Dynamic Special Interest Politics with Relational Contracts

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Abstract

We develop a dynamic repeated election model in which office-seeking political parties repeatedly compete in a winner-take-all campaign-spending game with an incumbency advantage. Special interests seek influence over the actions of governmental agencies and may make campaign contributions to the political parties. Slack-maximizing agency bureaucrats seek funding which is chosen by the in-party. All political actors are long-lived and may only form agreements based on non-binding relational contracts. We characterize a relevant refinement of perfect public equilibrium. We find that political parties and special interests form stable alliances that generate influence over agency decisions—agencies that acquiesce to the demands of political allies are rewarded in the agency funding process and shirking ones are punished.

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1 Introduction

Dynamic models of repeated elections typically involve the electorate setting a standard for performance and reelection of the politician hinges on whether or not the politician’s outcome meets the standard (Ashworth, Bueno de Mesquita, and Friedenberg 2010). These standards may be set in order to create incentives as in the dynamic political agency literature following Barro (1973) and Ferejohn (1986) or to select good types as in the repeated citizen-candidate model (Duggan 2000).1 In this paper, we develop a dynamic repeated elections model in which political parties repeatedly compete in a campaign spending game with an incumbency advantage, along the lines of the one-shot contest-theoretic campaign-spending model (Erikson and Palfrey 2000 and Meirowitz 2008),2 and examine a formulation of special interest politics that combines special interests seeking subgovernment influence with office-seeking political parties and slack-maximizing bureaucrats in an environment where all political actors are long-lived and agreements are based on non-binding relational contracts.

In the “capture model” of subgovernment special interest influence (Bernstein 1955; Lowi 1969; Stigler 1971; Peltzman 1976), the goals of special interests are best served within the inner reaches of the bureaucracy. All influence activity involves a direct transaction — influence over bureaucratic actions (e.g., the design of regulations) in return for side-payments or other benefits such as future employment — between agency bureaucrats and special interests. As formulated in models such as Tirole (1986) and Laffont and Tirole (1993), the assumption is that unless legislative action of some sort prevents it, the natural inclination is for agencies to fall under the influence of special interests. The role of the benevolent government is to maximize society’s welfare by limiting agency capture through the design of appropriate incentives for agency bureaucrats. In contrast to the conventional wisdom that legislatures seek to prevent agency capture by special interests, we explore the converse. That is, legislatures work to promote just such influence.

In this paper we examine a theory of subgovernment special interest politics that centers on the transactions among interest groups, political parties, and bureaucrats. Our approach

1See also Bernhardt, Dubey, and Hughson (2004), Banks and Duggan (2008), Bernhardt, Campuzano, Squintani, Câmera (2009), and Bernhardt, Câmera, and Squintani (2011). Most closely related is Snyder and Ting (2008) who examine a dynamic repeated elections model with special interest influence. Our model differs from theirs in a number of ways: our special interest expenditures are in the form of campaign contributions rather than side-payments, our agreements are non-binding rather than enforceable contracts, our special interest influence is over agency actions rather than over a spatial policy choice, to name a few.

2Also related are Herrera, Levine, and Martinelli (2008), Ashworth and Bueno de Mesquita (2009), Zakharov (2009).
combines the campaign spending model, special interest politics with campaign contributions, the economic model of bureaucracy (Niskanen 1971, Migué and Bélanger 1974, Breton and Wintrobe 1975), and relational contracting with hidden information (Levin 2003).\(^3\) Elections are winner-take-all; slack-maximizing agency bureaucrats want funding which is chosen by the in-party and take actions that affect special interests; special interests want influence over agency actions and may make campaign contributions; and office-seeking political parties want campaign contributions and choose agency funding levels when in office. In this context, we find that under a relevant refinement of perfect public equilibrium, which includes sequentially efficient agreements within each party-special interest alliance, special interests have incentive to enter into an agreement with a political party who provides programmatic influence over the in-house bureaucratic decisions that the special interest values most. Political parties receive campaign contributions from special interests, and agencies that acquiesce to the demands of political allies are rewarded in the agency funding process, while shirking ones are punished.

As our model features special interests that provide political parties with campaign contributions and in return political parties promise to provide special interests with influence over the policies that matter most to them, in the event that the political party wins, our approach builds upon static models of special interest politics such as Austen-Smith (1987), Baron (1989, 1994), and Grossman and Helpman (1996, 2001). Campaign contributions indirectly buy votes (via advertising, the distribution of campaign materials, valence accumulation, etc.), and political parties compete for vote share by spending the campaign contributions that are raised via the agreements with special interests. Motivated by questions that have been raised concerning commitment issues in political agreements (Morton and Cameron 1992; McCarty and Rothenberg 1996) and recent empirical and experimental work emphasizing the importance of long-term relationships in political influence activities (e.g. Bertrand, Bombardini, and Trebbi 2011 and Großer, Reuben, and Tymula 2010), we examine a dynamic repeated elections model in which agreements between political parties, special interests and agencies are not enforceable contracts but, rather, informal relational agreements that are supported by the repeated interactions of the relevant actors.\(^4\)

In the campaign spending model, each party allocates sunk resources to the campaign and, subject to distortions arising from the incumbency advantage, the party with the higher effective campaign expenditure wins the election. Our dynamic campaign spending game

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\(^3\)For more on the relational contracting literature see the recent survey: Malcomson (2010).

\(^4\)See Fox (2006) who takes a similar approach in a two-stage model.
with an incumbency advantage and endogenously determined incumbency status is closely related to the theoretical literature on dynamic contests.\textsuperscript{5} Konrad (2010) delineates this literature into three branches: (i) the racing contest and the tug of war,\textsuperscript{6} (ii) the elimination contest,\textsuperscript{7} and (iii) the iterated incumbency fight.\textsuperscript{8} Our model is most closely related to the iterated incumbency fight in which challengers repeatedly attack the incumbent until the incumbent loses, at which point the game ends for the incumbent, and the winning challenger becomes the incumbent. In our model, as in Mehlum and Moene (2007), Kovenock and Roberson (2009a), and Sela (2012), there are only two players who repeatedly compete for incumbency status which is endogenously determined by the outcome of each period’s contest and in which incumbency provides a strategic advantage. Kovenock and Roberson (2009a) and Sela (2012) examine a complete information two-stage resource allocation game\textsuperscript{9} and Mehlum and Moene (2007) examine an infinitely repeated complete information Tullock contest. In this paper we provide a dynamic infinite-horizon extension of the incomplete information all-pay auction (e.g., Weber 1985, Hillman and Riley 1989, Amann and Leininger 1996, Moldovanu and Sela 2001, Morath and Münster 2009, Parreiras and Rubinchik 2009 among others) that features an incumbency advantage and endogenous time-varying incumbency status. In addition, our model provides a framework for examining dynamic contests that include contestants and supporting players, where agreements between contestants and supporting players are supported by non-binding relational contracts.

Our model of subgovernment influence also builds upon the economic model of bureaucracy as modeled in Banks (1989), and Banks and Weingast (1992). In particular, we assume that: (i) agency bureaucrats are experts in their field and have informational advantages relative to their political overseers and (ii) agency bureaucrats are slack-maximizing and can exploit their informational advantage in order to maximize their expected budget surplus. In contrast to the “capture model,” this model emphasizes the role of politicians in providing incentives for agency officials to take actions which generate value for a particular constituency, but, in the previous literature, politicians are assumed to be benevolent from,

\textsuperscript{5}For a survey of dynamic contests see Konrad (2009a).
\textsuperscript{7}See for example Rosen (1986), Gradstein and Konrad (1999), and Fu and Lu (2012).
\textsuperscript{9}In Kovenock and Roberson (2009a) there are a finite number of all-pay auctions at each date and, within each auction, the margin of victory in the first-period generates an advantage, for the first-period’s winner, in the second period. In contrast, Sela (2012) examines a single all-pay auction at each date, and the players may have increasing or decreasing returns from winning more than one auction.
the point of view of the constituency, and there are no electoral considerations. Our approach modifies this framework in that special interests and parties form agreements involving the exchange of a campaign contribution to the party in return for the party using the budgetary process to provide incentives for agencies to respond to the interests of in-party political allies, in the event that the party wins the election. As agreements are supported by the repeated interactions of the relevant actors and the in-party’s contracting problem entails hidden information, our model builds upon the relational contracting approach developed in Levin (2003). To summarize, our dynamic repeated elections model with subgovernment special interest politics shows how campaign contributions map into political influence in an environment where contracts are not perfectly enforceable, special interests seek influence over agency actions, and agencies are slack-maximizing.

The remainder of the paper proceeds as follows. Section 2 presents the model. In section 3 we characterize the entire set of perfect public equilibrium of the dynamic special interest politics game, and then examine a relevant refinement of perfect public equilibrium. Lastly, section 4 concludes.

2 The Model

We first provide a brief overview of the three-stage extensive-form game that is played in each period of the infinite-horizon model and then examine each of the three stages in detail. To keep the focus on the new issues, we make a number of stylistic assumptions, and following the overview of the model, we examine the assumptions in each of the three stages in detail.

2.1 Overview

The seven long-lived players include a representative voter and for each $i \in \{a, b\}$, a political party $P_i$, a special interest $S_i$, and a governmental agency $A_i$. Each agency $A_i$ is involved in a specific governmental function that affects special interest $S_i$. For simplicity, we assume that each $S_i$ has a single supplier of influence, namely $P_i$. To summarize, $S_i$ and $A_i$ are linked in that the actions of $A_i$ affect $S_i$, and $P_i$ and $S_i$ are linked.

Time is discrete and there is an infinite sequence of identical periods each consisting of three stages: a campaign stage, an election stage, and a governing stage. In the campaign stage, each special interest $i$ privately observes its stochastic valuation parameter $v_{i,t}$.

\footnote{The assumption of long-lived political parties can be relaxed. See section 3.3 for more details.}


which is distributed according to a common time-invariant distribution $F$, which is continuously differentiable and strictly increasing over $(0, 1)$, with $F(0) = 0$. Then, each $S_i$ simultaneously makes a campaign contribution $x_{i,t}$ to its party. In the election stage, the parties play a campaign spending game with expenditures $x_t$, and the winner of the election, $P_I$, is determined according to an electoral contest with incumbency advantage which is described below. For simplicity, we assume that only the in-party and the in-party agency move in the governing stage. The governing stage includes: (i) the in-party agency $A_I$ privately observes its stochastic cost parameter $\theta_{I,t} \in \Theta \subset \mathbb{R}_+^+$ which is distributed according to a common time-invariant distribution $G$, which is continuously differentiable, strictly increasing over $\Theta \equiv [\bar{\theta}, \bar{\theta}]$ and concave\(^\dagger\) with $G(\bar{\theta}) = 0$, and (ii) the in-party agency $A_I$ chooses a managerial effort level $e_{I,t} \in \mathbb{R}_+$, and (iii) $P_I$, makes a funding decision $y_{I,t} \in \mathbb{R}_+$ and payoffs are realized.

A campaign contribution of $x_{i,t}$ imposes a cost of $x_{i,t}$ on special interest $i$. Each special interest $i$’s total benefit depends on agency effort, $e_{i,t}$, and the private valuation parameter, $v_{i,t}$, and is given by $v_{i,t}\pi_S(e_{i,t})$. We assume that the parties are office-seeking, and each party has a fixed benefit of $\pi_P$ for winning an election. Allocating $y_{i,t}$ resources to agency $A_i$ imposes a cost of $y_{i,t}$ on the in-party. The bureaucrat’s benefit from receiving a budget of $y_{i,t}$ is $y_{i,t}$. Agency effort $e_{i,t}$ imposes a cost of $\theta_{i,t}c_A(e_{i,t})$ on the agency.

Let $\iota_{i,t+1} \in \{0, 1\}$ be an incumbency indicator function, that takes a value of 1 if $P_i$ wins the time $t$ election and will be the in-party at the beginning of time $t + 1$. For $i \in \{a, b\}$, the end of period $t$ payoffs for special interest $i$, party $i$, and agency $i$ are given, respectively, by

$$u_{S_i}(v_{i,t}, x_{i,t}, \iota_{i,t+1}, e_{i,t}) = \iota_{i,t+1}(v_{i,t}\pi_S(e_{i,t})) - x_{i,t}$$

$$u_{P_i}(\iota_{i,t+1}, y_{i,t}) = \iota_{i,t+1}(\pi_P - y_{i,t})$$

$$u_{A_i}(\iota_{i,t+1}, \theta_{i,t}, e_{i,t}, y_{i,t}) = \iota_{i,t+1}(y_{i,t} - \theta_{i,t}c_A(e_{i,t}))$$

The quasi-linearity of the stage game payoffs for the parties and agencies allows us to use standard techniques to deal with the agencies’ hidden information. The linearity of the special interest’s costs is not necessary for our results. We make the following assumptions regarding payoff functions.

**Assumption 1.** (i) $c_A(e)$ is continuously differentiable, strictly increasing, strictly convex, with $c_A(0) = 0$ and $c_A'(0) = 0$ (ii) $\pi_S(0) = 0$, $\pi_S(e)$ is continuously differentiable, strictly

\(^\dagger\)This strong regularity assumption is standard in the literature on contracts with hidden information but can be relaxed. See Levin (2003) for more details.
increasing and strictly concave, and (iii) there exists an $\varepsilon < \infty$ such that $\pi_S(e) - \theta c_A(e) < 0$ for all $\theta \in \Theta$.

The parties, agencies and special interests have a common discount factor of $\delta$. For each $j \in \{A, P, S\}$ and $i \in \{a, b\}$ player $j_i$’s payoff for the entire game is the sum of the discounted payoffs in each of the periods. The extensive-form stage game is summarized in Figure 1 below.

![Figure 1: Extensive-Form Stage-Game Schematic](image-url)

* only the in-party agency chooses actions and the in-party chooses funding

Figure 1: Extensive-Form Stage-Game Schematic
2.2 Campaign Stage

For each special interest $i$, we assume that the total benefit derived from agency actions, depends on a private time-varying valuation parameter, $v_{i,t} \in [0, 1]$ for special interest $i$ in period $t$, and agency effort, $e_{i,t}$, with the total benefit given by $v_{i,t} \pi_S(e_{i,t})$. This formulation captures the fact that each special interest prefers more to less agency effort, but that the total benefit from a given level of agency effort may vary over time. In the campaign stage, each special interest $i$ privately observes $v_{i,t}$ and then makes a publicly observable campaign contribution $x_{i,t} \in \mathbb{R}_+ \cup \emptyset$ to its party, where a campaign contribution of $\emptyset$ denotes that a special interest is making an observable break from the relationship with its party in a given period. The issue here is that a campaign contribution of zero may arise because of a low valuation $v_{i,t}$ or because the special interest wants to make public that it does not support its party. As these two events have different meanings, we allow the special interest to signal which of these events is occurring, by 0 and $\emptyset$ respectively. Campaign contributions have no use other than the campaign and may not be stored for future elections.

2.3 Election Stage

In each period ($t \geq 1$), the election stage is modeled as a dynamic incomplete-information extension of the static complete-information electoral contest examined in Meirowitz (2008). Thus, electoral competition is primarily over valence accumulation, and we assume that there is an incumbency advantage that is described as follows. Letting $I$ denote the incumbent or in-party and $O$ the out-party, if the in-party raises $x_{I,t} \in \mathbb{R}_+$ in period $t$ campaign contributions, and the out-party raises $x_{O,t} \in \mathbb{R}_+$, then with with probability $\mu \in (0, 1]$ the representative voter reelects the in-party if $x_{I,t} > x_{O,t}$ and with probability $(1 - \mu)$ the in-party wins the time $t$ election regardless of the campaign contribution levels $x_{I,t}, x_{O,t} \in \mathbb{R}_+$. In the event of a tie ($x_{I,t} = x_{O,t}$ where $x_{I,t}, x_{O,t} \in \mathbb{R}_+$), any tie-breaking rule which assigns a strictly positive probability of winning to each party is sufficient.

To emphasize the dependence of political parties on special interests we assume that the in-party cannot win reelection without being allied with its special interest. There are several tie-breaking rules which would be sufficient for our purposes. One such rule is described as follows, in the case that only one party is allied with a special interest ($\exists!\ i \in \{a, b\} \ s.t.\ x_i \neq \emptyset$), the party with the special interest ($i$) wins the election with certainty. If both special interests break off their relationships with the parties ($x_i = \emptyset$, $i = a, b$), then the out-party is assumed to win the election.
Our game features an incumbency advantage which is modeled as the stochastic persistence \((1 - \mu)\) of the in-party. Handicapping in contests originates with Lein (1990) and is frequently used in the literature on unfair contests (see for instance Clark and Riis 2000; Konrad 2002; Kovenock and Roberson 2008, 2009b; Meirowitz 2008; Polborn 2006; Sahuguet and Persico 2006, Feess, et. al. 2008, and Kirkegaard 2010). However, handicapping typically involves an affine transformation of one or both players’ bids. Our stochastic formulation of the incumbency advantage differs in that it generates a smooth incumbency persistence effect that provides a tractable way to examine handicapping in an infinite-horizon model with endogenous time-varying incumbency status.

As the election stage is assumed to involve only valence competition, agency actions may affect the representative voter but the representative voter does not attribute those actions to the in-power party. This benchmark emphasizes that the “independence” of governmental agencies results in a less than transparent government and that political parties may use the non-transparency of the machinery of government to their advantage.\footnote{Alternatively, this assumption is consistent with agency actions providing a private non-rivalrous benefit, which does not impose a cost on others, to a special interest.}

That is, agencies may help political parties achieve the difficult and often conflicting objectives of claiming credit for outcomes political allies favor, while shifting the blame to the faceless bureaucracy for costs incurred by the electorate.

### 2.4 Governing Stage

As in the economic model of bureaucracy (Niskanen 1971, Migué and Bélanger 1974, Breton and Wintrobe 1975 and as modeled in Banks 1989, Banks and Weingast 1992), the agency provides a valuable service, but the agency has hidden information regarding the cost of providing the service. We assume that factors such as task difficulty or the opportunity cost of time, represented by the parameter \(\theta_{i,t}\) for agency \(i\) in period \(t\), may vary over time. There are two publicly observably governing-stage actions: the in-party agency effort \(e \in \mathbb{R}^+\), and the in-party funding decision \(y \in \mathbb{R}^+\).

### 3 Equilibrium

There are typically a plethora of equilibria in this type of dynamic game, and, in applications, it is common to focus on a refinement such as stationary strategies or efficient equilibria. In our case stationary strategies are too restrictive, in that they eliminate the
possibility of relational agreements, and the set of equilibria which are efficient across all seven players is not appropriate either, in that this would eliminate (socially wasteful) campaign contributions. Here, we focus on a refinement of perfect public equilibrium that is motivated by what Williamson considered the market transaction in the one-shot hold-up game and Che and Hausch (1999) term the Williamson game. In the one-shot Williamson game, the buyer and seller do not write explicit contracts at date zero but make independent relation-specific investments and then ex post efficiently bargain over the division of the joint surplus. Similarly, we will focus on the set of perfect public equilibria in which each special interest independently makes a campaign contribution given that the relational agreement between each agency-party-special interest triple results in efficient governing stage actions. This refinement’s combination of independent campaign contributions and sequentially efficient relational agreements provides a simple, transparent environment in which both special interests’ campaign-stage incentive-compatibility constraints and the in-party agency’s governing-stage incentive-compatibility constraints are all satisfied.

From the special interests’ stage game payoffs, $u_{i,t}$, we see that the special interests are playing a repeated private-values all-pay auction. The fact that tacit collusion in repeated private-values auctions can be sustained even if public monitoring is limited to the history of the identity of the winner (Skrzypacz and Hopenhayn 2004) creates an issue in our environment. As campaign-stage collusion between the two agency-party-special interest triples is not a relevant outcome in our setting, we restrict our focus to perfect public equilibria which feature non-collusive stationary electoral competition with relational agreements which are sequentially efficient, in the sense that the governing stage actions of the in-party and the in-party agency are Pareto efficient. We henceforth refer to this refinement as the Williamson game refinement. Note that in this refinement equilibria are stationary. Following Levin (2003), it is straightforward to show that if an optimal relational contract between the in-party and the in-party agency exists, then there exists a stationary contract that is optimal. The Williamson game refinement simplifies the analysis by focusing on stationary relational contracts and requiring that campaign contributions are stationary and non-collusive.

We consider equilibria in which each player conditions his strategy on past public information. Such strategies are called public strategies and such equilibria are called perfect public equilibria (henceforth, denoted PPE). We begin by completely characterizing the set of PPE and then examine the Williamson game refinement. Recall that $i_t \in \{0, 1\}$ is an

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13Pareto efficiency is more natural in our setting than maximizing the joint surplus, but the results are similar under either assumption.
incumbency indicator function, that takes a value of 1 if \( P_i \) won the time \( t - 1 \) election and is the time \( t \) in-party, and let \( \iota_t = (\iota_{a,t}, \iota_{b,t}) \). Define \( h_t = \{ t', x_{t-1}, y_{t-1} \} \) as the history of public information at the beginning of time \( t \), where without loss of generality \( y_{i,t} = e_{i,t} = 0 \) if \( \iota_{i,t} = 0 \). Let \( \mathcal{H} \) denote the set of all possible public histories. We denote information sets as follows. The information set of special interest \( i \) at the beginning of the campaign stage is \( h_{S_i,t} = \{ h_t, \iota_{i,t} \} \). The information set of agency \( i \) at the beginning of the governing stage is \( h_{A_i,t} = \{ h_t, x_t, \iota_{t+1}, \theta_{i,t} \} \). The information set of party \( i \) at the end of the governing stage is \( h_{P_i,t} = \{ h_t, x_t, \iota_{t+1}, e_t \} \). A strategy profile \( \sigma \) of the game is given by

\[
\sigma = \{ \{ x_{i,t}(h_{S_i,t}), e_{i,t}(h_{A_i,t}), y_{i,t}(h_{P_i,t}) \} \}_{i \in \{a,b\}} \}
\]

The parties, agencies and special interests have a common discount factor of \( \delta \). We now examine the continuation payoffs for the player(s) who moves at each of the information sets. In a PPE, the equilibrium continuation payoffs of the players depend on the common beliefs regarding the probability, in the time \( t \) governing stage, that the time \( t \) in-party wins reelection at time \( t + 1 \). Let \( p_{i,t} \) denote the common time \( t \) governing stage beliefs that the current in-party, \( i \), wins reelection at time \( t + 1 \).

Given a continuation strategy \( \sigma|_{h_t} \), define special interest \( i \)'s equilibrium continuation value in the period \( t \) campaign stage as

\[
U^C_{S_i}(\sigma|_{h_{S_i,t}}) = \text{Prob}(i \text{ wins}|x_{i,t}, \iota_t) \left[ v_{i,t} E(\pi_{S_i}(e_{i,t}(h_{A_i,t}))) + \delta E(U^C_{S_i}(\sigma|_{h_{S_i,t+1}})|_{i,t+1} = 1) \right] + (1 - \text{Prob}(i \text{ wins}|x_{i,t}, \iota_t)) \delta E(U^C_{S_i}(\sigma|_{h_{S_i,t+1}})|_{i,t+1} = 0) - x_{i,t}(h_{S_i,t})
\]

where \( \text{Prob}(i \text{ wins}|x_{i,t}, \iota_t) \) is the probability that party \( i \) wins the time \( t \) election given campaign expenditures \( x_{i,t} \) and the identity \( \iota_t \) of the in-party at the beginning of period \( t \).

Agency \( i \)'s in-party equilibrium continuation value in the governing stage is given by

\[
U^I_{A_i}(\sigma|_{h_{A_i,t}}) = y_{i,t}(h_{P_i,t}) - \theta_{i,t} c_A(e_{i,t}(h_{A_i,t})) + \delta \left[ p_{i,t} E(U^I_{A_i}(\sigma|_{h_{A_i,t+1}})) + (1 - p_{i,t}) E(U^O_{A_i}(\sigma|_{h_{A_i,t+1}})) \right].
\]

Finally, party \( i \)'s in-party equilibrium continuation value in the governing stage is given by

\[
U^I_{P_i}(\sigma|_{h_{P_i,t}}) = \pi_P - y_{i,t}(h_{P_i,t}) + \delta \left[ p_{i,t} E(U^I_{P_i}(\sigma|_{h_{P_i,t+1}})) + (1 - p_{i,t}) E(U^O_{P_i}(\sigma|_{h_{P_i,t+1}})) \right].
\]

Party \( i \)'s out-party equilibrium continuation payoff, \( U^O_{P_i} \), and agency \( i \)'s out-party equilibrium
continuation payoff, \( U_{A_i}^O \), follow directly.

**Definition 1.** A strategy profile \( \sigma \) is a PPE of the dynamic special interest game if and only if

1. in the campaign stage for each \( S_i, i \in \{a, b\} \), the continuation strategy \( \sigma_{S_i|h_{S_i,t}} \) that \( S_i \) chooses is a feasible strategy and a best response to \( E(\sigma_{-S_i|h_{-S_i,t}}) \) for each possible public history \( h_t \) and state of the valuation parameter \( v_{i,t} \).

2. in the governing stage for each \( j \in \{A, P\} \) and \( i \in \{a, b\} \), the continuation strategy \( \sigma_{j|i|h_{ji,t}} \) that \( j \) chooses, at his information set in the time \( t \) governing stage, is a feasible strategy and a best response to \( E(\sigma_{-j|i|h_{-ji,t}}) \), for each possible history \( h_{ji,t} \).

In the dynamic game examined here, the special interests first provide campaign resources in the campaign stage, and then in the governing stage the in-party agency and the in-party choose an action. In a PPE of this game each player chooses, at each of the information sets at which he moves, a best response given the public history of actions and any private parameters known by the player.

Proposition 1 characterizes the set of PPE.

**Proposition 1.** A strategy profile \( \sigma \) is a PPE of the dynamic special interest politics game, if and only if for each \( i \) and all possible \( h_t \) and \( v_{i,t} \),

\[
U_{S_i}^C(\sigma_{|h_{S_i,t}}) \geq 0 \tag{2}
\]

\[
U_{S_i}^C(\sigma_{|h_{t},v_{i,t}}) \geq \max_{v_{i,t}' \in [0,1]} \{U_{S_i}^C(\sigma_{|h_{t},v_{i,t}'})\} \tag{3}
\]

and for each \( i \) and all \( h_{P_i,t} \) and \( h_{A_i,t} \), such that \( i_{t+1} = 1 \),

\[
U_{A_i}^I(\sigma_{|h_{A_i,t}}) \geq 0 \tag{4}
\]

\[
U_{A_i}^I(\sigma_{|h_{t,x_{t+1},\theta_{t},\theta_{t+1}}}) \geq \max_{\theta_{t}', \theta_{t+1}' \in \Theta} \{U_{A_i}^I(\sigma_{|h_{t,x_{t+1},\theta_{t}',\theta_{t+1}'})\} \tag{5}
\]

\[
U_{P_p}^I(\sigma_{|h_{P_i,t}}) \geq \pi_p \tag{6}
\]

The proof of Proposition 1 is provided in the Appendix. Given that the incentive compatibility constraints for the special interests and the in-party agency are satisfied, the in-party is the only player that could possibly gain from deviating. However, there exists a playerspecific punishment that can be used to punish observable deviations from any strategy
profile that provides continuation payoffs satisfying the conditions specified in Proposition 1, and each player can ensure himself of the minimal continuation payoffs specified in Proposition 1. Let $\Gamma$ denote the set of PPE in the dynamic special interest politics game. We now examine the subset of PPE which are stationary and non-collusive in the campaign stage and feature sequentially efficient governing stage agreements.

The Williamson game PPE seems like a natural refinement in our context. Each special interest has a private benefit from its agency’s actions and the in-party agency has a private cost of effort. But, the combination of the independent relation-specific investments, in the form of a campaign contributions, and the efficient governing stage relational agreements create a reasonable environment in which both special interests’ campaign-stage incentive-compatibility constraints and the in-party agency’s governing-stage incentive-compatibility constraints are satisfied.

**Definition 2.** A PPE $\sigma$ is stationary if on the equilibrium path, $x_i(t_t, v_i,t), e_i(t_{t+1}, \theta_i,t)$, and $y_i(t_{t+1}, e_{i,t})$ at every date $t$, for some $x_i : \{0,1\}^2 \times [0,1] \to \mathbb{R}_+ \cup \emptyset, e_i : \{0,1\}^2 \times \Theta \to \mathbb{R}_+, y_i : \{0,1\}^2 \times \mathbb{R}_+ \to \mathbb{R}_+$.

By a stationary equilibrium, we mean that at each point on the equilibrium path in which agency-party-special interest triple $i$ reaches the state $(t_t, v_i,t, \theta_i,t), S_i$’s campaign contribution $x_i,t$ and, in the event that $P_i$ wins the time $t$ election, the pair of actions $(e_{i,t}, y_{i,t})$, are the same. If the in-party fails to take the action specified by his schedule, then there is no loss of generality in assuming that the worst in-party punishment is used as the threat point by the in-party special interest and the in-party agency.

We now formally define the refinement that is our focus in the continuation.

**Definition 3.** $\sigma \in \Gamma$ is a Williamson game PPE if $\sigma$ is stationary and at every possible history $h_t$ reached along an equilibrium path,

1. each special interest makes a non-collusive campaign contribution, that is for each $i \in \{a,b\}$ and each possible $t_t$ and $v_{i,t}$, $x_i(t_t, v_{i,t}) \in \mathbb{R}_+$ is a static best response to $E(x_{-i}(t_t, v_{-i,t}))$.

2. the governing stage actions of the in-party and the in-party agency are sequentially efficient, that is given that $t_{i,t+1} = 1$, $\exists \, \sigma' \in \Gamma$ such that (i) $\sigma'$ satisfies condition (1), (ii) $\sigma'_{j_i} |_{h_{j_i,t}} \neq \sigma_{j_i} |_{h_{j_i,t}}$ for some player $j_i \in \{A_I, P_I\}$ at an information set $h_{j_i,t}$ in the governing stage, and (iii) $U_{S_I}^C(\sigma'|_{h_{S_I,t}}) > U_{S_I}^C(\sigma|_{h_{S_I,t}}), U_{P_I}^I(\sigma'|_{h_{P_I,t}}) \geq U_{P_I}^I(\sigma|_{h_{P_I,t}}), \text{ and } U_{A_I}^I(\sigma'|_{h_{A_I,t}}) \geq U_{A_I}^I(\sigma|_{h_{A_I,t}})$. 

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As we are interested in the relationship between parties and special interests we focus on
the Williamson game PPE in which the in-party agency is driven to its reservation utility,
but it is straightforward to relax this. We also restrict our focus to the case that each party
has the same in-party expected equilibrium continuation payoff,
\( V_{IP} \equiv E(U_{IA}(\sigma|h_{Ai,t+1})) \),
when the in-party, but this symmetry can be relaxed. From Assumption 1 and Proposition
1, it follows that a Williamson game PPE is a stationary equilibrium \( \sigma \in \Gamma \) that is a solution
to the following program

\[
x_{i}(t_t, v_{it}) = \arg \max_{x_{i,t} \in \mathbb{R}^+} \text{Prob}(i \text{ wins}|x_{i,t}, t_t)J(V_{Pi})
\]

\[
+ (1 - \text{Prob}(i \text{ wins}|x_{i,t}, t_t)) \delta E(U_{SI}^{C}(\sigma|h_{Si,t+1})|t_{i,t+1} = 0) - x_{i,t} \tag{7}
\]

where

\[
J(V_{Pi}) = \max_{e_i, y_i} v_{it}E(\pi_S(e_{i,t}(t_{t+1}, \theta_{i,t}))) + \delta E(U_{SI}^{C}(\sigma|h_{Si,t+1})|t_{i,t+1} = 1) \tag{8}
\]

subject to the dynamic enforcement, incentive-compatibility, efficiency and monotonicity
constraints

\[
U_{IA}^{I}(\sigma|h_{Ai,t}) \geq 0
\]

\[
U_{IA}^{I}(\sigma|h_{i,t,x_{i,t+1},e_{i,t+1}, \theta_{i,t+1}}) \geq \max_{\theta_{i,t}} \{U_{IA}^{I}(\sigma|h_{i,t,x_{i,t+1}, \theta_{i,t}})\}
\]

\[
\dot{e}_i(\theta_{i,t}) \leq 0
\]

\[
E(U_{Pi}^{I}(\sigma|h_{Pi,t+1})) = V_{Pi}
\]

\[
U_{Pi}^{I}(\sigma|h_{Pi,t}) \geq \pi_p
\]

The interesting case is when the symmetric in-party expected continuation payoff, \( V_{Pi} \),
is large enough that the dynamic enforcement constraint is satisfied \( V_{Pi} \geq \pi_p \) but not so
large that agency funding is not possible in equilibrium. As we will show, this last condition
requires \( V_{Pi} < \pi_p/[1 - \delta + (\delta \mu/2) - (\delta^2 (\mu/2)^2/(1 - \delta + (\delta \mu/2)))\]}. The Pareto frontier, with
agency actions, is characterized by allowing \( V_{Pi} \) to vary between these two extremes.

**Assumption 2.** \( V_{Pi} \in [\pi_p, \pi_p/[1 - \delta + (\delta \mu/2) - (\delta^2 (\mu/2)^2/(1 - \delta + (\delta \mu/2)))]) \)

We now characterize the set of Williamson game PPE. We begin with the problem of
maximizing the special interest’s net benefit from winning and the dynamic enforcement
constraints that arise in the governing stage, and then move back through the game tree.
3.1 Governing Stage

In a Williamson game PPE, the actions of the players depend on the stationary beliefs regarding the probabilities of each party $i = a, b$ winning reelection when the in-party. For $i = a, b$, let $p_i$ denote the common belief, which is correct in equilibrium, regarding the probability that party $i$ wins reelection when the in-party. Given the symmetry between the agency-party-special interest triples, we focus on the symmetric case in which $p_a = p_b$, and denote the common belief that, in any period, the in-party wins reelection as $p_I$.

As $y_i(t_{i+1}, e_{i,t}) = e_i(t_{i+1}, \theta_{i,t}) = 0$ if $t_{i,t+1} = 0$, let $y_i(e_i)$ denote $P_i$’s in-party agency funding schedule, where we have suppressed the time subscript. Similarly, denote $A_i$’s in-party effort schedule by $e_i(\theta_i)$. We begin with the individual rationality and dynamic enforcement constraints for the in-party. Party $i$’s in-party and the out-party equilibrium continuation values at the end of the governing stage are given by

$$U_{P_i}^I(\theta_i) = \pi_p - y_i(e_i(\theta_i)) + \delta p_I V_{P_i}^I + \delta (1 - p_I) U_{P_i}^O$$

and

$$U_{P_i}^O = \delta (1 - p_I) V_{P_i}^I + \delta p_I U_{P_i}^O$$

The in-party’s dynamic enforcement and efficiency constraints are given, respectively, by

$$V_{P_i}^I \left( \delta p_I + \frac{\delta^2 (1 - p_I)^2}{1 - \delta p_I} \right) \geq y_i(e_i(\theta_i)).$$

for all $\theta_i \in \Theta$ and

$$\pi_p - E(y_i(e_i(\theta_i))) + V_{P_i}^I \left( \delta p_I + \frac{\delta^2 (1 - p_I)^2}{1 - \delta p_I} \right) = V_{P_i}^I$$

Similarly, the incentive compatibility and dynamic enforcement constraints for the in-party agency are

$$e_i(\theta_i) \in \arg \max_e [y(e) - \theta_i c_A(e)]$$

$$y_i(e_i(\theta_i)) - \theta_i c_A(e_i(\theta_i)) \geq 0$$

for all $\theta_i \in \Theta$. From standard incentive compatibility arguments it follows that for the in-party funding schedule $y_i(e)$ to induce the agency effort schedule $e_i(\theta_i)$, it must be the case
that

\[ y_i(e_i(\theta_i)) = \theta_i c_A(e_i(\theta_i)) + \int_{\theta_i}^{\bar{\theta}} c_A(e_i(\bar{\theta})) d\bar{\theta} \]

and as in-party funding covers the agency’s cost of effort and provides an informational rent, the agency’s dynamic enforcement constraint is satisfied.

To solve for the effort schedule, we take an optimal control theory approach that extends the optimal relational contracts with hidden information framework examined in Levin (2003) to our dynamic special interest politics game. Let \( \dot{e}_i(\theta_i) = c(\theta_i) \) be the control variable and let \( e_i(\theta_i) = s_1(\theta_i) \) be a state variable. Let \( p_1(\theta_i) \) be the adjoint variable associated with \( s_1(\theta_i) \). Let

\[ \dot{s}_2(\theta_i) = \left[ -c_A(s_1(\theta_i)) \left( \theta_i + \frac{G'(\theta_i)}{G''(\theta_i)} \right) \right] G'(\theta_i) \]

be a second state variable.\(^{14}\) The initial condition is \( s_2(\theta) = 0 \). The efficiency constraint can be written as a terminal condition on the state variable \( s_2 \)

\[ \pi_p + s_2(\bar{\theta}) - V_p^I \left( 1 - \delta p_I - \frac{\delta^2(1 - p_I)^2}{1 - \delta p_I} \right) = 0. \]

Let \( p_2(\theta) \) be the adjoint variable associated with \( \dot{s}_2(\theta) \). Let \( \dot{s}_3(\theta) = c_A(s_1(\theta_i)) \) be a third state variable with initial condition \( s_3(\theta) = 0 \). The dynamic enforcement constraint can be written as a combination of an initial condition on \( s_1 \) and a terminal condition on \( s_3 \),

\[ V_p^I \left( \delta p_I + \frac{\delta^2(1 - p_I)^2}{1 - \delta p_I} \right) - \theta c_A(s_1(\theta)) - s_3(\bar{\theta}) \geq 0. \]

Let \( p_3(\theta_i) \) be the adjoint variable associated with \( \dot{s}_3(\theta_i) \), and let \( \lambda \) denote the multiplier on the dynamic enforcement constraint. Lastly, let \( q(\theta_i) \) denote the multiplier on the monotonicity constraint, \( c(\theta_i) \leq 0 \).

Regardless of the realization of \( v_{i,t} \), the special interest prefers more to less agency effort. Thus, the Hamiltonian function may be written as

\[ H(s, c, p, \theta_i) = \pi_S(s_1(\theta_i)) G'(\theta_i) + p_1(\theta_i) \dot{s}_1(\theta_i) + p_2(\theta_i) \dot{s}_2(\theta_i) + p_3(\theta_i) \dot{s}_3(\theta_i). \]

Letting \( L \) denote the generalized Hamiltonian, the solution satisfies

\[ \frac{\partial L}{\partial c} = p_1(\theta_i) - q(\theta_i) = 0, \quad p_1(\theta) - \lambda c_A'(s_1(\theta_i)) = 0 \quad \& \quad p_1(\theta) = 0 \quad (12) \]

\(^{14}\)Observe that after an integration by parts, \( s_2(\bar{\theta}) = -E(y_i(e_i(\theta_i))). \)
\[
\frac{\partial L}{\partial s_1} = \left[ \pi'_S(s_1(\theta_i)) - c'_A(s_1(\theta_i)) \left( p_2(\theta_i)\theta_i + \frac{p_2(\theta_i)G(\theta_i) - p_3(\theta_i)}{G'(\theta_i)} \right) \right] G'(\theta_i) = -\dot{p}_1(\theta_i) \quad (13)
\]

\[
\frac{\partial L}{\partial s_2} = -\dot{p}_2(\theta_i) = 0 \quad (14)
\]

\[
\frac{\partial L}{\partial s_3} = -\dot{p}_3(\theta_i) = 0 \quad & \quad p_3(\theta) = \lambda \geq 0 \quad (15)
\]

boundary conditions

\[
s_2(\theta) = 0, \quad \pi_P + s_2(\bar{\theta}) - V'_P \left( 1 - \delta p_I - \frac{\delta^2(1-p_I)^2}{1 - \delta p_I} \right) = 0, \quad s_3(\theta) = 0,
\]

\[
V'_P \left( \delta p_I + \frac{\delta^2(1-p_I)^2}{1 - \delta p_I} \right) - \theta c_A(s_1(\theta)) - s_3(\bar{\theta}) \geq 0 \quad (16)
\]

and the complimentary slackness condition on the monotonicity constraint

\[
q(\theta_i) \geq 0, \quad c(\theta_i) \leq 0 \quad \text{and} \quad q(\theta_i)c(\theta_i) = 0 \quad (17)
\]

There are two cases depending on whether the in-party’s dynamic enforcement constraint is binding. Consider first the case that the in-party’s dynamic enforcement constraint is satisfied and \( \lambda = 0 \). In this case, we will focus on the relaxed problem — ignoring the monotonicity constraint, \( q(\theta_i) = 0 \) for all \( \theta_i \) — and then show that the monotonicity constraint is satisfied at the unconstrained optimum. Given this focus, it follows from (12) that \( \dot{p}_1(\theta_i) = \dot{q}(\theta_i) = 0 \). From (14), \( p_2(\theta_i) = \bar{p}_2 \) a constant for all \( \theta_i \). Combining these two facts with (15) and (13), we see that \( s_1(\theta_i) \) solves

\[
\pi'_S(s_1(\theta_i)) - c'_A(s_1(\theta_i)) \left( \bar{p}_2\theta_i + \frac{\bar{p}_2G(\theta_i)}{G'(\theta_i)} \right) = 0 \quad (18)
\]

and the existence of a solution to (18) requires that \( \bar{p}_2 > 0 \). As the efficiency constraint is binding, \( \bar{p}_2 \) is chosen so that the terminal condition on the state variable \( s_2 \) is binding. Applying the implicit function theorem to (18), we see that \( s_1 \) satisfies the monotonicity constraint. If

\[
V'_P \left( \delta p_I + \frac{\delta^2(1-p_I)^2}{1 - \delta p_I} \right) \geq \theta c_A(s_1(\theta)) + \int_{\bar{\theta}}^{\tilde{\theta}} \theta c_A(s_1(\tilde{\theta}))d\tilde{\theta} \quad (19)
\]
then $s_1(\theta_i)$ is the equilibrium effort schedule for all $\theta_i \in \Theta$. Furthermore, note that in this case, with $p_1(\theta_i) = q(\theta_i) = p_3(\theta_i) = \lambda = 0$ and $p_2(\theta_i) = \overline{p}_2 > 0$ for all $\theta_i \in \Theta$, the generalized Hamiltonian is concave with respect to $(s,c)$. Combining this with the fact that the boundary conditions are concave in $\{s_i(\theta), s_i(\theta)\}_{i=1}^{3}$, the Pontryagin Maximum Principle provides necessary and sufficient conditions for a global maximum.\(^{15}\)

Second, if the dynamic-enforcement constraint is satisfied and (19) does not hold, then $\lambda > 0$, from (12)\(^{16}\) $q(\theta_i) = \lambda c_A'(s_1(\theta_i)) > 0$, and from complementary slackness $c(\theta_i) = s_1(\theta_i) = 0$. Thus, pooling must involve the most efficient types. Note that from (15), $p_3(\theta_i) = \lambda$ for all $\theta_i$, and consider the case of partial pooling. Over any interval in which $c(\theta_i) < 0$, it follows from complimentary slackness that $q(\theta_i) = 0$ and $s_1(\theta_i)$ implicitly solves

$$
\pi'_S(s_1(\theta_i)) - c_A'(s_1(\theta_i)) \left( \overline{p}_2 \theta_i + \frac{\overline{p}_2 G(\theta_i) - \lambda}{G'(\theta_i)} \right) = 0. \tag{20}
$$

Existence of such a solution requires that

$$
\left( \overline{p}_2 \theta_i + \frac{\overline{p}_2 G(\theta_i) - \lambda}{G'(\theta_i)} \right) > 0 \tag{21}
$$

and the monotonicity constraint requires that

$$
\frac{\partial \left( \overline{p}_2 \theta_i + \frac{\overline{p}_2 G(\theta_i) - \lambda}{G'(\theta_i)} \right)}{\partial \theta_i} \geq 0. \tag{22}
$$

Thus, on any interval where $s_1(\theta_i)$ is decreasing, $s_1(\theta_i)$ solves (20) and $(\overline{p}_2, \lambda)$ satisfy (21) and (22).

Lemma 1 shows that there exists a “cut-off” type $\hat{\theta}$ such that $s_1(\theta_i)$ is constant for $\theta_i \in [\hat{\theta}, \hat{\theta})$ and decreasing for $\theta_i \in [\hat{\theta}, \overline{\theta}]$.

**Lemma 1.** *In any optimum, if there exists a $\hat{\theta} \in \Theta$ such that $c(\hat{\theta}) < 0$, then $c(\theta) < 0$ almost everywhere for $\theta \in [\hat{\theta}, \overline{\theta}]$.***

**Proof.** If $c(\hat{\theta}) < 0$ then by complimentary slackness, $q(\hat{\theta}) = 0$. By way of contradiction, suppose that there exists an optimum in which there is a $\theta_0 \in \Theta$ such that $c(\theta_0) < 0$ and there exists a region $\Omega \subseteq [\theta_0, \overline{\theta}]$, with strictly positive measure, such that $c(\theta) = 0$ for all $\theta \in \Omega$. At $\theta_0$, $(\overline{p}_2, \mu)$ satisfy (21) and (22). Combining this with (13), we see that $\dot{q}(\theta) \leq 0$.


\(^{16}\)Note that if $s_1(\theta_i) = 0$ then the monotonicity constraint implies that $s_1(\theta_i) = 0$ for all $\theta_i$, which is clearly suboptimal. Thus, in any optimum $s_1(\theta_i) > 0$. 

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for all \( \theta \in \Omega \), with equality for at most one value of \( \theta \in \Omega \). From (12) \( q(\theta) = p_1(\theta) \) for all \( \theta \), and, in any optimum, the adjoint variable \( p_1 \) is a continuous function. Because \( q(\hat{\theta}) = 0 \) and \( q \) is a continuous nonincreasing function, \( \dot{q}(\theta) < 0 \) for almost every \( \theta \in \Omega \) contradicts the constraint that \( q(\theta) \geq 0 \) for all \( \theta \in \Theta \).

Given a “cut-off” type \( \hat{\theta} \) such that \( s_1(\theta_i) \) is constant for \( \theta_i \in [\hat{\theta}, \tilde{\theta}] \) and decreasing for \( \theta_i \in [\tilde{\theta}, \hat{\theta}] \) and the continuity of \( s_1, s_1(\theta_i) = s_1(\hat{\theta}) \) for all \( \theta_i \in [\hat{\theta}, \tilde{\theta}] \) such that \( \tilde{p}_2 \) and \( \lambda \) are chosen so that the terminal condition on the state variable \( s_2 \) is binding. As in the first case, the combination of the concavity of the boundary conditions with respect to \( \{s_i(s), s_i(c)\}_{i=1}^3 \) and the concavity of the generalized Hamiltonian with respect to \( (s, c) \) for \( p_1(\theta_i) = q(\theta_i) \geq 0, p_3(\theta_i) = \lambda \geq 0 \) and \( p_2(\theta_i) = \tilde{p}_2 > 0 \) for all \( \theta_i \in \Theta \) implies that the Pontryagin Maximum Principle provides necessary and sufficient conditions for a global maximum.

Now, consider a pooling equilibrium in which all types \( \theta_i \) choose the same effort. That is, \( e_1(\theta_i) = \hat{e} \) where \( \hat{e} \) is given implicitly by the dynamic enforcement constraint,

\[
y_i(\hat{e}) = -\theta c_A(\hat{e}) = V_P(\delta p_I + \frac{\delta^2(1 - p_I)^2}{1 - \delta p_I}).
\]

Furthermore, the efficiency constraint must also be binding, and as \( y_i(\hat{e}) = E(y_i(\hat{e})) \), it follows that

\[
y_i(\hat{e}) = \pi_p - V_P(1 - \delta p_I - \frac{\delta^2(1 - p_I)^2}{1 - \delta p_I}).
\]

Thus, a pooling equilibrium can only arise in the special case that \( V_P = \pi_p \), otherwise the efficiency constraint is slack and the equilibrium governing-stage agreement must feature either partial pooling or full separation.

Given the efficient effort schedule \( e^*_i(\theta_i) = s_1(\theta_i) \), the maximized value of the special interest’s net benefit is \( v_i \pi_S(e^*_i(\theta_i)) \) with expectation

\[
v_i E(\pi_S) = v_i \int_{\theta}^{\bar{\theta}} \pi_S(e^*_i(\theta)) G'(\theta) d\theta
\]

The Pareto efficient governing-stage actions are summarized below.

**Proposition 2.** Suppose Assumptions 1 and 2 hold. Given beliefs \( p_I \) regarding each party’s probability of reelection when the incumbent, the Pareto efficient governing-stage actions are summarized as follows. For agency-party-special interest triple in-party agency effort \( e^*_i(\theta_i) \) is given implicitly by equations (12)-(16) with complimentary slackness condition (17) and
in-party funding is given by

\[ y_i^*(e_i^*(\theta)) = \theta_i c_a(e_i^*(\theta)) + \int_{\theta_i}^{\bar{\theta}} c_a(e_i^*(\theta)) d\tilde{\theta} \]  

(24)

The special interest’s expected net benefit is

\[ v_i E(\pi_{S_i}) = v_i \int_{\theta_i}^{\bar{\theta}} \pi_s(e_i^*(\theta)) G'(\theta) d\theta \]  

(25)

3.2 Campaign Stage

We now turn to the special interests’ campaign contribution schedules. Let \( V_{S_i}^C(I, v_i) \) denote \( S_i \)'s campaign stage continuation payoff when \( P_i \) is the in party and \( V_{S_i}^C(O, v_i) \) denote \( S_i \)'s continuation payoff when \( P_i \) is the out party. Recall that in the election stage the in-party wins with probability \((1 - \mu)\) and with probability \(\mu\) the party with the higher campaign expenditure wins. The maximization problems for each \( S_i \) are

\[
V_{S_i}^C(O, v_i) = \max_{x_i^O} \mu(v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_{i+1}))) Pr(x_i^O > x_{-i}^O) \\
+ \delta E(V_{S_i}^C(O, v_{i+1}))(1 - \mu Pr(x_i^O > x_{-i}^O)) - x_i^O \\
= \max_{x_i^O} \mu \left[ v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_{i+1})) - \delta E(V_{S_i}^C(O, v_{i+1})) \right] Pr(x_i^O > x_{-i}^O) \\
+ \delta E(V_{S_i}^C(O, v_{i+1})) - x_i^O
\]

(26)

and

\[
V_{S_i}^C(I, v_i) = \max_{x_i^I} (v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_{i+1}))(1 - \mu + \mu Pr(x_i^I > x_{-i}^I)) \\
+ \mu \delta E(V_{S_i}^C(O, v_{i+1}))(1 - Pr(x_i^I > x_{-i}^O)) - x_i^I \\
= \max_{x_i^I} \mu \left[ v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_{i+1})) - \delta E(V_{S_i}^C(O, v_{i+1})) \right] Pr(x_i^I > x_{-i}^O) \\
+ \delta E(V_{S_i}^C(O, v_{i+1})) \mu + (1 - \mu)(v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_{i+1}))) - x_i^I
\]

(27)

Note that the special interests’ campaign contribution problems differ by only a constant. Recalling that in a Williamson game PPE the campaign stage is non-collusive and for each \( S_i \), \( x_i(t_i, v_{i,t}) \) is a static best to \( E(x_{-1}(t_i, v_{-i,t})) \), it follows from (26) and (27) that along any
Williamson game PPE path,

\[
E(V_{S_i}^C(I, v_i)) - E(V_{S_i}^C(O, v_i)) = \frac{(1 - \mu)E(v_i)E(\pi_S)}{1 - (1 - \mu)\delta}.
\] (28)

Let \(\phi\) be defined as

\[
\phi = \frac{\delta(1 - \mu)E(v_i)}{1 - (1 - \mu)\delta}
\] (29)

Then (26) and (27) can be rewritten as

\[
V_{S_i}^C(O, v_i) = \max_{x_i^O} \mu E(\pi_S)[v_i + \phi] \Pr(x_i^O > x_i^I) - x_i^O + \delta E(V_{S_i}^C(O, v_i+1))
\] (30)

and

\[
V_{S_i}^C(I, v_i) = \max_{x_i^I} \mu E(\pi_S)[v_i + \phi] \Pr(x_i^I > x_i^O) - x_i^I - \mu E(\pi_S)\phi + (1 - \mu)v_i E(\pi_S) + \delta E(V_{S_i}^C(I, v_i+1))
\] (31)

From equations (30) and (31), each special interest \(i\)'s value for winning the election consists of three parts: (i) a common scaling component \(\mu E(\pi_S)\), (ii) a private valuation \(v_i\), and (iii) a common additive component \(\phi\). Furthermore, it follows that, each \(S_i\)'s campaign contribution schedule is independent of incumbency status.

Let \(x(v) \equiv x(\iota, v)\) denote a symmetric campaign contribution schedule. From either (30) or (31), the first-order condition is,

\[
\mu E(\pi_S)[v + \phi]F'(x^{-1}(x(v))) \frac{dx^{-1}(x(v))}{dx(v)} - 1 = 0
\] (32)

Then because \(x^{-1}(x(v)) = v\), it follows that

\[
\frac{dx^{-1}(x(v))}{dx(v)} = \frac{1}{\frac{dx(v)}{dv}}
\] (33)

Inserting (33) into (32), we have that

\[
\frac{dx(v)}{dv} = \mu E(\pi_S)[v + \phi]F'(v)
\] (34)
The equilibrium campaign contribution schedules are given by

\[ x(\iota, v) = \int_0^v \mu E(\pi_S)(\tilde{v} + \phi) F'(\tilde{v}) d\tilde{v} \tag{35} \]

In this symmetric equilibrium, the probability that the in-party wins reelection is

\[ p_I = (1 - \mu) + \frac{\mu}{2} = 1 - \frac{\mu}{2} \tag{36} \]

As this is an incomplete information all-pay auction in which the value of the prize is given by an affine transformation of a random variable, the proof of uniqueness of the campaign-stage contributions follows along the lines of Amann and Leininger (1996).

The characterization of the symmetric Williamson game PPE is summarized below.

**Theorem 1.** Suppose Assumptions 1 and 2 hold. The unique symmetric Williamson game PPE consists of: beliefs \( p_I = 1 - \frac{\mu}{2} \) regarding each party’s probability of reelection when the in-party, efficient governing-stage actions, \( e^*(\theta) \) and \( y^*(e) \), given by Proposition 2 and the campaign-stage contribution schedule

\[ x^*(\iota, v) = \int_0^v \mu E(\pi_S)(\tilde{v} + \phi) F'(\tilde{v}) d\tilde{v} \tag{37} \]

Contrary to conventional arguments about the allure of agency-group collusion, Theorem 1 demonstrates how the interactions within a party-special interest alliance result in a dynamically enforceable relational contract in which: (i) slack-maximizing agency bureaucrats take the actions that special interests want because agency funding is tied to the level of agency effort directed towards activities that benefit the interests of majority-party political allies, (ii) office-seeking political parties honor commitments to provide agency funding when in office because they need special interests to make campaign contributions in future elections, and (iii) special interests make campaign contributions because they know that their party has incentive to influence agency actions if they win the election.

Theorem 1 also provides insight into how incumbency persistence \( (1 - \mu) \) affects the equilibrium governing stage agreements.

**Corollary 1.** \( e^*(\theta) \) and \( y^*(e) \) are weakly decreasing in \( \mu \).

**Proof.** Clearly, \( e^*(\theta) \) is increasing in \( y^*(e) \). From Proposition 2 and Theorem 1, \( P_1 \)’s dynamic
enforcement constraint is,

\[ V_P^I \left( \delta \left( 1 - \mu \right) - \frac{\delta^2 \left( \mu \right)^2}{1 - \delta \left( 1 - \frac{\mu}{2} \right)} \right) \geq y^* \left( e^* \left( \theta \right) \right) \]

for all \( \theta \in \Theta \). Note that the left-hand side of this inequality is strictly decreasing in \( \mu \) for all \( \delta \in (0, 1) \) and \( \mu \in (0, 1] \). That is, as \( \mu \) increases \( P_i \)'s dynamic enforcement constraint becomes more restrictive.

As \( \mu \) increases, i.e. the incumbency advantage \( (1 - \mu) \) decreases, and the value, to each party, of holding office (weakly) decreases. Each party now offers a (weakly) lower level of agency funding in return for campaign contributions. Lower agency funding translates directly into lower agency effort. A standard argument for term limits is that by eliminating career politicians the power of special interests is curbed. To the extent that term limits make elections more competitive (i.e., increase \( \mu \)) in that the in-party must periodically replace incumbent candidates, our model shows that term limits do in fact generate lower levels of special interest influence.

The effect of incumbency persistence \( (1 - \mu) \) on equilibrium campaign contributions is slightly more nuanced. The standard result in campaign spending games is that the equilibrium campaign expenditures increase as the incumbency advantage decreases. The intuition is that as the playing field becomes more level, the challenger exerts more effort. Here, the level playing field effect of a higher \( \mu \) is countered by the decrease in \( E(\pi_S) \) that results from lower agency funding and lower agency effort. If \( E(\pi_S) \) is inelastic with respect to \( \mu \), then the level playing field effect dominates and the equilibrium campaign contributions are increasing in \( \mu \). Otherwise, the lower agency funding and lower agency effort effect dominates and the equilibrium campaign contributions are decreasing in \( \mu \).

**Corollary 2.** \( x^*(\iota, v) \) is weakly increasing in \( \mu \) if \( E(\pi_S) \) is inelastic with respect to \( \mu \). Otherwise, \( x^*(\iota, v) \) is weakly decreasing in \( \mu \).

**Proof.** From Theorem 1,

\[
\frac{\partial x^*(\iota, v)}{\partial \mu} = \left( E(\pi_S) + \mu \frac{\partial E(\pi_S)}{\partial \mu} \right) \int_{0}^{v} (\tilde{v} + \phi) F'(\tilde{v})d\tilde{v}.
\]

As \( (\partial E(\pi_S)/\partial \mu) \leq 0 \), \( (\partial x^*(\iota, v)/\partial \mu) \geq 0 \) if and only if \( 1 \geq (-\mu/E(\pi_S))(\partial E(\pi_S)/\partial \mu) \).
3.3 Discussion

Our dynamic repeated election model with subgovernment influence provides a new framework for thinking about repeated elections, subgovernment influence activities, and, more generally, dynamic contests that include supporting players and non-binding relational agreements. Rather than examine extensions of the model, we now briefly examine some of the distinctive features of our model in more detail. However, it should be pointed out that it is possible to relax assumptions such as two long-lived political parties\footnote{This can be replaced by the assumption that there are two distinct sets of citizen candidates as in Bernhardt et al. (2009).} and that the election depends on only valence competition.

A key premise of our model is that each party is linked with a special interest and the competition between special interests, via campaign contributions, is not over a single-dimensional policy space, but rather for influence over the actions of disjoint agencies. In (Parker and Parker 2012) we demonstrate that the two major parties have used the assignment of advisory committee membership rights to create durable linkages with special interest allies whose policy interests are clustered in disjoint agencies. For example, if we define a linked special interest as a special interest PAC giving at least two-thirds of their campaign contributions to the same party from 1990-2008, then less than 5 percent of the advisory committees in the Departments of Energy and Agriculture represent Democratic Party interests; likewise, in the Departments of Education and Labor there are no advisory committees representing the interests of Republican Party supporters.

A novel feature of our arguments is that they imply greater friction in agency-group relations than traditional treatments of bureaucratic politics. In the “agency capture” model of subgovernment special interest influence the goals of special interests are best served within the inner reaches of the bureaucracy, but all influence activity involves a direct relationship between agency bureaucrats and special interests. In this framework, legislators seek to constrain such capture. Conversely, our model shows that parties actually work to promote just such influence. There are a number of distinctions between our model and the standard “capture model,” but one key difference is that, in our model, special interests and agencies face a two-sided incomplete information trade problem (private benefits from agency action for the special interest and private costs of action for the agency). Things are further complicated by the fact that an agency must be the in-party agency in order to take actions that benefit its special interest. The independent relation-specific investment aspect of our Williamson game refinement of PPE provides a simple, plausible environment in which it is
incentive compatible for special interests to act on their private information (in the form of a
campaign contribution) and the in-party willingly provides incentive-compatible agreements
for the agency (via the funding process) to act on their private information.

Although we have focused on a dynamic repeated election application, our model provides
a general framework for examining infinite-horizon dynamic contests with contestants and
supporting players in a world where agreements between contestants and supporting players
are supported by non-binding relational contracts and is clearly relevant for a host of appli-
cations. Consider for example a repeated research and development game in which each firm
has its own research department. If promises of future departmental funding levels are non-
binding and each research department is slack-maximizing and has a private time-varying
cost of research effort, then a variation of our model in which each firm provides relational
incentives via a departmental funding decision and the firm with the larger effective research
output in a given period wins that period’s R&D game would apply directly.\(^{18}\)

Another distinctive feature of our model is that it provides a dynamic repeated election
model in which the campaign matters. In the existing literature on dynamic repeated elec-
tions, voting is usually a referendum on the incumbent (Ashworth et al. 2010). That is, the
electorate sets a performance standard for the incumbent and reelection depends on whether
the performance standard has been met. In our model the campaign matters in that both
parties expend campaign resources in order to improve their odds of winning the election.
Furthermore, it is straightforward to nest our dynamic model of campaign spending into
a standards based model. In our model the representative voter is assumed to reelect the
incumbent with probability \((1 - \mu)\) regardless of the campaign spending levels, but this can
be modified so that the representative voter looks to the campaign with probability \(\mu\) and
looks to a performance standard with probability \((1 - \mu)\).

4 Conclusion

In this paper we have examined a theory of special interest politics that centers on the
transactions among interest groups, political parties, and bureaucrats. For the equilibrium
refinement that we examine, we find that political parties, rather than preventing the capture
of governmental agencies by special interests, have incentive to form an alliance with special
interests involving the sale of political influence over agency actions in return for campaign

\(^{18}\)This corresponds to a four-player—the 2 parties and the 2 agencies—version of our model in which
both parties and agencies take actions in the campaign stage.
contributions. Such an alliance arises because the special interest benefits from the in-party’s power over agency funding, which generates the necessary incentives for the agency to act on the demands of the in-party’s political allies, and the party benefits form the special interest’s campaign contributions, which improve its odds of winning the election.

Appendix

This Appendix contains the proof for Proposition 1.

Proof of Proposition 1

First, we show that the dynamic special interest game allows for in-party punishments. This includes: (i) showing that there exists a worst equilibrium of breaking off trade, and (ii) the worst equilibrium can be used as a punishment in the event that a party deviates from its continuation strategy (Abreu 1988).

Lemma 2 (Worst Equilibrium). There exists a PPE $\sigma^W$ of the dynamic special interest game, in which party $I$ is the in-party in every period and party $O$ is the out-party in every period and

$$\sigma^W = \{x_{I,t} = 0, x_{O,t} = \emptyset, \{e_{i,t} = 0, y_{i,t} = 0, \}i \in \{I,O\}\}_{t=0}^\infty.$$ 

for all possible information sets $h_{j_{i,t}}$, $j \in \{A, P, S\}$ and $i \in \{I, O\}$.

Proof. If $\sigma^W$ is a PPE, then there exist no unilateral one-shot payoff increasing deviations in either the campaign contribution or the governing stages. As the out-party does not submit a campaign contribution, the period 0 in-party, denoted $P_I$, is the in-party in every period and the remaining party, $P_O$, is always the out-party. It follows directly that there are no profitable one-shot deviations for $A_i$ and $P_{i_t}$, $i \in \{I, O\}$, in the governing stage.

In the campaign stage, $S_O$ can deviate and win the election but doing so has a continuation value of zero, given the continuation strategies of the other players and is, thus, not a payoff increasing deviation. Similarly, $S_I$ always wins the election, receiving a net benefit of 0 in each period, and thus has no incentive to deviate from $x_{I,t} = 0$ in the campaign stage. 

We now show that given any strategy $\sigma$, with continuation strategy $\sigma|_{h_t}$ after history $h_t$, an in-party that observably deviates from an agreement can be punished with a continuation value of 0.
Lemma 3. Given any strategy \( \sigma \) a time \( t \) deviation from \( \sigma|_{h_{P_I,t}} \) by the in-party in the governing stage, \( y_{I,t} \neq y_{I,t}(h_{P_I,t}) \), can be punished with a time \( t + 1 \) continuation value of 0.

Proof. If \( \sigma = \sigma^W \), then all players except the in-party have time \( t + 1 \) continuation values of 0. If the in-party deviates from \( \sigma|_{h_{P_I,t}} \), then \( S_I \) can choose \( x_{I,t+1} = \emptyset \) and the tie-breaking rule implies that the time \( t \) in-party loses the time \( t + 1 \) election with certainty. Once the time \( t \) out-party wins the time \( t + 1 \) election, there is no loss in assuming that the time \( t \) out-party chooses a campaign contribution of 0, instead of \( \emptyset \), from time \( t + 1 \) on. Thus, the time \( t \) in-party becomes the out-party in period \( t + 1 \) and remains the out-party in every subsequent period, which results in a time \( t + 1 \) continuation value of 0.

If \( \sigma \neq \sigma^W \), then if in the time \( t \) governing stage \( P_i \) is the in-party and \( P_i \) deviates from its agreement, \( y_{I,t} \neq y_{I,t}(h_{P_I,t}) \), then \( S_I \) may begin a punishment phase that breaks off trade at time \( t + 1 \) with \( x_{i,t+1} = \emptyset \). Such a punishment results in a reversion to the worst equilibrium and time \( t + 1 \) continuation values of 0 for all members of the time \( t \) in-party agency-party-special interest triple.

We now complete the proof of Proposition 1. For the necessity of (2), \( S_i \) can choose \( x_{i,\tau} = 0 \) for all \( \tau \geq t \) yielding a minimum continuation value of 0. For the necessity of (3), note that if (3) is violated, then it must be the case that \( x_{i,t}(h_t, v_{i,t}') \neq x_{i,t}(h_t, v_{i,t}) \) and \( S_i \) can unobservably deviate and increase his continuation payoff by choosing \( x_{i,t}(h_t, v_{i,t}') \). For the necessity of (4), \( A_i \) can choose \( e_{i,\tau} = 0 \) for all \( \tau \geq t \) yielding a minimum continuation value of 0. The necessity of (5) follows from the fact that if (3) is violated, then \( A_i \) can unobservably deviate and increase his continuation payoff by choosing \( e_{i,t}(h_t, x_{i,t}, \omega_{t+1}, \theta_{i,t}) \neq e_{i,t}(h_t, x_{i,t}, \omega_{t+1}, \theta_{i,t}) \). For the necessity of (6), \( P_i \) can choose \( y_{i,\tau} = 0 \) for all \( \tau \geq t \) yielding a minimum continuation value of \( \pi_p \).

For sufficiency, the conditions in Proposition 1 eliminate the possibility of a unilateral one-shot payoff increasing deviation from \( \sigma|_{h_t} \), given any history \( h_t \) and any realization of \( v_t \) and \( \theta_t \), given that such deviations are either observable and may be punished with a time \( t + 1 \) continuation value of 0 or are not incentive compatible. In particular, given (3) and (5) a deviation from \( x_{i,t}(h_t, v_{i,t}) \) or \( e_{i,t}(h_t, x_{i,t}, \omega_{t+1}, \theta_{i,t}) \), by \( S_i \) or \( A_I \) respectively, is weakly dominated. A deviation by \( P_I \) from \( \sigma|_{h_{P_I,t}} \) is observable, to both the in-party agency and the in-party special interest, and from Lemma 2 may be punished with a time \( t + 1 \) continuation value of 0. Thus, such a deviation is weakly dominated by (6). This completes the proof of Proposition 1.
References


