot price discriminate and 65% when they can.

Support: Table 8 provides the estimation results for linear mixed effects models for the minimum good A price. The results in Table 7 identify the typical price paid by uninformed consumers.

*****Insert Table 8 here*****

Impact of Comparison Shopping on Price for Good B: Sellers who set the lowest prices for good A do not charge substantially different prices for good B as compared to other sellers.

Support: Table 8 also provides the estimation results for the linear mixed effect model for the price of good B charged by sellers who set the lowest price for good A. These are comparable to the results in Table 6 for the typical uninformed consumer.

With correlated values there are no adverse welfare implications due to sequential pricing, just as in the two previous cases.

Welfare Implications of Sequential Pricing: Sellers' ability to sequentially price items does not change the welfare outcomes in these markets.

Support: Figure 7 plots buyer surplus and seller profit for each session in the two correlated values conditions. Using the session average as the unit of measure, one cannot reject the null hypothesis of no treatment effect on buyer surplus, seller profit, or efficiency based upon the Wilcoxon Rank Sum test.

*****Insert Figure 7 here*****

5. Conclusions

New technologies will enable sellers to engage in new pricing strategies and it is important to anticipate how these strategies are likely to affect sellers and customers. Currently, there is a growing trend in retail markets to track individual items. RFID tags or similar technologies can be used to identify which items a buyer intends to purchase at a given price, just as placing an item in an electronic shopping cart does for an e-tailer. Currently, sellers openly use this information to manage inventory and make recommendations regarding other products. However, this information could also be used to adjust prices on items a shopper is likely to purchase.

What are the likely implications of sellers being able to set prices sequentially and discriminate based upon previous actions? As a first step, this paper presents a theoretical model that can be used to answer this question for monopoly markets. The results indicate that the ability to set prices sequentially, absent the ability to discriminate, increases profits relative to a pure components framework where the monopolist sets a price for each good simultaneously when the goods are substitutes. Further, sequential pricing with discrimination is more profitable than mixed bundling when the goods are either close substitutes or when the goods are highly positively correlated.

The technology to engage in sequential pricing exists in competitive markets too and the implications may be very different. Theoretically, the related concept of bundling has been shown to be an effective method of extracting surplus in monopoly markets. However, Foubert and Gisbrecht (2007) find that contrary to intuition, promotional bundles are far more useful at inducing switching brands than at boosting category sales. Aloysius and Deck (2008) show that sellers use bundling as a competitive weapon rather than a tool for extracting surplus. Therefore, this paper reports a series of experiments designed to explore sequential pricing in competitive markets. The results indicate the ability to set conditional sequential prices does not impact social welfare or harm consumers. It does

however shift some of the benefits between those who comparison shop and those who do not depending on the underlying relationship of the goods. The ability to price discriminate does not impact the price of the item initially offered for sale, but does impact the price of the second item depending on the underlying structure of the goods. When the goods are complements, those who are identified as not having purchased the first good receive a substantial price discount. When a buyer's values for the two goods are highly correlated, those who could be identified as having not bought the initial good received a lower price than they would have if they could not have been identified. Across all treatments, there was evidence of leader pricing behavior as a large proportion of sellers priced at very close to marginal cost, regardless of whether or not they could price discriminate. To our knowledge this is the first evidence of people following a leader pricing strategy in controlled laboratory market experiments.

A word of caution is due. This research is meant to be forward looking and as such represents a first step in identifying how technological changes may affect pricing strategies. One important area for future exploration is how buyers will react to such practices. For example, Coca-Cola developed a vending machine which would adjust price with temperature. However, the negative attention this attracted thwarted implementation. More generally, Kahneman, et al. (1986) report that people deem it "unfair to exploit shifts in demand by raising prices." (p. 728). Would buyers be less likely to visit sellers who engage in sequential pricing? How would buyers attempt to strategically shop? It is important to note that the ability to set sequential prices does not only help the seller, but it is also potentially beneficial to buyers. In the numerical example presented in the introduction, under sequential pricing with discrimination, person L is able to purchase good B , which would not be the case otherwise. It is also easy to imagine situations where the B product is one with which the buyer was unfamiliar initially. The desire to generate profits will lead sellers to make buyers aware of more potentially valuable products. Ayres (2007) lists many naturally occurring examples of collaborative filtering generating recommended products that the buyer would not have been aware of otherwise, thus increasing a buyer's overall satisfaction.

References

- Acquisti, A., Varian, H. R. (2005). Conditioning price on purchase history. *Marketing Science* 24(3), 367-381.
- Adams, W. J., Yellen, J. (1976). Commodity bundling and the burden of monopoly. *Quarterly Journal of Economics* 40, 475-498.

- Aloysius, J., Deck, C. (2008). Price bundling in competitive retail markets. Working Paper, University of Arkansas.
- Ansari, A., Essegai, S., Kohli, R. (2000). Internet recommendation systems. *Journal of Marketing Research* 37(3), 363-375.
- Ayres, I. (2007). *Super Crunchers: why thinking-by-numbers is the new way to be smart*. Bantam Dell.
- Basenko, D., Dube J-P., Gupta, S. (2003). Competitive price discrimination strategies in a vertical channel using aggregate retail data. *Management Science* 49(9), 1121-1138.
- Bodapati, A. V. (2008). Recommendation systems with purchase data. *Journal of Marketing Research* 45(1), 77-93.
- Chen, Y. (1997). Equilibrium product bundling. *The Journal of Business* 70(1), 85-103.
- Chen, Y., Narasimhan, C., Zhang, Z. J. (2001). Individual marketing with imperfect targetability. *Marketing Science* 20(1), 23-41.
- Cope, E. (2006). Bayesian strategies for dynamic pricing in e-commerce. *Naval Research Logistics* 54, 265-281.
- Cromhout, D., Hardgrave, B. C., Armstrong, D. J. (2008). RFID item-level tagging for apparel/footwear: A feasibility study. Working paper, Information Technology Research Institute, Walton College of Business, University of Arkansas.
- Davis, D., Holt, C. (1993). *Experimental Economics*. Princeton: Princeton University Press.
- De Bruyn, A., Liechty, J. C., Huizingh, E. K. R. E., Lilien, G. L. (2008). Offering online recommendations with minimum customer input through conjoint-based decision aids. *Marketing Science* 27(3), 443-460.
- Deck, C., Wilson, B. (2006). Tracking customers to price discriminate. *Economic Inquiry* 44(2): 280-295.
- Draganska, M., Jain, D. C. (2006). Consumer preferences and product-line pricing strategies: An empirical analysis. *Marketing Science* 25(2), 164-174.
- Ellison, G. (2006). Bounded Rationality in Industrial Organization. In *Advances in Economics and Econometrics: Theory and Applications*. Edited by Richard Burnell, Whitney Newey, and Torsten Persson. Ninth World Congress. New York, NY: Cambridge University Press.
- Foubert, B., Gijsbrechts, E. (2007). Shopper response to bundle promotions for packaged goods. *Journal of Marketing Research* 44(4), 647-662.
- Fudenberg, D., Villas-Boas, J. Miguel. (2006). Behavior based price discrimination and customer recognition. T.J. Hendershott, Ed. *Handbook on Economics and Information Systems*, 377-436.

- Hanson, W. A., Martin, R. K. (1990). Optimal bundle pricing. *Management Science* 36 (2), 155–174.
- Hess, J. D., Gerstner, E. (1987). Loss leader pricing and rain check policy. *Marketing Science* 6 (4), 358-374.
- Huang, Z., Zeng, D. D., Chen, C. (2007). Analyzing consumer-product graphs: Empirical findings and applications in recommender systems. *Management Science* 53(7), 1146-1164.
- Kahneman, D., Knetsch, J., Thaler, R. (1986). Fairness as a constraint on profit seeking: entitlements in the market. *American Economic Review* 76(4), 728-741.
- Maglaras, C., Meissner, J. (2006). Dynamic pricing strategies for multi-product revenue management problems. *Manufacturing and Service Operations Management* 8(2), 136-148.
- McAfee, R. P., McMillan, J., Whinston, M. D. (1989). Multiproduct monopoly, commodity bundling, and correlation of values. *Quarterly Journal of Economics* 103, 371–383.
- Montgomery, A. L., Li, S. B., Srinivasan, K., Liechty, J. C. (2004). Modeling online browsing and path analysis using clickstream data. *Marketing Science* 23(4), 579-595.
- Moon, S., Russell, G. J. (2008). Predicting product purchase from inferred customer similarity: An autologistic model approach. *Management Science* 54(1), 71-82.
- Mulhern, F. J., Leone, R. P. (1991). Implicit price bundling of retail products: A multiproduct approach to maximizing store profitability. *Journal of Marketing* 55(4), 63-76.
- Natter, M., Reutterer, T., Mild, A., Taudes, A. (2007). An assortmentwide decision-support system for dynamic pricing and promotion planning in DIY retailing. *Marketing Science* 226(4), 576-583.
- Netessine, S., Savin, S., Xiao, W. (2006). Revenue management through dynamic cross selling in e-commerce retailing. *Operations Research* 54(5), 893-913.
- Pigou, A. C. (2006). *The Economics of Welfare: Vol. 1*. New York: Cosimo.
- Rossi, P. E., McCulloch, R. E., Allenby, G. M. (1996). The value of purchase history data in target marketing. *Marketing Science* 15(4), 321-340.
- Rust, R. T., Chung, T. S. (2006). Marketing models of service and relationships. *Marketing Science* 25(6), 560-580.
- Schmalensee, R. (1984). Pricing of product bundles. *Journal of Business* 57(1), 211-230.
- US Census Bureau (2008). Quarterly retail e-commerce sales 4th quarter 2007, downloaded from <http://www.census.gov/mrts/www/data/html/07Q4.html>, on 9/11/08.
- Varian, H. R. (1980). A model of sales. *American Economic Review* 70(4), 651-659.

Venkatesh, R., Kamakura, W. (2003). Optimal bundling and pricing under a monopoly: Contrasting complements and substitutes from independently valued products. *Journal of Business* 76(2), 211-231.

Weier, M. H. (2008). Walmart gets tough on RFID with Sam's Club suppliers. *InformationWeek* 1169, p. 26.

Zhang, J., Krishnamurthi, L. (2004), Customizing promotions in online stores. *Marketing Science* 23(4), 561-578.

Table 1. Numerical Comparison of Sequential Pricing with and without Discrimination

θ	ρ	Sequential Pricing without Discrimination			Sequential Pricing with Discrimination			Distribution of Values	
		P_A	P_B	$E(\Pi)$	P_A	$P_B q_A=1$	$P_B q_A=0$		$E(\Pi)$
-0.5	0	63	50	39.45	62	7	50	39.66	$V_A \sim U[0,100]$ $V_B \sim U[0,100]$
-0.4	0	63	50	39.45	61	14	50	40.69	
-0.3	0	58	37	40.32	58	23	50	42.47	
-0.2	0	56	41	43.81	56	32	50	44.72	
-0.1	0	53	46	47.17	53	42	50	47.39	
0	0	50	50	50.50	50	50	50	50.50	
0.1	0	48	55	53.46	48	59	50	53.63	
0.2	0	46	59	56.45	46	68	50	57.11	
0.3	0	43	63	59.32	44	76	50	60.72	
0.4	0	40	68	62.23	41	84	50	64.60	
0.5	0	37	73	65.13	39	93	50	68.64	
0.6	0	35	78	67.89	38	101	50	72.62	
0.7	0	32	83	70.80	36	110	50	76.86	
0.8	0	24	116	74.05	34	117	50	81.27	
0.9	0	22	124	78.83	33	126	50	85.67	
1	0	21	131	83.93	31	133	50	90.38	
0	-1.0	50	50	50.50	43	29	58	57.71	$V_A, V_B \in [0,100] \mid V_A + V_B = 100$
0	-0.85	47	47	50.73	46	27	58	54.72	$V_A, V_B \in [0,100] \mid 75 \leq V_A + V_B \leq 125$
0	-0.75	46	46	50.91	47	30	56	53.63	$V_A, V_B \in [0,100] \mid 67 \leq V_A + V_B \leq 133$
0	-0.5	45	45	51.36	46	39	50	51.89	$V_A, V_B \in [0,100] \mid 50 \leq V_A + V_B \leq 150$
0	-0.25	48	48	50.67	48	45	50	50.75	$V_A, V_B \in [0,100] \mid 33 \leq V_A + V_B \leq 167$
0	-0.15	49	49	50.55	49	47	50	50.58	$V_A, V_B \in [0,100] \mid 25 \leq V_A + V_B \leq 175$
0	0	50	50	50.50	50	50	50	50.50	$V_A, V_B \in [0,100]$
0	0.15	49	49	50.55	49	50	47	50.58	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 75$
0	0.25	48	48	50.67	48	50	45	50.76	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 67$
0	0.5	45	45	51.36	45	50	36	52.08	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 50$
0	0.75	46	46	50.91	47	54	28	53.61	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 33$
0	0.85	47	47	50.73	49	56	25	54.52	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 25$
0	1	50	50	50.50	58	58	29	57.71	$V_A, V_B \in [0,100] \mid V_A - V_B = 0$

$V_A, V_B \in [0,100] \mid \xi$ indicates that the pair (V_A, V_B) is drawn uniformly from the subset of $[0,100] \times [0,100]$ that satisfies condition ξ .

Table 2. Numerical Comparison of Simultaneous Pricing with Pure Components and Mixed Bundling

θ	ρ	Mixed Bundling			Pure Components		Distribution of Values
		P_A, P_B	P_{AB}	Π	P_A, P_B	Π	
-0.5	0	58	58	38.87	58	38.87	$V_A \sim U[0,100]$ $V_B \sim U[0,100]$
-0.4	0	59	61	39.96	58	38.87	
-0.3	0	61	70	42.68	58	38.87	
-0.2	0	63	75	46.26	55	39.02	
-0.1	0	64	80	50.51	49	44.06	
0	0	67	87	55.27	50	50.50	
0.1	0	80	88	60.05	52	56.40	
0.2	0	80	96	65.51	54	62.54	
0.3	0	83	109	70.85	57	68.43	
0.4	0	80	112	76.43	60	74.39	
0.5	0	82	123	81.97	63	80.50	
0.6	0	82	131	87.30	67	86.16	
0.7	0	83	141	92.82	67	92.03	
0.8	0	80	144	98.26	72	98.09	
0.9	0	80	152	103.72	76	103.66	
1	0	82	164	109.29	82	109.29	
0	-1.0	100	100	100.00	50	50.50	$V_A, V_B \in [0,100] \mid V_A + V_B = 100$
0	-0.85	75	75	75.00	47	50.73	$V_A, V_B \in [0,100] \mid 75 \leq V_A + V_B \leq 125$
0	-0.75	67	73	67.64	46	50.91	$V_A, V_B \in [0,100] \mid 67 \leq V_A + V_B \leq 133$
0	-0.5	66	79	59.87	45	51.36	$V_A, V_B \in [0,100] \mid 50 \leq V_A + V_B \leq 150$
0	-0.25	66	83	56.83	48	50.67	$V_A, V_B \in [0,100] \mid 33 \leq V_A + V_B \leq 167$
0	-0.15	67	85	56.10	49	50.55	$V_A, V_B \in [0,100] \mid 25 \leq V_A + V_B \leq 175$
0	0	67	87	55.27	50	50.50	$V_A, V_B \in [0,100]$
0	0.15	62	87	54.65	49	50.55	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 75$
0	0.25	59	88	54.22	48	50.67	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 67$
0	0.5	52	88	52.70	45	51.36	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 50$
0	0.75	51	92	50.91	46	50.91	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 33$
0	0.85	51	95	50.92	47	50.73	$V_A, V_B \in [0,100] \mid V_A - V_B \leq 25$
0	1	50	100	50.50	50	50.50	$V_A, V_B \in [0,100] \mid V_A - V_B = 0$

$V_A, V_B \in [0,100] \mid \xi$ indicates that the pair (V_A, V_B) is drawn uniformly from the subset of $[0,100] \times [0,100]$ that satisfies condition ξ .

Table 3. Linear Mixed Effects Estimation for Average Prices in “Independent Values” Treatment

Model: $P_{Aijt} = \alpha_0 + \alpha_1 PD_j + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD	
Estimate	18.0	3.6	
t-statistic	6.23	0.88	
p-value	<0.01	0.41	

Model: $P_{Bijt} = \beta_0 + \beta_1 PD_j \times BoughtA + \beta_2 PD_j \times (1-BoughtA) + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	33.2	6.3	6.1
t-statistic	8.28	1.11	1.09
p-value	<0.01	0.27	0.28

The unit of observation is at the individual level each period. Each session and period is modeled as having a random effect while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Table 4. Linear Mixed Effects Estimation for Minimum Price of Good A and Low Price Seller’s Good Price in “Independent Values” Treatment

Model: $MinPA_{jt} = \gamma_0 + \gamma_1 PD_j + \varepsilon_j + \varepsilon_{jt}$		
	Intercept	PD
Estimate	6.1	5.6
t-statistic	1.91	1.24
p-value	0.06	0.22

Model: $P_B MinPA_{jt} = \omega_0 + \omega_1 PD_j \times BoughtA + \omega_2 PD_j \times (1-BoughtA) + \varepsilon_j + \varepsilon_{jt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	32.0	5.6	4.8
t-statistic	8.05	0.99	0.86
p-value	<0.01	0.32	0.39

The unit of observation is at the market level each period. Each session and period is modeled as having a random effect, while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Table 5. Linear Mixed Effects Estimation for Average Prices in “Complements” Treatment

Model: $P_{Aijt} = \alpha_0 + \alpha_1 PD_j + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD	
Estimate	13.0	3.4	
t-statistic	5.33	0.98	
p-value	<0.01	0.37	

Model: $P_{Bijt} = \beta_0 + \beta_1 PD_j \times BoughtA + \beta_2 PD_j \times (1-BoughtA) + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	51.8	-1.0	-17.7
t-statistic	10.80	-0.14	-2.60
p-value	<0.01	0.89	<0.01

The unit of observation is at the individual level each period. Each session and period is modeled as having a random effect while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Table 6. Linear Mixed Effects Estimation for Minimum Price of Good A and Low Price Seller’s Good Price in “Complements” Treatment

Model: $MinP_{Ajt} = \gamma_0 + \gamma_1 PD_j + \varepsilon_j + \varepsilon_{jt}$		
	Intercept	PD
Estimate	5.4	3.7
t-statistic	2.29	1.11
p-value	0.02	0.27

Model: $P_B MinP_{Ajt} = \omega_0 + \omega_1 PD_j \times BoughtA + \omega_2 PD_j \times (1-BoughtA) + \varepsilon_j + \varepsilon_{jt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	53.4	-0.5	-19.6
t-statistic	8.80	-0.05	-2.28
p-value	<0.01	0.96	0.02

The unit of observation is at the market level each period. Each session and period is modeled as having a random effect while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Table 7. Linear Mixed Effects Estimation for Average Prices in “Correlated Values” Treatment

Model: $P_{Aijt} = \alpha_0 + \alpha_1 PD_j + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD	
Estimate	19.7	-0.3	
t-statistic	4.32	-0.05	
p-value	<0.01	0.96	

Model: $P_{Bijt} = \beta_0 + \beta_1 PD_j \times BoughtA + \beta_2 PD_j \times (1 - BoughtA) + \varepsilon_i + \varepsilon_j + \varepsilon_{ijt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	39.5	-6.4	-7.7
t-statistic	10.15	-1.17	-1.40
p-value	<0.01	0.24	0.16

The unit of observation is at the individual level each period. Each session and period is modeled as having a random effect while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Table 8. Linear Mixed Effects Estimation for Minimum Price of Good A and Low Price Seller’s Good Price in “Correlated Values” Treatment

Model: $MinPA_{jt} = \gamma_0 + \gamma_1 PD_j + \varepsilon_i + \varepsilon_{jt}$		
	Intercept	PD
Estimate	11.1	-2.5
t-statistic	3.19	-0.50
p-value	<0.01	0.61

Model: $P_B MinPA_{jt} = \omega_0 + \omega_1 PD_j \times BoughtA + \omega_2 PD_j \times (1 - BoughtA) + \varepsilon_i + \varepsilon_{jt}$			
	Intercept	PD×BoughtA	PD×(1-BoughtA)
Estimate	38.4	-5.1	-8.4
t-statistic	9.32	-0.88	-1.44
p-value	<0.01	0.38	0.15

The unit of observation is at the market level each period. Each session and period is modeled as having a random effect while the treatments are modeled as a fixed effect. PD is a dummy variable that equals 1 if the seller was operating in a market in which price discrimination was possible and 0 otherwise. BoughtA is a dummy variable that equaled 1 if the price was targeted to people who had purchased good A and 0 otherwise.

Figure 1. Onscreen Pricing Tool – Correlated Values with Good B Price Discrimination

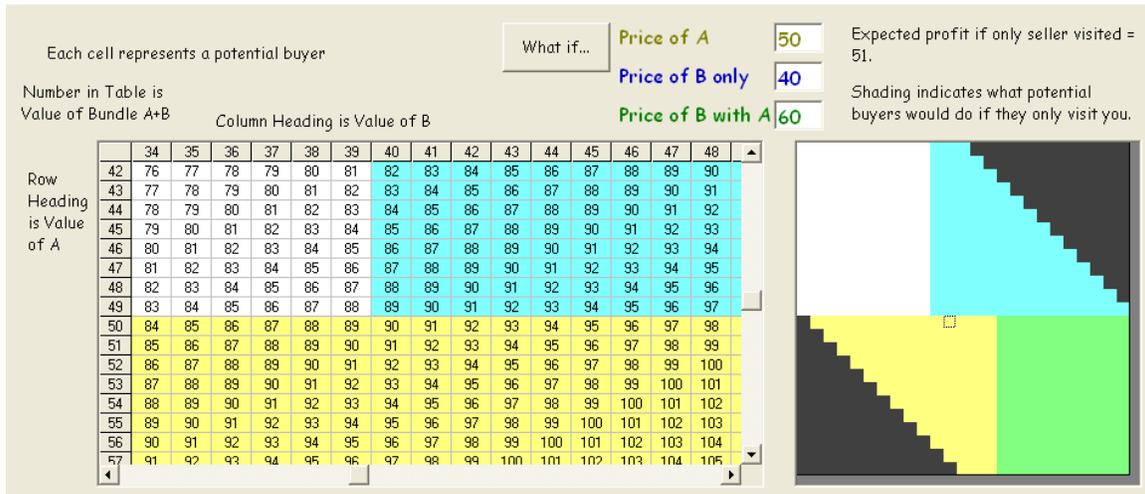


Figure 2. Frequency of Observed Prices in Baseline Treatments

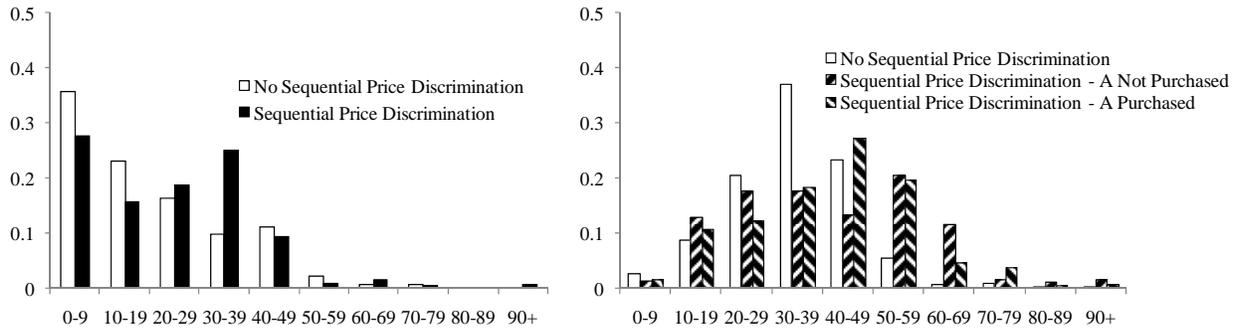


Figure 3. Welfare Implications of Sequential Pricing (Independent Values) by Session

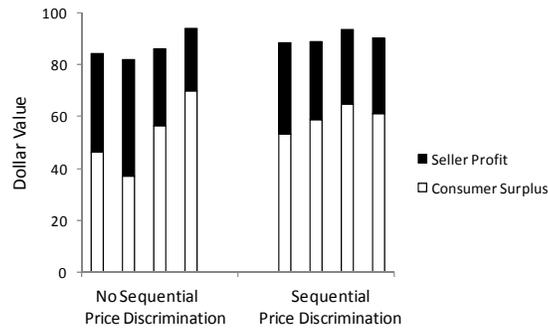


Figure 4. Frequency of Observed Prices in Complementary Goods Treatments

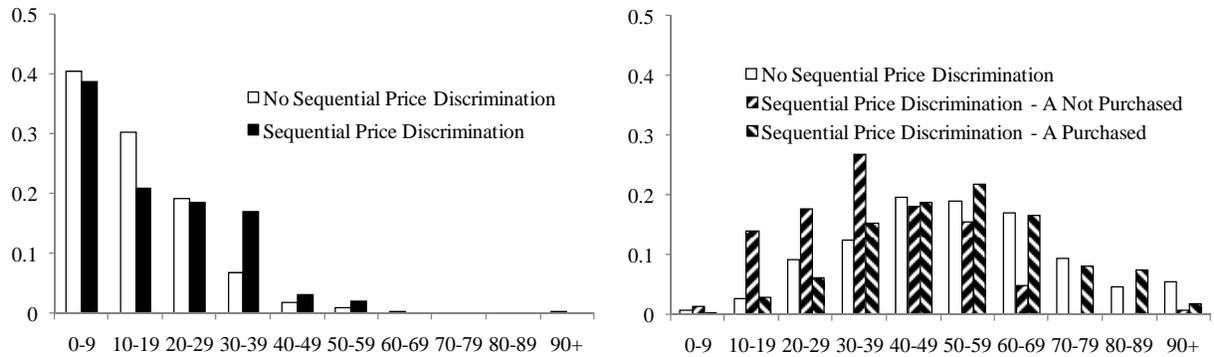


Figure 5. Welfare Implications of Sequential Pricing (Complementary Goods) by Session

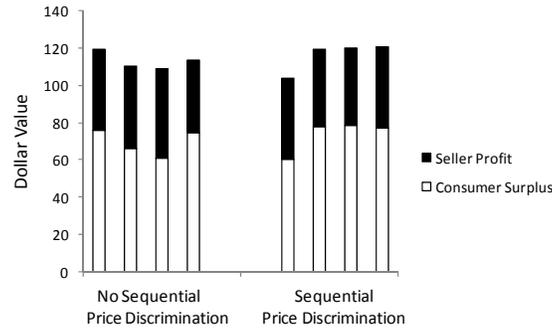


Figure 6. Frequency of Observed Prices in Correlated Values Treatments

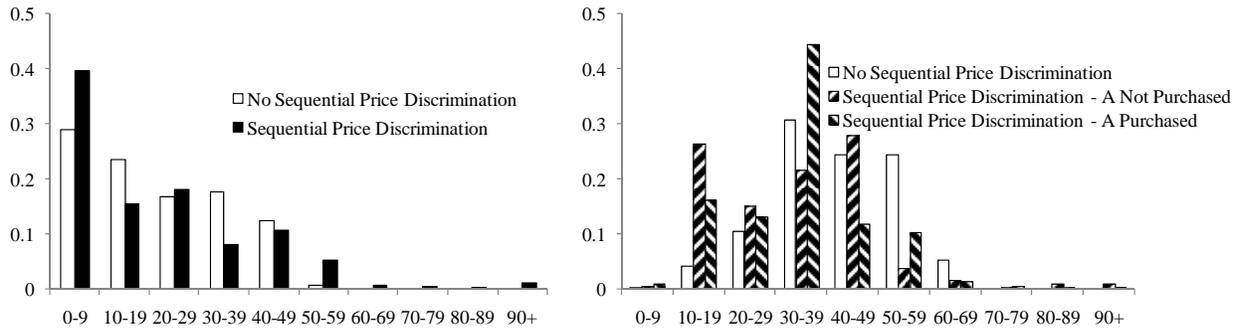


Figure 7. Welfare Implications of Sequential Pricing (Correlated Values) by Session

