Ambiguity Framed

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In his exposition of subjective expected utility theory, Savage (1954) proposed that the Allais paradox could be reduced if it were recast into a format which made the appeal of the independence axiom of expected utility theory more transparent. Recent studies consistently find support for this prediction. We consider a salience-based choice model which explains this frame-dependence of the Allais paradox and derive the novel prediction that the same type of presentation format which was found to reduce Allais-style violations of expected utility theory will also reduce Ellsberg-style violations of subjective expected utility theory since that format makes the appeal of Savage's "sure thing principle" more transparent. We design an experiment to test this prediction and find strong support for such frame dependence of ambiguity aversion in Ellsberg-style choices. In particular, we observe markedly less ambiguity-averse behavior in Savage's matrix format than in a more standard 'prospect' format. This finding poses a new challenge for the leading models of ambiguity aversion.

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1. Introduction

Expected utility (EU) theory (Von Neumann and Morgenstern, 1947) and subjective expected utility (SEU) theory (Savage, 1954) are widely recognized as the standard models of rational decision making under risk and uncertainty. Both models have also been applied as descriptive theories of actual behavior, although persistent empirical challenges were raised soon after the models were introduced. Allais (1953) devised pairs of choices, one involving a certain outcome and a risky prospect and the other a choice between two risky prospects where people frequently[1] violate the independence axiom of EU. Ellsberg (1961) presented pairs of choices each involving a risky prospect (whose probabilities are given) and an uncertain prospect (whose probabilities are unknown) where people frequently violate the 'sure-thing' principle of SEU.

In his exposition of subjective expected utility, Savage (1954) digressed to address the Allais-type violations of the independence axiom. He conjectured that these violations might be reduced if the choice situations were reframed in a transparent format. Tests of this prediction, discussed below, have consistently found that the Allais paradox is susceptible to framing, with significantly fewer violations in Savage's proposed presentation format. Since the Ellsberg paradox also violates an independence condition, we ask whether applying Savage's presentation format to Ellsberg-style choices leads to fewer violations of SEU: To our knowledge this question has not been previously investigated. We ground our investigation in new and rigorous theory formalizing the notion of a transparent frame (Leland and Schneider 2016) and recent theory formalizing salience (e.g., Bordalo et al. 2012; Koszegi and Szeidl 2013).

2. Motivation

Consider Savage's (1954) version of the Allais paradox: Figure 1 presents this version in two different frames (payoffs are in thousands of dollars). The left panel of Figure 1 presents it in what we call *minimal* or *efficient* frames.[2] In Savage's version, a decision maker chooses between lotteries p and q and then chooses between lotteries p' and q'. Lottery p offers \$500,000

[1] We are speaking here of the classic Allais example, which is a thought experiment involving very large hypothetical outcomes which no experimenter can actually pay out. When its outcomes are proportionally scaled down to an experimentally feasible size for actual payment, similar behavior does not always occur (e.g. Conlisk 1989; Fan 2002). This could be either a payoff magnitude effect or a hypothetical versus real incentives effect. In incentivized experiments, the generalized Allais example (known as the common consequence effect) does not always occur (Burke et al. 1996) and sometimes occurs in 'non-classic' ways (Starmer 1992).

[2] A minimal frame is a matrix presentation of choice alternatives which (among other properties) has the smallest dimension (fewest number of columns) needed to represent those alternatives. See Leland and Schneider (2016) for formal property lists which uniquely define minimal and transparent frames.

with certainty, whereas q offers a 10% chance of \$2.5 million, an 89% chance of \$500,000, and a 1% chance of \$0. The independence axiom (and Savage's sure-thing principle) imply that a decision maker with strict preferences will choose either p and p' or q and q' (in accord with the decision maker's attitude toward risk). Yet Savage himself reports expressing a preference for p over q and for q' over p' (Savage, 1954), in violation of his own theory!

Figure 1. The Allais Paradox in Minimal and Transparent Frames

The Allais Paradox in Minimal Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p	500	0.10	500	0.89	500	0.01
q	2500	0.10	500	0.89	0	0.01

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
p′	500	0.11	0	0.89
q′	2500	0.10	0	0.90

The Allais Paradox in Transparent Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p	500	0.10	500	0.89	500	0.01
q	2500	0.10	500	0.89	0	0.01

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p′	500	0.10	0	0.89	500	0.01
q′	2500	0.10	0	0.89	0	0.01

Troubled by his own expressed preferences, Savage (1954) invites consideration of an alternative presentation of the same choices: We show a similar presentation in the right panel of Figure 1. In this presentation, it is clear that p and q each offer an 89% chance of \$500,000 and that p' and q' each offer an 89% chance of \$0. Savage proposes that this change in framing may enhance the appeal of the independence axiom and produce more consistent choices: Following his suggestion, we say that this presentation employs *transparent* frames. In particular, a transparent frame isolates the common consequences of the lotteries under consideration and focuses attention on the differences between lotteries as prescribed by the independence axiom.

A number of recent studies (Leland, 2010; Bordalo et al., 2012; Incekara-Hafalir and Stecher, 2012; Birnbaum and Schmidt, 2015; Harman and Gonzalez, 2015) have investigated whether observed behavior is more consistent with Savage's theory when the Allais paradox choices are presented to subjects in transparent frames. All of these studies find support for Savage's conjecture. Incekara-Hafalir and Stecher (2012) conclude that "given a transparent presentation, expected utility theory performs surprisingly well."

Leland and Schneider (2016) formalize matrix presentations of lotteries (like those used by Savage) and develop a salience-based decision algorithm that operates over frames. Their theoretical work implies that ambiguity aversion is also susceptible to framing: The same transparent framing that reduces Allais-style violations of EU should reduce Ellsberg-style violations of SEU. Figure 2 illustrates this with choice situations from our new experiment—as these were presented to our subjects. The top pair shows a choice between two acts (state-contingent lotteries) in minimal frames. The lottery the decision maker plays depends on her choice (act A or act B) and the realization of an ambiguous state of the world (a 'red ticket' state or a 'blue ticket' state). The decision maker does not know the probability that the true state is red or blue. As in Ellsberg's classic paradox, one act (A) is merely risky because it yields the same lottery regardless of the state: In keeping with relevant theoretical work (Grant and Polak 2013), we call these "constant acts." The other act (B) is an "ambiguous act" (it yields different lotteries in different states). The decision maker is also given a similar choice in which the lotteries assigned to the red and blue ticket states are reversed. This construction resembles that in Ellsberg's (1961) two-color paradox.

Figure 2. Ellsberg's Paradox in Minimal Frames (top) and Transparent Frames (bottom)

	You Draw a Red Ticket				You Draw a Blue Ticket			
	$	N/12	$	N/12	$	N/12	$	N/12
A	$25	6/12	$0	6/12	$25	6/12	$0	6/12
B	$25	9/12	$0	3/12	$25	3/12	$0	9/12

	You Draw a Red Ticket						You Draw a Blue Ticket					
	$	N/12	$	N/12	$	N/12	$	N/12	$	N/12	$	N/12
A	$25	6/12	$0	3/12	$0	3/12	$25	3/12	$25	3/12	$0	6/12
B	$25	6/12	$25	3/12	$0	3/12	$25	3/12	$0	3/12	$0	6/12

The SEU model predicts that a decision maker who strictly prefers A to B in the top panel of Figure 2 will also strictly prefer B to A when the lotteries assigned to red and blue states are reversed—acting as if that agent assigns a subjective probability distribution over states. However, in similar types of choices, Ellsberg (1961) found that many people preferred A to B regardless of whether the assignment of lotteries to states is reversed. Since A offers a known probability of winning a prize, whereas the probability of winning in B is ambiguous, the strict preference for A is termed *ambiguity aversion*.

The choices between A and B could, like the Allais lotteries, be presented in the ‘transparent’ frames shown in the bottom of Figure 2. In this ‘Savage’ presentation, for the choice between A and B, the common consequences in each state-contingent lottery are isolated, encouraging the decision maker to focus on the differences between A and B (the 3/12 chance of A paying \$0 and B paying \$25 in the red state and the 3/12 chance of A paying \$25 and B paying \$0 in the blue state). A decision maker who focuses only on these differences and assigns a uniform prior over states will then be indifferent between A and B, regardless of whether the assignment of lotteries to states is reversed. This reasoning suggests that transparent framing of the Ellsberg paradox will produce behavior closer to ambiguity neutrality. However, such frame-dependence of ambiguity aversion is not consistent with any of the leading models of ambiguity aversion in the literature. The following section shows that Leland and Schneider’s (2016) model predicts this under fairly general conditions, and introduces a variant of their model generalizing the prediction in an empirically useful way. Our new experiment finds strong support for this generalized version of the prediction.

There is but a small and very recent literature on the possibility that ambiguity attitudes are susceptible to framing effects. Chew et al. (2017) examine whether presentation of choice alternatives as text description versus payoff tables (so as to make the ambiguity inherent in the choices more or less explicit) influences the degree of ambiguity aversion observed. They find that for subjects who do not recognize ambiguity in some tasks, emphasizing ambiguity produces greater ambiguity aversion. However, subjects that recognized ambiguity in each task were more ambiguity-averse than those who did not recognize ambiguity in some tasks, regardless of whether the ambiguity is emphasized. Trautmann and van der Kuijlen (2014) examine attitudes toward ambiguity for gains versus losses. They report results suggesting that ambiguity aversion varies according to whether the outcomes are gains or losses, as has been observed for attitudes toward risk. Finally, Voorhoeve et al. (2016) test the findings in Chew et al. (2017) and in Trautmann and van der Kuijlen (2014), and fail to find significant support for the hypotheses that emphasizing ambiguity, or reframing gains as losses, alters the prevalence of ambiguity aversion. With these mixed findings, we think there is room for more experimental work. Additionally, the next section provides a highly focused theoretical motivation for our new experiment.

3. Two Salience-based Models of Choice under Uncertainty

Leland and Schneider (2016) employ a matrix representation of the attributes (e.g. payoffs and probabilities in lotteries) of pairs of alternatives: We follow this for pairs of acts. A generic frame for simple Ellsberg-style choices which encompasses the basic pairs of acts used in our experiment is shown in Figure 3 in which there are two possible states $\omega \in \{r, b\}$ –"red ticket" and "blue ticket" states r and b. The decision maker does not know the underlying state ω. In Figure 3, act X offers lottery $\{x_1^r, p_1^r; \ldots; x_{n_r}^r, p_{n_r}^r\}$ when $\omega = r$ and offers lottery $\{x_1^b, p_1^b; \ldots; x_{n_b}^b, p_{n_b}^b\}$ when $\omega = b$. Likewise, act Y offers lottery $\{y_1^r, q_1^r; \ldots; y_{n_r}^r, q_{n_r}^r\}$ when $\omega = r$ and lottery $\{y_1^b, q_1^b; \ldots; y_{n_b}^b, q_{n_b}^b\}$ when $\omega = b$. All frames in the experiment presented outcomes that monotonically decrease (from left to right) in each state-contingent lottery: In Figure 3, this entails that $x_1^r \geq \cdots \geq x_{n_r}^r$ and $x_1^b \geq \cdots \geq x_{n_b}^b$ for act X and analogous monotonicity for act Y. Note that the index $i \in \{1,2, \ldots, n_\omega\}$ in Figure 3 denotes the location of the i^{th} column vector in each state ω's frame.

Figure 3. A Generic Frame under Ambiguity

	Red Ticket State									Blue Ticket State							
X	x_1^r	p_1^r	...	x_i^r	p_i^r	...	$x_{n_r}^r$	$p_{n_r}^r$		x_1^b	p_1^b	...	x_i^b	p_i^b	...	$x_{n_b}^b$	$p_{n_b}^b$
Y	y_1^r	q_1^r	...	y_i^r	q_i^r	...	$y_{n_r}^r$	$q_{n_r}^r$		y_1^b	q_1^b	...	y_i^b	q_i^b	...	$y_{n_b}^b$	$q_{n_b}^b$

3.1 Salience-Weighted Utility of Presentations (SWUP) Derived from SEU

Given the notion of a frame as a matrix representation of state-contingent lotteries, we can model the behavior of a frame-sensitive decision maker by developing a computational decision algorithm which operates over frames. To do so, following Leland and Schneider (2016), we start with the SEU model of Anscombe and Aumann (1963).

More generally, index the possible states of the world by $\omega \in \Omega = \{1,2, \ldots, m\}$. Denote ambiguous acts by X and Y, where X assigns lottery $X(\omega)$ with corresponding payoff and probability vectors $(\mathbf{x}^{\boldsymbol{\omega}}, \boldsymbol{p}^{\boldsymbol{\omega}})$ to each state. Likewise, Y assigns lottery $Y(\omega)$ with payoff and probability vectors $(\mathbf{y}^{\boldsymbol{\omega}}, \boldsymbol{q}^{\boldsymbol{\omega}})$ to each state. In the classic alternative-based evaluation model, there is a unique subjective probability distribution π over states (Anscombe and Aumann, 1963) such that X is chosen over Y if and only if

(1) $$\sum_{\omega}^{m}\sum_{i}^{n^{\omega}}\pi^{\omega}[p_i^{\omega}u(x_i^{\omega})] > \sum_{\omega}^{m}\sum_{i}^{n^{\omega}}\pi^{\omega}[q_i^{\omega}u(y_i^{\omega})].$$

We may equivalently rewrite eq. 1 as an attribute-based comparative evaluation model:

(2) $$\sum_{\omega}^{m}\sum_{i}^{n^{\omega}}\pi^{\omega}[(p_i^{\omega}-q_i^{\omega})(u(x_i^{\omega})+u(y_i^{\omega}))/2 + (u(x_i^{\omega})-u(y_i^{\omega}))(p_i^{\omega}+q_i^{\omega})/2] > 0.$$

Leland and Schneider (2016) note that this "attribute-based evaluation computes probability differences associated with outcomes weighted by the average utility of those outcomes plus utility differences of outcomes weighted by their average probability of occurrence." Agents who choose according to eq. 2 will make the same choices as agents who choose according to the SEU model in eq. 1. But drawing on recent work which highlights the role of salience perception in decision making (e.g., Bordalo et al., 2012; Koszegi and Szeidl, 2013), suppose that when comparing state-contingent lotteries, agents focus more on large differences in payoffs or probabilities and systematically overweight them as a consequence. To formalize this intuition, Leland and Schneider place weights $\psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})$ on probability differences and $\psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})$ on payoff differences, yielding a model in which X is strictly preferred to Y if and only if

(3) $$\sum_{\omega}^{m}\sum_{i}^{n^{\omega}}\pi^{\omega}[\psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})(p_i^{\omega}-q_i^{\omega})(u(x_i^{\omega})+u(y_i^{\omega}))/2$$
$$+\psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})(u(x_i^{\omega})-u(y_i^{\omega}))(p_i^{\omega}+q_i^{\omega})/2] > 0.$$

Leland and Schneider (2016) call this representation of preferences "salience weighted utility over presentations" or SWUP: The weights $\psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})$ and $\psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})$ are "salience functions" satisfying two critical properties of salience perception noted in Bordalo et al. (2012; 2013):

Definition 1 (Salience Function): A *salience function* $\psi(a_i, b_i)$ is any (non-negative), symmetric and continuous function that satisfies the following two properties:

1. **Ordering:** If $[a_i', b_i'] \subset [a_i, b_i]$ then $\psi(a_i', b_i') < \psi(a_i, b_i)$.
2. **Diminishing Sensitivity:** for any $a_i, b_i, \epsilon > 0$, $\psi(a_i+\epsilon, b_i+\epsilon) < \psi(a_i, b_i)$.

SWUP explains the Allais paradox framing effect conjectured by Savage. In the transparent frame in Figure 1, a decision maker who acts in accordance with SWUP chooses p over q if and only if she chooses p' over q', consistent with the independence axiom. In contrast, the salience evaluations in the two choice pairs can differ under minimal frames, enabling the model to accommodate the Allais paradox. SWUP not only explains the Allais framing effect but also predicts a novel framing effect in the context of Ellsberg's paradox. We can now apply the SWUP model to demonstrate this prediction.

3.2 The Ellsberg Paradox in Minimal and Transparent Frames

We illustrate SWUP with basic pair 1 from our experiment: Figure 4 shows two minimal frame versions of this pair. Set $u(\$25) = 1$ and $u(\$0) = 0$, and let π^r denote the subjective probability that the true state is red. Then SWUP predicts that A is chosen over B if

$$\pi^r \psi_{\mathrm{P}}(0.5{,}0.75)(-0.25) + (1 - \pi^r)\psi_{\mathrm{P}}(0.5, 0.25)(0.25) > 0.$$

As observed by Leland and Schneider (2016), symmetry and diminishing sensitivity of ψ_{P} imply that $\psi_{\mathrm{P}}(0.5, 0.25) > \psi_{\mathrm{P}}(0.5{,}0.75)$. Thus, under a uniform prior, a SWUP decision maker chooses constant act A over ambiguous option B, and likewise chooses constant act A' over ambiguous option B', for *any* salience function ψ_{P}. Hence, SWUP predicts ambiguity aversion in minimal frames. In the minimal frames of Figure 4, all payoff differences within each column vector are zero, so that behavior under SWUP depends solely on the subjective prior over states and the probability salience function.

Figure 4. The Ellsberg Paradox in Minimal Frames

	Red Ticket State				Blue Ticket State			
A	\$25	0.50	\$0	0.50	\$25	0.50	\$0	0.50
B	\$25	0.75	\$0	0.25	\$25	0.25	\$0	0.75

	Red Ticket State				Blue Ticket State			
A′	\$25	0.50	\$0	0.50	\$25	0.50	\$0	0.50
B′	\$25	0.25	\$0	0.75	\$25	0.75	\$0	0.25

Figure 5 shows two transparent frame versions of basic pair 1: Here the probability differences within each column vector are zero, so behavior is determined solely by the subjective prior and payoff salience. In particular, SWUP now predicts that A is chosen over B if

$$\pi^r \psi_{\mathrm{X}}(0{,}25)(-1) + (1-\pi^r)\psi_{\mathrm{X}}(25{,}0)(1) > 0.$$

However, under a uniform prior over states and by symmetry of ψ_{X}, the left side of this expression is identically zero, so the decision maker is predicted to be indifferent between A and B (and is likewise predicted to be indifferent between A' and B'). Thus, this version of SWUP (derived from SEU) predicts ambiguity aversion in minimal frames and ambiguity-neutrality in transparent frames (for any utility function, and any salience function).

Figure 5. The Ellsberg Paradox in Transparent Frames

	Red Ticket State						Blue Ticket State					
A	\$25	0.50	\$0	0.25	\$0	0.25	\$25	0.25	\$25	0.25	\$0	0.50
B	\$25	0.50	\$25	0.25	\$0	0.25	\$25	0.25	\$0	0.25	\$0	0.50

	Red Ticket State						Blue Ticket State					
A′	\$25	0.25	\$25	0.25	\$0	0.50	\$25	0.50	\$0	0.25	\$0	0.25
B′	\$25	0.25	\$0	0.25	\$0	0.50	\$25	0.50	\$25	0.25	\$0	0.25

3.3 Unifying Frame-Independent and Frame-Sensitive behavior toward Ambiguity

The SWUP model derived above from SEU explains the Allais and Ellsberg paradoxes and predicts that they are sensitive to framing, and does so with one coherent subjective prior over states. It formalizes the intuition of frame-dependent decision making in a simple manner. However, its predictions are too restrictive to accommodate the variety of subject behavior observed in many experiments. For data analysis, we need a version of SWUP that allows for individual differences in ambiguity attitudes independent of frames. We do so by embedding SWUP's comparative form in a simple and well-known model of ambiguity attitudes—the

Hurwicz (1951) optimism criterion—instead of SEU.[3] In the Anscombe-Aumann framework, the Hurwicz criterion evaluates ambiguous acts according to the convex combination of the best and worst-case expected utilities generated by the act across all states of the world: Act X is weakly preferred to act Y if and only if $H(X) \geq H(Y)$, where $H(X)$ is

$$(4) \qquad \alpha \max_{\omega\in\Omega} \sum_i^{n^\omega} p_i^\omega\, u(x_i^\omega) + (1-\alpha) \min_{\omega\Omega} \sum_i^{n^\omega} p_i^\omega\, u(x_i^\omega).$$

We propose an analogous 'Hurwicz-SWUP' criterion that allows for frame-independent heterogeneity in ambiguity attitudes. Let X be 'more ambiguous'[4] than Y. Then under the Hurwicz-SWUP criterion, X is preferred to Y if and only if $S(X,Y) > 0$, where

$$(5) \quad S(X,Y) = \alpha \max_{\omega\in\Omega} \sum_i^{n^\omega} [\frac{\psi_{\mathrm{P}}(p_i^\omega,q_i^\omega)(p_i^\omega-q_i^\omega)\big(u(x_i^\omega)+u(y_i^\omega)\big)}{2} + \frac{\psi_{\mathrm{X}}(x_i^\omega,y_i^\omega)\big(u(x_i^\omega)-u(y_i^\omega)\big)(p_i^\omega+q_i^\omega)}{2}]$$

$$+ (1-\alpha) \min_{\omega\in\Omega} \sum_i^{n^\omega} [\frac{\psi_{\mathrm{P}}(p_i^\omega,q_i^\omega)(p_i^\omega-q_i^\omega)\big(u(x_i^\omega)+u(y_i^\omega)\big)}{2} + \frac{\psi_{\mathrm{X}}(x_i^\omega,y_i^\omega)\big(u(x_i^\omega)-u(y_i^\omega)\big)(p_i^\omega+q_i^\omega)}{2}].$$

The above formulation computes a weighted average of the best-case and worst-case SWUP comparisons between the more and less ambiguous acts. This formulation decomposes behavior toward ambiguity into a frame-independent ambiguity attitude, α, and a frame-dependent component determined by salience functions $\psi_{\mathrm{P}}(p_i^\omega, q_i^\omega)$ and $\psi_{\mathrm{X}}(x_i^\omega, y_i^\omega)$ and the frame of the decision. Proposition 1 below follows from eq. 5 (Section A4 of our Appendix shows this).

Proposition 1: Let $\succ$ ($\sim$) denote strict preference (indifference) as determined by the Hurwicz-SWUP criterion in eq. 5. For the choice situations shown in Figures A1 and A2 of Appendix Section A4 (Figures 4 and 5 are examples), with constant act Y and ambiguous act X:

(i) If $X \sim Y$ in the minimal frame then $X \succ Y$ in the transparent frame.

(ii) If $X \sim Y$ in the transparent frame then $Y \succ X$ in the minimal frame.

[3] Another SWUP variant would add a probability $\vartheta \in [0,1]$ that the agent naturally re-frames transparent presentations as minimal ones. This is particularly plausible if people naturally think in minimal frames. ϑ then governs the strength of the framing effect for that agent (agents with $\vartheta = 1$ are frame insensitive and agents with $\vartheta = 0$ conform to SEU in transparent frames but exhibit ambiguity aversion in minimal frames). This SWUP variant accommodates reduced ambiguity aversion (without requiring ambiguity neutrality) in transparent frames, but still rules out ambiguity seeking behavior. Hurwicz-SWUP allows any ambiguity attitude, but requires less ambiguity aversion (or more ambiguity seeking) in transparent frames than in minimal frames.

[4] While there is not yet a general consensus for ranking all pairs of ambiguous acts by their level of ambiguity, one natural approach is given by the family of f-divergences which measures the distance between two probability distributions. In Section A3 of our Appendix we show that two well-known f-divergences – the Hellinger distance (Hellinger, E., 1909; Sengar, 2009) and the total variation distance (Levin et al., 2009) predict the same ranking of ambiguous acts for each of the basic pairs in our experimental design.

Thus Hurwicz-SWUP makes the comparative statics prediction that subjects will be less ambiguity averse in transparent frames than in minimal frames for appropriately constructed choice situations shown in Appendix A4: We test this prediction.

4. The Experiment

Within the experiment, and henceforth, we use the layman's term "options" instead of the theorist's term "acts." Our experiment consists of $j = 1, 2, \ldots, 11$ "basic pairs" of options, where each pair involves a choice between a more ambiguous and a less ambiguous option. Repeated trials of each basic pair, with variations of presentation, create a total of $s = 1, 2, \ldots, 60$ choice situations presented to each subject. In each situation, subjects chose just one of three responses: "I prefer Option A," "I prefer Option B," or "I am indifferent between Option A and Option B." Table 1 presents the basic pairs and all variations of them. After a subject made all 60 choices, she drew a card from a deck of cards numbered from 1 to 60, selecting the subject's chosen option from one situation for payment. (If she chose indifference in that situation, the experimenter flipped a coin to choose either option A or option B for the subject.) Uncertainty in the chosen option was then resolved in two stages. In the first stage, the subject drew a ticket from a opaque bag containing ten paper raffle tickets in an unknown mixture of red and blue tickets. As shown in Table 1, the drawn ticket color determined a lottery to be played out. In the second stage the subject rolled a twelve-sided die to determine her payment from the lottery. This payment was $25 or $0 ($11.01 averaged across the 79 subjects)[5] which, when added to a flat $15 participation fee, yielded average subject earnings of $26.01.

The subjects were seventy-nine[6] undergraduate students at a U.S. university.[7] Subjects were seated in visually isolated carrels in a laboratory. From the beginning to the end of each experimental session, each subject had an opaque bag hanging in the corner of his or her carrel. Subjects were truthfully told that each bag contained an unknown mixture of ten red and/or blue

[5] Consider a hypothetical noiseless subjective EV maximizer Bob with equal priors over ticket colors: His expected probability of receiving $25 in our design would have been 0.518056 (and otherwise zero). Now assume a sample of 79 Bobs: Simulation of a million such samples show that average earnings of those 79 Bobs will exceed $11.01 in 93% of those samples. Now consider a hypothetical random chooser Ted: His probability of receiving $25 would be 0.492361 in our design, and the average earnings of 79 Teds will exceeded $11.01 in 84% of samples.

[6] The planned sample was 80 subjects. One subject failed to show for the final session.

[7] Each of the five experimental sessions lasted approximately 90 minutes (in keeping with recruitment promising that sessions would be less than two hours).

Table 1. Summary of Experimental Design Pair Variations

	Option A				Option B				Trials of each basic pair		
	Red ticket		Blue ticket		Red ticket		Blue ticket		Minimal frame		Transparent frame
basic pair #	$25	$0	$25	$0	$25	$0	$25	$0	A on top	B on top	A on top
1	1/2	1/2	1/2	1/2	3/4	1/4	1/4	3/4	2	1	1
2	1/2	1/2	1/2	1/2	1	0	0	1	1	0	1
3	2/3	1/3	2/3	1/3	1	0	1/3	2/3	2	1	1
4	1/3	2/3	1/3	2/3	2/3	1/3	0	1	2	0	1
5	1/2	1/2	1/2	1/2	1	0	1/4	3/4	2	1	0
6	1/2	1/2	1/2	1/2	1	0	1/3	2/3	3	0	0
7	1/3	2/3	1/3	2/3	1/2	1/2	0	1	2	1	0
8	2/3	1/3	1/3	2/3	1	0	0	1	2	0	0
9	2/3	1/3	1/2	1/2	1	0	1/3	2/3	1	0	0
10	1/2	1/2	1/3	2/3	1	0	0	1	2	1	0
11	1/2	1/2	1/3	2/3	3/4	1/4	0	1	1	1	0

Notes: The first eight columns show the state-contingent lotteries associated with each of the two ticket color states within each pair of options. Fractions below each outcome in each state-contingent lottery are outcome probabilities. The total trials in each variation, shown in the right three columns, sum to thirty choice situations: For each one of these situations, there was a corresponding situation with the options' state-contingent lottery assignment of red and blue ticket colors reversed. This totals sixty choice situations.

raffle tickets, and that the mixture could differ across their bags. Subjects were never permitted to look in their bag, and made one blind draw from their bag at the end of their session.[8]

Experimental instructions were read aloud to subjects while they followed along in their own copies of the instruction booklet. Figure 2 (from Section 2) shows Basic Pair 1 in minimal and transparent frames, exactly as these were presented to subjects in the experiment. As shown in the figure, both minimal and transparent frames were monotonic in that payoffs decreased (weakly) monotonically from left to right. All presentations used the same table format with the column "N/12" denoting the number of die rolls (from a twelve-sided die) yielding the payoff in the column to the left: Die rolls corresponding to each payoff increased from left to right.[9] After explaining all facts concerning the decision representation, subjects were quizzed for their understanding of how random events (ticket draws and die rolls) would determine payouts given

[8] For interpretation of results and estimation we assume that any prior probabilities subjects place on the red and blue ticket states are constant across their choice situations. Our placement of the bags with the subjects, from the start to the finish of their session, is meant to make this assumption plausible.

[9] For instance, in Option B in Basic Pair 1, any die roll between 1 and 9 paid $25 and any die roll between 10 and 12 paid $0 if a red ticket was drawn. Likewise, any die roll between 1 and 3 paid $25 and any die roll between 4 and 12 paid $0 if a blue ticket was drawn.

choices: Subjects' answers were individually checked, and any errors explained to them. Then subjects were quizzed once more, and any errors (very rare at that point) were again individually explained to each subject. An attendant then read a final overview of the events that would take place during the session, and the session commenced as described above.

4.1 Explanation of Design Features and Assumptions

For every choice situation with the assignment of lotteries to states as shown in the eight left-hand columns of Table 1, there was a corresponding choice situation with this assignment of lotteries to states (ticket colors) reversed. This counter-balancing serves two purposes. First, it helps neutralize any suspicion a subject might have that the contents of their ticket bag is 'rigged' to minimize experimenter payout. Second, the counter-balancing of lotteries to states is needed to infer whether the subject acts as if she assigns coherent probabilities to the red ticket and blue ticket states (in the same manner Ellsberg's two-color paradox tests SEU). For instance, in pair 1 an SEU agent who prefers B to A when the preferred lottery in Option B (the 75% chance of winning $25) is assigned to the red ticket state, is acting as-if her subjective probability of the red ticket state is greater than 0.50. The same agent should then prefer A to B when the preferred lottery in Option B is instead assigned to the blue ticket state.

In Table 1, basic pairs $j = 1, 2, 3$ and 4 are Ellsberg-style choices in that (i) they involve a choice between a constant act A (which yields the same lottery regardless of the state) and an ambiguous act B (which assigns different lotteries to different states) and (ii) both options in each of these pairs have the same expected payout if the decision maker assigns a coherent uniform prior over states. These four basic pairs are our focus: 26 of the 60 situations s are trials of these pairs (18 minimal frame trials and 8 transparent frame trials). Section A2 of our Appendix shows that, for basic pairs 1 to 4, the Hurwicz-SWUP model predicts decreased ambiguity aversion in transparent (versus minimal) frames. Choice pairs 5 through 11 are only presented in minimal frames: These provide extra information needed for structural estimation[10]

[10] In the econometrics of risk and uncertainty, structural estimation at the individual subject level benefits strongly from choice problems which challenge the boundaries of each subject's attitudes toward risk and/or ambiguity. Thus the inclusion of a range of choice problems, some that tempt even relatively ambiguity-averse subjects to choose the ambiguous option (e.g. Option B in basic pair 6) add significant information concerning preferences. Minimal frames predominate in our design. This enables the "generalization criterion" analysis we perform below in Section 5.3. Additionally, note that transparent frames provide no information about probability salience functions ψ_{P}: In other work based on these data, we plan to test properties of these particular salience functions.

and act as spacing trials between repeated trials of the central pairs 1 to 4. As shown in Table 1, there were two possible outcomes for each subject: They could receive either \$25 or \$0. Restricting payouts to two possible outcomes, as in Ellsberg's paradoxes, allows us to test some hypotheses without knowledge of risk attitudes.[11]

Especially in Basic Pairs 1, 2, 3 and 4, we wish to interpret the indifference response, which is a 50:50 randomization between options A and B, as actual indifference between them. To do so, we need the "certainty betweenness" property described by Grant and Polak (2013). Using our experimental term "options" rather than the more usual term "acts," we can state this property as follows. Let Ω denote the set of states ω and let Z denote the set of outcomes z. An objective lottery is a known probability distribution p on Z. Denote the set of objective lotteries by $\mathcal{P}(Z)$. An option, X, is a mapping $X: \Omega \to \mathcal{P}(Z)$ which assigns an objective lottery $X(\omega)$ to each state ω. A "constant option" K assigns the same objective lottery to every state, and an "ambiguous option" assigns distinct objective lotteries to at least two states. Denote the set of all options by $\mathbb{O}$. Certainty betweenness assumes that indifference between a constant option K and any other option X implies indifference between the constant option and any probabilistic mixture of the constant option and the other option:

Axiom (Certainty Betweenness): For any option $X \in \mathbb{O}$, and any constant option $K \in \mathbb{O}$, and any $\delta \in (0,1)$: $X \sim K \;\Rightarrow \alpha X + (1-\alpha)K \sim K$.

In the presence of their other axioms, Certainty Betweenness is implied by the certainty independence axiom Gilboa and Schmeidler (1989) assumed in their multiple priors model. The subset of Grant and Polak's monotone mean-dispersion preferences which satisfy certainty betweenness and Gilboa and Schmeidler's (1989) uncertainty aversion axiom is the class of multiple priors preferences (Grant and Polak 2013, p. 1369, Corollary 3).

Certainty betweenness can be applied to basic pairs 1 through 7 in Table 1 since they involve a choice between a constant option (option A) and an ambiguous option (option B). Since our focus is on basic pairs 1 through 4, we will treat certainty betweenness as a maintained hypothesis throughout our study. In our data analyses, we therefore restrict attention to theories

[11] Computation of ambiguity premia requires knowledge of von Neumann and Morgenstern (vNM) functions, but below we find that some interesting features of ambiguity premia are fairly robust to curvature of vNM functions.

of ambiguity aversion which satisfies certainty betweenness: In particular, our structural econometrics compares Hurwicz-SWUP to both Hurwicz preferences and the "mean-standard deviation" or MSD preferences described in Grant and Kajii (2007) and Grant and Polak (2013).[12] In Section A2 of our Appendix we demonstrate that SWUP satisfies certainty betweenness if the salience functions exhibit homogeneity of degree 0, a property that Bordalo et al. (2013) argue is plausible for a salience function and which they invoke in their analysis of salience effects in consumer choice. Since the framing effect between minimal and transparent frames is predicted under general conditions by SWUP (for any salience function) it also holds for the class of salience functions exhibiting homogeneity of degree zero.

Random selection of just one of each subject's several choices by means of a random device (such as a draw from a card deck) goes under various names: We call it random task selection. Currently, we think the balance of experimental evidence suggests that when each of several tasks is presented separately (on its own page of a booklet, or on its own computer screen—this is important), random task selection produces incentive-compatible choices.[13] Brown and Healy (2018) discuss existing evidence and their own new experimental evidence supporting this claim.

5. Results

Our experimental design varied the assignment of ticket colors to objective lotteries to better test SEU; additionally, we also vary whether the more ambiguous act is in the top or bottom row of the choice table displayed to subjects (to check for response set), and the order in which the choice situations are presented to subjects (to check for learning and/or fatigue). Appendix A1.2 describes an initial estimation that checks on these matters. We find no significant evidence of row placement of options or situation order, so we proceed ignoring these things. The assignment of ticket color to objective lotteries has no systematic effect across subjects. This does not mean all individual subjects have equal subjective priors of each ticket state: Different subjects could

[12] MSD preferences include many other preferences. Grant and Polak show that since the standard deviation dispersion function is non-negative, convex, symmetric, and satisfies certainty betweenness, MSD preferences have a corresponding representation in the vector expected utility model (Siniscalchi, 2009), the invariant biseparable representation (Ghirardato et al., 2004), and the multiple prior representation (Gilboa and Schmeidler, 1989) provided that the mean-standard deviation preferences are monotone (a property we impose on our estimations of MSD preferences, though this monotonicity constraint rarely binds at the level of individual subjects).

[13] By "incentive-compatible" we mean that the subject's choice in any one of the several tasks will be equivalent to the choice the subject would have made in that task if that task had been the sole task presented to the subject.

still believe that a red (or blue) ticket draw is more likely than blue (or red). So where we estimate any structural model containing subjective priors we will still estimate those priors.

Section 5.1 describes results for pairs 1, 2, 3 and 4, plotting the data and performing very simple statistics, to compare results in minimal and transparent frames. Section 5.2 estimates an aggregate Hurwicz model of the data, allowed to depend on frames, to contrast "ambiguity premia" in transparent and minimal frames. We then perform disaggregated, subject-by-subject econometrics in Section 5.3, showing that the Hurwicz-SWUP model outperforms two alternatives (Hurwicz criterion and MSD) in predicting choices in transparent frames using only choices in minimal frames.

5.1 Establishing the Treatment Effect: Minimal Versus Transparent Frames.

For the purpose of comparing subjects' choice behavior between minimal and transparent frames in basic pairs $j = 1$ to 4, define a dependent variable c_{lj}^e taking values

$$c_{lj}^e = \begin{cases} 1 & \text{if subject } e \text{ chose the constant option in trial } l \text{ of pair } j; \\ 0.5 & \text{if subject } e \text{ chose indifference in trial } l \text{ of pair } j; \text{ and} \\ 0 & \text{if subject } e \text{ chose the ambiguous option in trial } l \text{ of pair } j. \end{cases}$$

Empirically, given trials l satisfying some condition $\mathbb{C}$, we take ambiguity neutrality to mean that $E\left(c_{lj}^e|\mathbb{C}\right) = 0.5$ in basic pairs $j = 1$ to 4. Similarly, we take $E\left(c_{lj}^e|\mathbb{C}\right) > 0.5$ and $E\left(c_{lj}^e|\mathbb{C}\right) < 0.5$ to mean ambiguity aversion and ambiguity seeking, respectively, given condition $\mathbb{C}$. Let M and T denote the sets of minimal and transparent frame trials l of pairs $j = 1$ to 4, respectively. The Hurwicz-SWUP prediction is that the framing effect $F^e = E\left(c_{lj}^e|lj \in M\right) - E\left(c_{lj}^e|lj \in T\right) > 0$.

The sample analogues of the expectations in F^e are $\bar{c}_M^e = \sum_{lj \in M} c_{lj}^e/18$ and $\bar{c}_T^e = \sum_{lj \in T} c_{lj}^e/8$, yielding an estimate $\hat{F}^e = \bar{c}_M^e - \bar{c}_T^e$ of F^e for each subject e. Figure 6 plots pairs $(\bar{c}_M^e, \bar{c}_T^e)$ for the 79 subjects ($\hat{F}^e > 0$ for pairs below the 45 degree line), and Figure 7 shows the cumulative sample distribution of $\hat{F}^e$: By a sign test (and other applicable tests),[14] this distribution's location is easily statistically different from zero. The Hurwicz-SWUP prediction easily holds.

[14] For every sign test result reported in this article, we also calculated results of the Wilcoxon signed rank test and a paired sample t-test: Those two tests always yield still smaller *p*-values than the sign tests do.

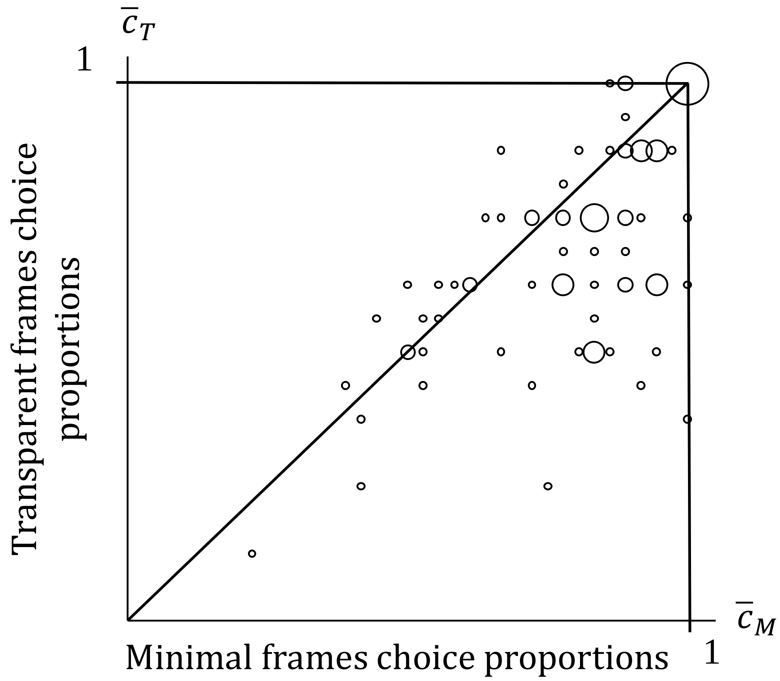


Figure 6. Minimal and transparent frame choice proportions, pairs 1 to 4.
Notes: Smallest (largest) bubbles are one (six) subjects.

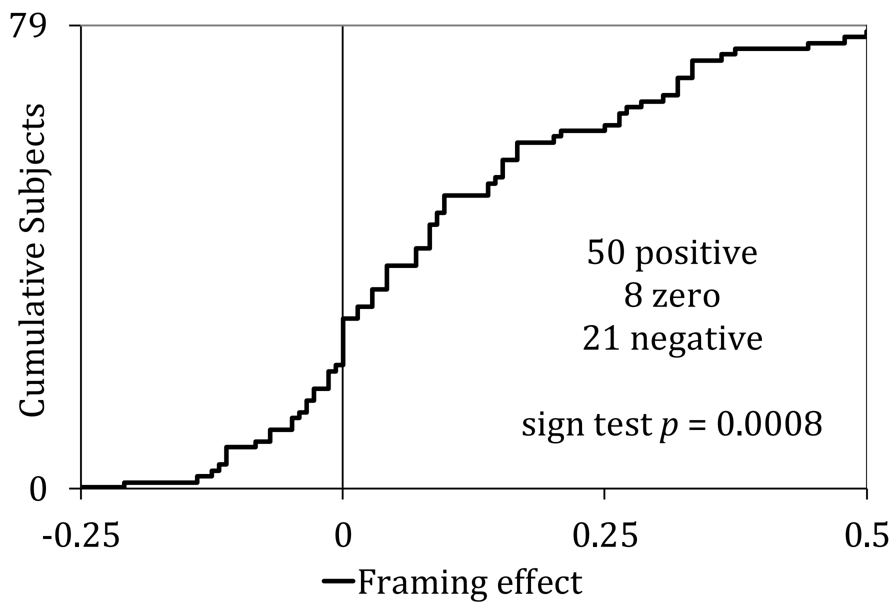


Figure 7. Cumulative distribution of estimated framing effects $\hat{F}^e$.

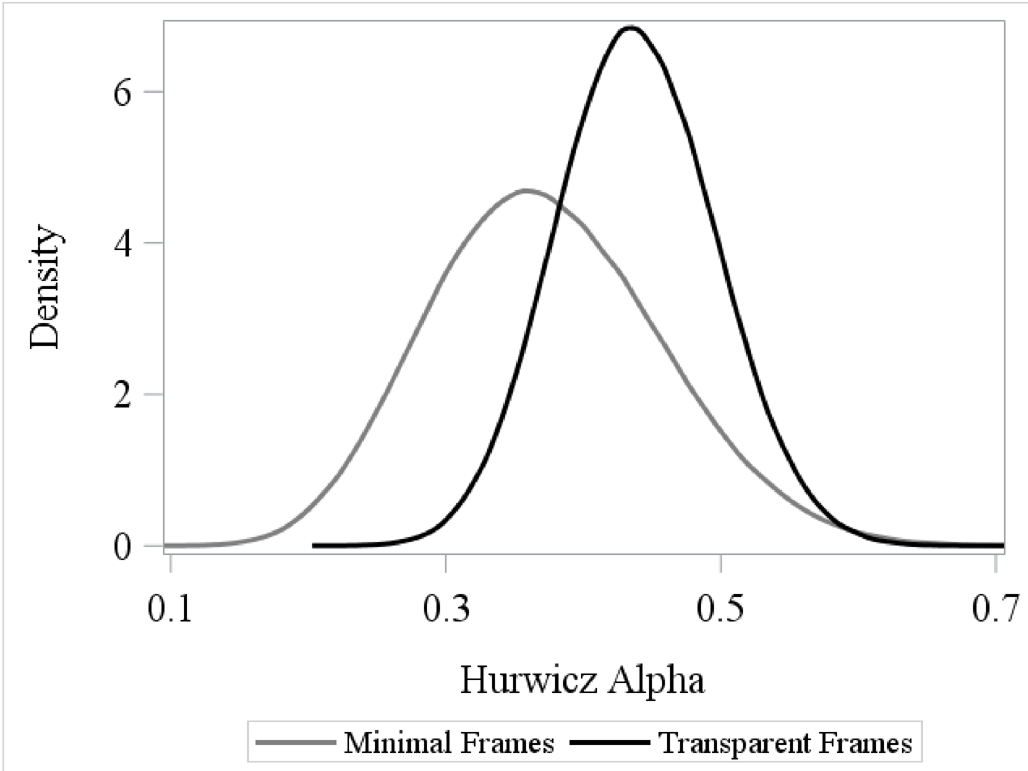


Figure 8. Estimated distributions of Hurwicz α in minimal and transparent frame trials: Random parameters estimation using the Hurwicz criterion representation.

5.2 Comparing Ambiguity Aversion and Premia Across Minimal and Transparent Frames

Structural estimation—that is, estimation of parameters found in decision-theoretic representations—provides another quantification of differences in ambiguity aversion across the two types of frames. We do this using eq. 4, the simple Hurwizc criterion, with a parameterization allowing the Hurwicz α to vary across the two types of frames and across subjects as well. Appendix section A1.3 discusses this random parameters estimation of the distribution of the Hurwicz α parameter in our sampled population, which is based on subjects' choices in all sixty choice situations. We estimate mean values of α equal to 0.370 (standard error 0.011) and 0.438 (standard error 0.010) in minimal and transparent frame choices, respectively. The random parameters method also produces estimates of the distributions of α conditional on frame type: Figure 8 shows these two estimated distributions.

Our experiment uses just two outcomes ($25 and $0) in all options, so we cannot estimate curvature of subjects' underlying vNM (von Neumann and Morgenstern, or Bernoulli) utilities of outcomes: Therefore estimates of cash equivalents of options is not possible. However, we can report some interesting characteristics of cash equivalents that hold across a wide range of utility curvatures (including those most scholars take seriously). In particular, the percentage increase in ambiguity premia due to minimal frames (versus transparent frames) is almost certainly quite substantial and fairly insensitive to assumed curvature of vNM utilities.

Using the Hurwicz criterion, implicitly define cash equivalents $CE(X|\alpha)$ of options X as

(6) $$u[CE(X|\alpha)] = \alpha \max_{\omega\in\Omega} \sum_i^{n^\omega} p_i^\omega\, u(x_i^\omega) + (1-\alpha) \min_{\omega\in\Omega} \sum_i^{n^\omega} p_i^\omega\, u(x_i^\omega).$$

Note that $\alpha = 0.5$ has special status. It weighs the best and worst expected utilities equally of course, but when there are just two states (as in all our options) and $\alpha = 0.5$, the Hurwicz criterion is mathematically identical to SEU with uniform priors over the two states. This implies that when there are just two states, $CE(X|0.5)$ is the cash equivalent of an ambiguity neutral SEU agent who regards the two states as equally likely. Therefore, we define an ambiguity premium $\varpi(X|\alpha) = CE(X|0.5) - CE(X|\alpha)$ that a Hurwicz criterion agent (with optimism parameter α) attaches to ambiguous option X.[15]

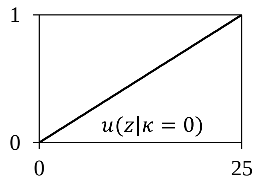
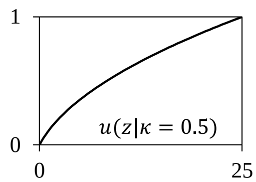
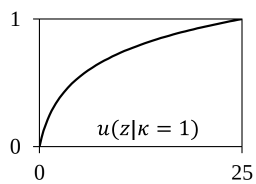
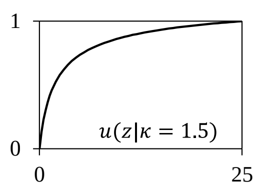
Table 2 shows this ambiguity premium for ambiguous option B in basic pairs 1 to 4, given our estimated mean values of α in minimal and transparent frame choices, and assuming four different amounts of vNM utility curvature. This table also shows the difference $\varpi(B|0.370) - \varpi(B|0.438)$ between the estimated ambiguity premia in minimal and transparent frame choices and a quantity we call the "minimal frame markup" $100 \cdot [\varpi(B|0.370) - \varpi(B|0.438)]/\varpi(B|0.438)$, our estimated percentage increase in ambiguity premia between minimal and transparent frames. Table 2 shows that although the ambiguity premia themselves vary quite a bit with changes in vNM utility curvature, the minimal frame markup stays in a reasonably small neighborhood of 100%. Regardless of vNM curvature, changing from transparent to minimal frames roughly doubles our subjects' estimated ambiguity premium for ambiguous options.

5.3 Predicting Transparent Frame Behavior from Minimal Frame Behavior

The Hurwicz-SWUP representation motivated the predictions we tested in Section 5.1 concerning differences between behavior in minimal and transparent frames. There is another

[15] Let $SEV(X)$ denote the subjective expected value of option X under equal priors $\pi^r = \pi^b = 0.5$. Define the total premium $SEV(X) - CE(X|\alpha)$ and the risk premium $SEV(X) - CE(X|0.5)$. Our definition of the ambiguity premium simply decomposes the total premium into the sum of the risk premium and ambiguity premium.

Table 2. Ambiguity Premia in Minimal and Transparent Frames (Basic Pairs 1-4)

		Option B in Pair $j = \ldots$			
		1	2	3	4
	ambiguous option subjective expected value[a]	\$12.50	\$12.50	\$16.67	\$8.33
vNM utility curvature[b]					
					
	ϖ (minimal frames)	\$1.63	\$3.26	\$2.17	\$2.17
	ϖ (transparent frames)	\$0.78	\$1.55	\$1.04	\$1.04
	difference	\$0.85	\$1.71	\$1.13	\$1.13
	minimal frame markup	109%	110%	109%	109%
					
	ϖ (minimal frames)	\$1.56	\$2.97	\$2.53	\$1.56
	ϖ (transparent frames)	\$0.76	\$1.49	\$1.24	\$0.77
	difference	\$0.80	\$1.48	\$1.29	\$0.79
	minimal frame markup	105%	99%	104%	103%
					
	ϖ (minimal frames)	\$0.98	\$1.76	\$2.16	\$0.73
	ϖ (transparent frames)	\$0.49	\$0.93	\$1.11	\$0.37
	difference	\$0.49	\$0.83	\$1.05	\$0.36
	minimal frame markup	100%	89%	95%	97%
					
	ϖ (minimal frames)	\$0.43	\$0.77	\$1.14	\$0.31
	ϖ (transparent frames)	\$0.22	\$0.41	\$0.60	\$0.16
	difference	\$0.21	\$0.36	\$0.54	\$0.15
	minimal frame markup	95%	88%	90%	94%

Notes: [a]Under the assumption of equal subjective priors ($\pi^r = \pi^b = 0.5$). [b]The assumed utility of outcomes is $u(z|\kappa) = \left[-1 + (1+z)^{(1-\kappa)}\right] / \left[-1 + 26^{(1-\kappa)}\right]$ for $\kappa \neq 1$ and $u(z|\kappa) = \ln(1+z)\,/\ln(26)$ at $\kappa = 1$: This maps the outcome range [\$0,\$25] onto the unit interval. This is a HARA (hyperbolic absolute risk aversion) utility function: For $\kappa > 0$, it exhibits declining absolute risk aversion but increasing relative risk aversion. This utility function has the property that $u(0|\kappa) = 0 \;\forall\; \kappa \in \mathbb{R}$, a property not shared by the CRRA (power) utility functions, as is required for SWUP, RDU and CPT.

way to test such predictions known as the "generalization criterion" (Busemeyer and Wang 2000). This test estimates parameters of two or more preference representations using only minimal frame observations and uses those estimates to predict transparent frame observations. Here we will index the dependent variable defined in Section 5.1 by situations s instead of trials and pairs (writing c_s^e instead of c_{ij}^e). Let $\mathcal{S} \in \{\mathcal{M}, \mathcal{T}\}$ index the two mutually exclusive and exhaustive subsets $\mathcal{M}$ (minimal frame situations) and $\mathcal{T}$ (transparent frame situations) of the sixty experimental situations s. Let $\tau \in \{H, HS, MSD\}$ index the theory representations (Hurwicz criterion, Hurwicz-SWUP, and Mean-Standard Deviation, respectively) we examine here; let θ^{τ} be the parameter vector of each theory representation τ; and let $\hat{\theta}_{\mathcal{M}}^{\tau,e}$ be an estimate of θ^{τ} based only on subject e's choices in the fifty-two minimal frame situations $s \in \mathcal{M}$. We estimate $\hat{\theta}_{\mathcal{M}}^{\tau,e}$ by maximizing subject e's log likelihood $\mathcal{L}_{\mathcal{M}}^{\tau,e}(\theta^{\tau})$ in a choice of θ^{τ} specific to each subject e. We refer to $\hat{\theta}_{\mathcal{M}}^{\tau,e}$ as the "in-sample estimate" and $\mathcal{L}_{\mathcal{M}}^{\tau,e}\left(\hat{\theta}_{\mathcal{M}}^{\tau,e}\right)$ as the "in-sample fit" for subject e, given theory τ. Appendix A1.3 discusses details of this estimation.

The generalization criterion computes the "out-of-sample fit" $\mathcal{L}_{\mathcal{T}}^{\tau,e}\left(\hat{\theta}_{\mathcal{M}}^{\tau,e}\right)$ using the in-sample estimate and compares theories τ_1 and τ_2 by the difference between their out-of-sample fits: That is, the generalization criterion for subject e is $G^e(\tau_1, \tau_2) = 2\left[\mathcal{L}_{\mathcal{T}}^{\tau_1,e}\left(\hat{\theta}_{\mathcal{M}}^{\tau_1,e}\right) - \mathcal{L}_{\mathcal{T}}^{\mathcal{Y},e}\left(\hat{\theta}_{\mathcal{M}}^{\mathcal{Y},e}\right)\right]$. Figures 9 and 10 show the cumulative distributions of $G^e(HS, H)$ and $G^e(HS, MSD)$, respectively, across our seventy-nine subjects. For comparison we also show cumulative distributions of $DAIC^e(HS, H)$ and $DAIC^e(HS, MSD)$ in the two figures. These are "in-sample fit" comparisons measured by differences between the Akaike (1973) Information Criterion: $DAIC^e(\tau_1, \tau_2) = 2\left[\mathcal{L}_{\mathcal{M}}^{\tau_1,e}\left(\hat{\theta}_{\mathcal{M}}^{\tau_1,e}\right) - \mathcal{L}_{\mathcal{M}}^{\tau_2,e}\left(\hat{\theta}_{\mathcal{M}}^{\tau_2,e}\right)\right] - 2\Delta k^{\tau_1,\tau_2}$, where $\Delta k^{\tau_1,\tau_2}$ is the difference between the number of parameters in θ^{τ_1} and θ^{τ_2}.[16] Judged by $DAIC^e$ (in-sample fit for minimal frame observations) there is no statistically significant difference between the Hurwicz-SWUP criterion and either the Hurwicz criterion or Mean-Standard Deviation preferences. However, the Hurwicz-SWUP criterion clearly outperforms these two competitors in out-of-sample prediction according to the generalization criterion.

6. Conclusions

Motivated by a new model of ambiguity aversion and by the success of Savage's conjecture in predicting the frame-dependence of the Allais paradox for choice under risk, we tested for an influence of framing on Ellsberg's paradox in decisions under uncertainty. We observed a highly significant framing effect in the direction predicted by the Hurwicz-SWUP criterion motivated by Leland and Schneider's (2016) SWUP criterion.

[16] One sometimes thinks of $\Delta k^{\mathcal{X},\mathcal{Y}}$ as a penalty for relative lack of parsimony. More accurately, it is due to a difference in degrees of freedom lost because the very observations used to compute fit were also used to estimate parameters. This explains why the generalization criterion has no such "penalty:" Observations used to calculate fit (those in $\mathcal{T}$) were not used to estimate $\theta^{\mathcal{R}}$ (that estimation used only observations in $\mathcal{M}$).

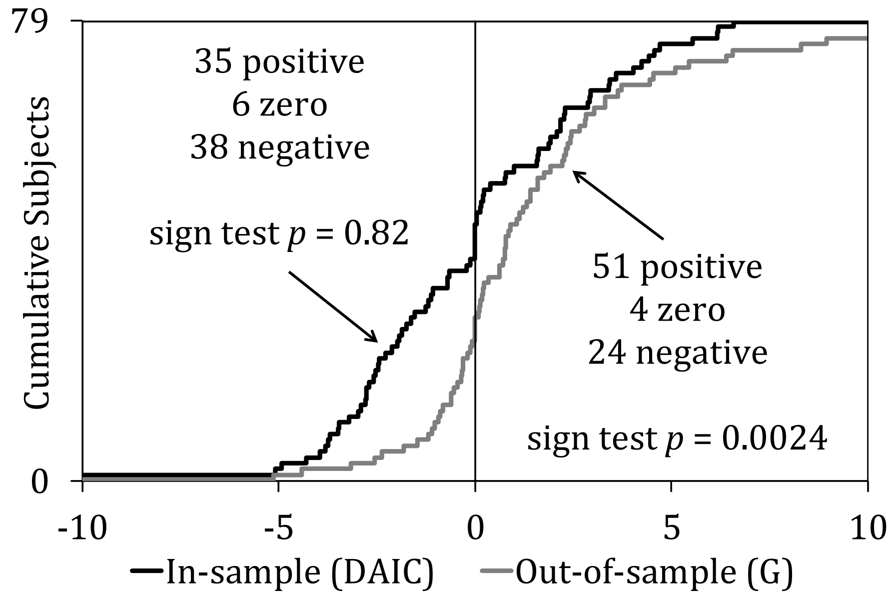


Figure 9. In-sample and out-of-sample fit comparison between Hurwicz-SWUP and Hurwicz criterion representations.

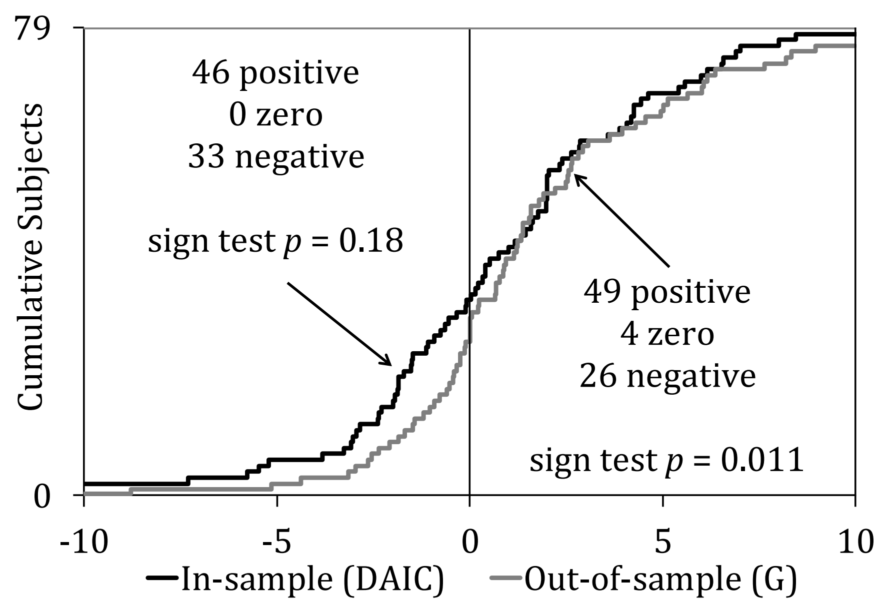


Figure 10. In-sample and out-of-sample fit comparison between Hurwicz-SWUP criterion and Mean-Standard Deviation representations.

One important question warranting further investigation concerns the precise locus of the treatment effect. Our hypothesis is that minimal frames hamper "true" preference expression (while transparent frames do not): Transparent frames make common consequences of all options clearly visible, focus attention on remaining differences between options, and therefore enhancing the sure-thing principle's descriptive drawing power. An opposite hypothesis, based on Chew et al. (2017), is that subjects are "truly" ambiguity averse and transparent frames hamper their recognition of ambiguity (while minimal frames do not). Put differently, it might be that our framing effect does not decrease ambiguity aversion, but rather increases the noise inherent in the decisions of subjects who are truly ambiguity-averse. Using econometric modeling similar to that discussed in Sections A1.1 and A1.3 in our Appendix, we explored this and think those results suggest that transparent frames do not increase decision noise.[17]

As noted in Section 2, a variety of fairly recent studies have investigated whether the Allais paradox is susceptible to framing. All of these studies (Leland, 2010; Bordalo et al., 2012; Incekara-Hafalir and Stecher, 2012; Birnbaum and Schmidt, 2015; Harman and Gonzalez, 2015) find significantly fewer violations of the independence axiom of expected utility theory, when the lotteries are recast from minimal frames (i.e., the standard 'prospect' presentation format) to transparent frames (i.e., the Savage matrix format). While the Ellsberg paradox violates a similar independence postulate, no such experiment has been conducted for ambiguity attitudes. In the present experiment, we find that the same types of frames which reduce Allais-type violations of objective expected utility theory also reduce ambiguity aversion in Ellsberg-type decision situations. This is in keeping with Leland and Schneider's (2016) formalization of different kinds of frames and the decision algorithms that may operate on those frames.

[17] This long analysis cannot be include here (the third author will provide it on request). Its essence is to allow both parameters governing attitude toward ambiguity, and probabilistic model parameters governing the noisiness of decisions, to depend on whether a situation is in a minimal or transparent frame. We did this using the Mean-Standard Deviation theory, both with and without random parameter controls for heterogeneity across subjects. No specification uncovers significant increases in the noisiness of our subjects' decisions in transparent frames and, in all of those specifications, we find a significant decrease in their ambiguity aversion in transparent frames.

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Appendix

A1. Econometric Analysis

A1.1 The Probability of Indifference Responses

Recall from Section 3 that in each choice situation s = 1 to 60, subjects may choose either Option A or Option B from a pair shown in Table 1, or may report indifference (which is resolved by a coin flip). Denote these three alternatives in situation s by $sk, k \in \{A, B, \sim\}$, and let $v_{sk}^{\tau} \in \{v_{sA}^{\tau}, v_{sB}^{\tau}, v_{s\sim}^{\tau}\}$ denote their values according to some deterministic theory τ's representation theorem. Then a Luce model of choice probabilities Q_{sk}^{τ} would be

(A1) $Q_{sA}^{\tau} = exp(\lambda v_{sA}^{\tau})/D_s^{\tau}$, $Q_{s\sim}^{\tau} = exp(\lambda v_{s\sim}^{\tau})/D_s^{\tau}$, and $Q_{sB}^{\tau} = exp(\lambda v_{sB}^{\tau})/D_s^{\tau}$,

where $D_s^{\tau} = exp(\lambda v_{sA}^{\tau}) + exp(\lambda v_{s\sim}^{\tau}) + exp(\lambda v_{sB}^{\tau})$,

and λ is a scale parameter, sometimes called "precision" or "sensitivity" (as $\lambda \to \infty$ the decision maker chooses the highest value alternative with certainty, and as $\lambda \to 0$ the decision maker chooses each of the three alternatives with one-third probability).

This approach to our data predicts far too many indifference responses. To solve this problem we reinterpret the Q_{sk}^{τ} in eq. A1 as "preference state probabilities"—probabilities that a subject finds herself in one of three "preference states" after choice deliberation (just before she responds). Conditional on being in the "indifference state," we assume she chooses the indifference response with probability ϕ, or chooses either A or B—each with probabilities $(1-\phi)/2$. This allows the subjects' observed probability of indifference responses to be rarer than the Luce model predicts but enforces the compelling notion that an agent in an indifferent state favors neither the A nor B response. However, conditional on being in the "prefer A state" or the "prefer B state," she responds by choosing her preferred option with certainty. Now let P_{sk}^{τ}, $k \in \{A, B, \sim\}$, denote "response probabilities:" Our assumptions yield a one-parameter generalization of the Luce choice model allowing for "rare" indifference responses:

(A2) $P_{sA}^{\tau} = Q_{sA}^{\tau} + 0.5(1-\phi)Q_{s\sim}^{\tau}$; $P_{s\sim}^{\tau} = \phi Q_{s\sim}^{\tau}$; and $P_{sB}^{\tau} = Q_{sB}^{\tau} + 0.5(1-\phi)Q_{s\sim}^{\tau}$.

Equations A1 and A2 are the basis of our probabilistic choice models for our econometrics—with some variations of eq. A1 described below.

A1.2 Checks for Significant Nuisance Variance

Recall that the experimental design features some standard variations meant to check for, and control, several potential artifactual effects: We switched the rows in which options were displayed, and also changed the order in which the pairs were presented. The hope is always that these kinds of variations have no statistically significant impact on choices. We also switched the assignment of ticket colors to states (for reasons described in Section 4). In this section we perform a statistical analysis meant to check whether any of these three variations have unanticipated effects.

Dividing all terms in eq. A1 by $exp(\lambda v_{sB}^{\tau})$ yields this new and useful expression of preference state probabilities:

(A3) $Q_{sA}^{\tau} = exp[\lambda(v_{sA}^{\tau} - v_{sB}^{\tau})]/D_s^{\tau}$, $Q_{s\sim}^{\tau} = exp[\lambda(v_{s\sim}^{\tau} - v_{sB}^{\tau})]/D_s^{\tau}$, and $Q_{sB}^{\tau} = 1/D_s^{\tau}$,

where $D_s^{\tau} \;=\; exp[\lambda(v_{sA}^{\tau} - v_{sB}^{\tau})] + exp[\lambda(v_{s\sim}^{\tau} - v_{sB}^{\tau})] \;+\; 1.$

We always assume that subjects' preferences obey the certainty betweenness axiom (Section 4.1 explained that the experimental design was predicated on this). In basic pairs 1 through 7 (where A is a constant option and B is an ambiguous option) this assumption (along with the experimental design) implies that indifference responses have value $v_{s\sim}^{\tau} = (v_{sA}^{\tau} + v_{sB}^{\tau})/2$. However, options A and B are both ambiguous in basic pairs 8 through 11 so the certainty betweenness axiom does not apply: In those pairs there may be a diversification motive giving indifference responses increased value. Here we approximate this motive by a constant "diversification utility" ζ common to pairs 8 through 11 (in structural estimations the diversification motive is modeled in an explicit theoretical way without ζ). Therefore, in this section's estimation, we have $v_{s\sim}^{\tau} = (v_{sA}^{\tau} + v_{sB}^{\tau})/2 \;+ \zeta$ and $v_{s\sim}^{\tau} - v_{sB}^{\tau} = \;(v_{sA}^{\tau} - v_{sB}^{\tau})/2 \;+ \zeta$ (where δ is zero in pairs 1 through 7). Also let $\Delta v_s^{\tau} \equiv v_{sA}^{\tau} - v_{sB}^{\tau}$, and eq. A3 may then be rewritten to give this modified version of the preference state probabilities:

(A4) $Q_{sA}^{\tau} = exp(\lambda\Delta v_{\tau}^{s})/D_s^{\tau}$, $Q_{s\sim}^{\tau} = exp[\lambda(\Delta v_{\tau}^{s}/2 \;+ \zeta)]/D_s^{\tau}$, and $Q_{sB}^{\tau} = 1/D_s^{\tau}$,

where $D_s^{\tau} \;=\; exp(\lambda\Delta v_s^{\tau}) + exp[\lambda(\Delta v_s^{\tau}/2 \;+ \zeta)] \;+\; 1$,

and $\zeta = 0$ in pairs 1 through 7.

This formulation (along with eq. A2) is appealing since Δv_s^{τ} is simply the difference between the values of Option A and Option B. The estimation in this section simply makes Δv_s^{τ} a linear function of pair indicators and indicators for all experimentally induced sources of variance in responses—along with a random (normally distributed, zero mean) effect to account for subject-specific heterogeneity. As is well-known, scale λ is not separately identifiable in linear latent variable formulations such as this; so here, we set $\lambda = 1$. The results of the first estimation are summarized below in Table A1.1, with interpretations of these results following the table.

Table A1.1 Analysis of Effects of Experimentally Induced Sources of Potential Variance

Meaning of estimated parameters	Estimates	Std. Error[a]	p-value[b]
standard deviation of subject-specific random effects	1.5	0.23	m[c]
A. effect of transparent frames (in pairs 1, 2, 3, and 4)	–1.0	0.20	< 0.0001
B. effect of switching red and blue ticket assignment	0.087	0.16	0.60
C. effect of switching top and bottom row assignment	–0.074	0.16	0.65
D1. order effect—pair presented in first (of three) booklets	–0.069	0.11	0.52
D3. order effect—pair presented in last (of three) booklets	0.0098	0.11	0.93
pair 1 indicator effect	2.3	0.42	< 0.0001
pair 2 indicator effect	2.6	0.44	< 0.0001
pair 3 indicator effect	2.2	0.39	< 0.0001
pair 4 indicator effect	2.1	0.40	< 0.0001
pair 5 indicator effect	–0.71	0.41	0.088
pair 6 indicator effect	–2.9	0.48	< 0.0001
pair 7 indicator effect	4.2	0.52	< 0.0001
pair 8 indicator effect	2.6	0.47	< 0.0001
pair 9 indicator effect	–0.20	0.43	0.64
pair 10 indicator effect	1.0	0.39	0.011
pair 11 indicator effect	4.0	0.46	< 0.0001
Indifference response probability ϕ (all pairs)	0.33	0.069	m[c]
Diversification utility ζ (in pairs 8, 9, 10, and 11)	–0.68	0.25	0.0097

Notes: [a]We use a standard robust “sandwich estimator” to estimate the covariance matrix of parameter estimates. [b]Against the hypothesis that the true coefficient equals zero. [c]The “m” means that a p-value would be misleading in this case, since the natural null hypothesis (that the parameter equals zero) lies on the boundary of the parameter’s allowable space (the parameters in question are first a variance and second a probability).

Row A. This row presents the estimated deviation from the pair indicators (restricted to basic pairs 1 through 4) due to transparent versus minimal framing. It is negative and highly significant: Transparent frames reduced ambiguity aversion as predicted.

Row B. Every pair was presented in two ways, with either the red ticket or the blue ticket being the better state in the relatively ambiguous option. The insignificance of this effect says that there is no mean effect on ambiguous choices of this manipulation, suggesting that on average, subjects have equal priors of red and blue ticket states.

Row C. In the presentations of the choice situations it is usually true that the top row of each presentation is option A while the bottom row is option B, but in twelve of the sixty choice situations this was reversed. Row C shows that this effect is statistically insignificant, suggesting that we have no empirically important “response set” issue in our experiment.

Rows D1 and D3. We grouped our sixty choice situations into three booklets of twenty situations each and systematically varied the order in which subjects encountered the three booklets, so that each booklet was either the first, second or third booklet subjects encountered. The insignificance of these two effects suggests that we have no appreciable order effects.

Pair indicator effects. All the effects described above, except the framing effect, are parameterized as deviations from the estimated pair indicator effects (when presented in minimal frames). So each pair indicator effect is interpreted as $\Delta v_s^\tau \equiv v_{sA}^\tau - v_{sB}^\tau$ under minimal framing. A significantly positive (negative) value of a pair intercept means that, on average, subjects prefer Option A (Option B) in that pair when presented in a minimal frame. The only pairs (5, 6 and 9) with negative estimates are pairs where Option B (the relatively ambiguous option) has an appreciably higher subjective expected value under the assumption of equal prior probabilities assigned to states (ticket colors).

Indifferent response probability. Across all subjects, this estimate suggests that when subjects find themselves in the indifferent preference state, they choose each of the three possible responses (A, B or ~) with nearly equal one-third probabilities.

Diversification utility. The negative sign and statistical significance of this estimate is not what most would expect; a diversification motive implies a positive sign for this effect.

A1.3 Details of Structural Estimations Underlying Sections 5.2 and 5.3

When estimating structural models of choice under risk and uncertainty, many behavioral econometricians now use one of several modifications of (or alternatives to, e.g. Fishburn 1978) the Luce model of preference state probabilities in eq. A3. This is due to special econometric issues, derived from decision-theoretic considerations, in the realm of choice under risk and uncertainty (Busemeyer and Townsend 1993; Wilcox 2008, 2011; Blavatskyy 2011; Blavatskyy 2014). The general form of all these modifications to eq. A3 is

(A5) $Q_{sA}^{\tau} = exp[\lambda(v_{sA}^{\tau} - v_{sB}^{\tau})/N_s^{\tau}]/D_s^{\tau}$, $Q_{s\sim}^{\tau} = exp[\lambda(v_{s\sim}^{\tau} - v_{sB}^{\tau})/N_s^{\tau}]/D_s^{\tau}$, and $Q_{sB}^{\tau} = 1/D_s^{\tau}$,

where $D_s^{\tau} = exp[\lambda(v_{sA}^{\tau} - v_{sB}^{\tau})/N_s^{\tau}] + exp[\lambda(v_{s\sim}^{\tau} - v_{sB}^{\tau})/N_s^{\tau}] + 1$,

and N_s^{τ} is a normalization specific to each choice set (situation) s.

Our N_s^{τ} is a small generalization of Blavatskyy's (2014) "Stronger Utility" normalization. For each situation s, derive two new options $s\overline{AB}$ and $s\underline{AB}$ from options A and B: option $s\overline{AB}$ is the stochastic dominance supremum of options sA and sB, while option $s\underline{AB}$ is the stochastic dominance infimum of options A and B. Put differently, $s\overline{AB}$ is the least desirable option that still stochastically dominates both options sA and sB, while $s\underline{AB}$ is the most desirable option that is nevertheless stochastically dominated by both options sA and sB. Blavatskyy's Stronger Utility normalization is $N_s^{\tau} = v_{s\overline{AB}}^{\tau} - v_{s\underline{AB}}^{\tau}$. Blavatskyy developed this normalization for use with decision-theoretic representations that assign values to alternatives. We generalize it to decision-theoretic representations that are comparative (such as SWUP and Hurwicz-SWUP): For instance the normalization becomes $N_s^{HS} = S(s\overline{AB}, s\underline{AB})$ for the comparative Hurwicz-SWUP representation. In fact, the plausibility of this minor extension of Stronger Utility is the reason we choose it for this particular work: We estimate both value representations and comparative representations, and prefer a common normalization for both.

An example aids understanding of the simple construction of Blavatskyy's "bounding options" $s\overline{AB}$ and $s\underline{AB}$. Rows 2 and 3 of Table A1.2 show basic pair 1 in a minimal frame, while rows 1 and 4 show $s\overline{AB}$ and $s\underline{AB}$ in this situation. One constructs $s\overline{AB}$ by assigning the best state-contingent lottery (offerred by either sA or sB in each state ω) to $s\overline{AB}$ in every state ω; similarly one constructs $s\underline{AB}$ by assigning the worst state-contingent lottery (offerred by either sA or sB in each state ω) to $s\underline{AB}$ in every state ω. Table A1.2 illustrates this construction.

Table A1.2 Example of Blavatskyy's Bounding Options (Basic Pair 1, Minimal Frame)

			$\omega = r$ (red ticket state)					$\omega = b$ (blue ticket state)			
row	option		money	prob	money	prob		money	prob	money	prob
1	$s\overline{AB}$		\$25	0.75	\$0	0.25		\$25	0.50	\$0	0.50
2	sA		\$25	0.50	\$0	0.50		\$25	0.50	\$0	0.50
3	sB		\$25	0.75	\$0	0.25		\$25	0.25	\$0	0.75
4	$s\underline{AB}$		\$25	0.50	\$0	0.50		\$25	0.25	\$0	0.75

To avoid clutter we have so far suppressed dependence on parameters when writing v_{sk}^{τ} and P_{sk}^{τ}. We now explicitly note this dependence. Values $v_{sk}^{\tau}(\beta^{\tau})$ depend on any parameters β^{τ} of theory representation τ. The response probabilities $P_{sk}^{\tau}(\beta^{\tau},\lambda,\phi)$ inherit dependence on β^{τ} from their dependence on the $v_{sk}^{\tau}(\beta^{\tau})$ and add dependence on the precision parameter λ and the indifferent response probability ϕ. So the parameter vector θ^{τ} introduced in Section 5.3 is identical to the vector $(\beta^{\tau},\lambda,\phi)$. We then have the likelihood

(A6) $$L^{\tau}(c_s^e|\theta^{\tau}) = \mathbf{1}(c_s^e = 1)P_{sA}^{\tau}(\theta^{\tau}) + \mathbf{1}(c_s^e = 0)P_{sB}^{\tau}(\theta^{\tau}) + \mathbf{1}(c_s^e = 0.5)P_{s\sim}^{\tau}(\theta^{\tau})$$

of observation c_s^e, given response probabilities $P_{sk}^{\tau}(\theta^{\tau})$ defined by θ^{τ} and eqs. A2 and A5. Recall that $\mathcal{S} \in \{\mathcal{M},\mathcal{T}\}$ indexes the sets of all minimal frame situations and all transparent frame situations: Let the vectors $c_{\mathcal{S}}^e$ denote all of subject e's responses in set $\mathcal{S}$, and let the vector $c^e = (c_{\mathcal{M}}^e, c_{\mathcal{T}}^e)$ be all sixty of subject e's responses. Assuming independence of observations c_s^e across s, the total likelihood of subject e's responses is

(A7) $$L^{\tau}(c^e|\theta^{\tau}) = \prod_{s=1}^{60} L^{\tau}(c_s^e|\theta^{\tau}) = L^{\tau}(c_{\mathcal{M}}^e|\theta^{\tau})L^{\tau}(c_{\mathcal{T}}^e|\theta^{\tau}),$$
$$\text{where } L^{\tau}(c_{\mathcal{S}}^e|\theta^{\tau}) = \prod_{s\in\mathcal{S}} L^{\tau}(c_s^e|\theta^{\tau}).$$

Section 5.2 employs a particular random parameters estimation of $\tau = H$, the Hurwicz criterion. Values $v_{sk}^{\tau}(\beta^{\tau})$ are as given by $H(X)$ in eq. 4. Since we have just two outcomes in the experiment (allowing us to normalize their vNM utilities zero or one), β^{τ} is just a single parameter here, the Hurwicz optimism parameter α. However we allow α to vary across subjects and frames. The logit of α is $x(\alpha) = \ln\left[\frac{\alpha}{1-\alpha}\right]$ and the logistic function $\Lambda(x) = [1+e^{-x}]^{-1}$ is the inverse of $x(\alpha)$. Let η be a Standard Normal random variable with cumulative distribution function $\Phi(\eta)$. Characterize each subject by η^e, a subject-specific draw of η that is fixed across the sixty situations in the experiment. The Section 5.2 estimation assumes that subject e's optimism parameter is $\alpha_{\mathcal{S}}^e = \Lambda[x(\alpha_{\mathcal{S}}) + \sigma_{\mathcal{S}}\eta^e]$, where $\mathcal{S} \in \{\mathcal{M},\mathcal{T}\}$ indexes either minimal frame or transparent frame trials. This allows both the mean $x(\alpha_{\mathcal{S}})$ and standard deviation $\sigma_{\mathcal{S}}$ of (the logit of) subjects' optimism parameters to vary with frames, but requires that each subject's position in those two different distributions remains unchanged across the two types of frames.

In random parameters estimation one estimates the underlying $\alpha_{\mathcal{S}}$ and $\sigma_{\mathcal{S}}$ rather than estimating individual subjects' η^e (as in a fixed effects model). To do this, the variation of η^e

across subjects must be integrated out of the total likelihood: Once that is done, we get a likelihood that depends on $\alpha_{\mathcal{S}}$ and $\sigma_{\mathcal{S}}$ rather than the η^e:

(A8) $$L^\tau(c^e|\alpha_{\mathcal{M}},\sigma_{\mathcal{M}},\alpha_{\mathcal{T}},\sigma_{\mathcal{T}},\lambda,\phi) = \int_{\mathbb{R}} L^\tau(c^e_{\mathcal{M}}|\Lambda[x(\alpha_{\mathcal{S}})+\sigma_{\mathcal{S}}y],\lambda,\phi)L^\tau(c^e_{\mathcal{T}}|\Lambda[x(\alpha_{\mathcal{S}})+\sigma_{\mathcal{S}}y],\lambda,\phi)d\Phi(y).$$

In practice, integrals like that shown on the right-hand-side of A8 rarely have analytical solutions and must be approximated using numerical methods such as simulation or quadrature. We use the Gauss-Hermite quadrature method. One then takes natural logarithms of these approximations of A8, sums them across subjects, and maximizes this sum in the parameters $(\alpha_{\mathcal{M}},\sigma_{\mathcal{M}},\alpha_{\mathcal{T}},\sigma_{\mathcal{T}},\lambda,\phi)$. This generates the estimates and standard errors reported in Section 5.2 as well as the estimated distributions of α shown in Figure 8.

In Section 5.3 all estimations are individual subject-by-subject estimations involving no assumptions about distributions of parameters across subjects: Every subject e gets her own estimated parameter vector on the basis of her own vector $c^e_{\mathcal{M}}$ of fifty-two responses to the choice situations $s \in \mathcal{M}$. The log likelihood introduced in Section 5.3 is just $\mathcal{L}^{\tau,e}_{\mathcal{M}}(\theta^\tau) \equiv \ln[L^\tau(c^e_{\mathcal{M}}|\theta^\tau)]$: When this is maximized in θ^τ, the solution is $\hat{\theta}^{\tau,e}_{\mathcal{M}}$. We do this for three theories τ. As discussed above, Hurwicz criterion values $v^H_{sk}(\beta^H)$ depend on just one parameter $\beta^H = \alpha$. Hurwicz-SWUP comparison functions $S(sA,sB)$ (defined by eq. 5) substitute for value differences $v^\tau_{sA} - v^\tau_{sB}$ in eq. A5 and also depend on just one parameter $\beta^{HS} = \alpha$ since we employ a parameter-free salience function for these estimations (see eq. A13 in Section A2 below).

Our third theory, the "Mean-Standard Deviation" theory, has this value representation for our two-state options X:

(A9) $MSD(X) = SEU(X|\pi^r) - \gamma SSD(X|\pi^r)$, where

where $SEU(X|\pi^r) = \pi^r EU[X(r)] + (1-\pi^r)EU[X(b)]$,

$$SSD(X|\pi^r) = \sqrt{\pi^r\big(EU[X(r)] - SEU(X|\pi^r)\big)^2 + (1-\pi^r)\big(EU[X(b)] - SEU(X|\pi^r)\big)^2},$$

and $EU[X(\omega)]$ is the state-contingent (objective) expected utility of option X in state ω.

Because our experimental design involves just two outcomes, calculation of state-contingent expected utilities $EU[X(\omega)]$ requires no parameter estimates. Therefore this theory has just two parameters $\beta^{MSD} = (\pi^r,\gamma)$ and $\theta^{MSD} = (\pi^r,\gamma,\lambda,\phi)$. One small complication flows from the fact that this theory only satisfies monotonicity when $\gamma^2 < \min\left(\frac{\pi^r}{1-\pi^r}, \frac{1-\pi^r}{\pi^r}\right)$, so this constraint on the relationship between π^r and γ must be imposed during maximization of subject's likelihood functions. This constraint rarely binds (and never binds with statistical significance).

The distribution of estimated subjective priors $\hat{\pi}^{r,e}_{\mathcal{M}}$ across subjects may interest some readers. The 10th, 25th, 50th, 75th and 90th centiles of this distribution are 0.465, 0.490, 0.500, 0.521 and 0.555, respectively (we think that with just 79 subjects, centiles further into the tails aren't too meaningful). This is a fairly tight distribution around uniform subjective priors.

A2: A Sufficient Condition for SWUP to Satisfy Certainty Betweenness

Consider the following frame involving a choice between a constant option K and an ambiguous act X, encompassing basic pairs 1 through 7 from our experiment as special cases.

	Red Ticket				Blue Ticket			
K	\$25	p	\$0	$1-p$	\$25	p	\$0	$1-p$
X	\$25	q_r	\$0	$1-q_r$	\$25	q_b	\$0	$1-q_b$

Let r and b denote the red and blue ticket states, respectively. Normalize $u(25) = 1$, and $u(0) = 0$. Under SWUP, $K \sim X \Leftrightarrow \pi^r \psi_{\mathrm{P}}(p, q_r)(p - q_r) + (1 - \pi^r)\psi_{\mathrm{P}}(p, q_b)(p - q_b) = 0$. Act K and mixture $W \coloneqq \delta K + (1-\delta)X$ are shown below in the red and blue ticket states:

	Red Ticket							
$K(r)$	\$25	δp	\$0	$\delta(1-p)$	\$25	$(1-\delta)p$	\$0	$(1-\delta)(1-p)$
$W(r)$	\$25	δp	\$0	$\delta(1-p)$	\$25	$(1-\delta)q_r$	\$0	$(1-\delta)(1-q_r)$

	Blue Ticket							
$K(b)$	\$25	δp	\$0	$\delta(1-p)$	\$25	$(1-\delta)p$	\$0	$(1-\delta)(1-p)$
$W(b)$	\$25	δp	\$0	$\delta(1-p)$	\$25	$(1-\delta)q_b$	\$0	$(1-\delta)(1-q_b)$

For basic pairs 1 through 7, certainty betweenness implies this condition for SWUP:

(A10) $$K \sim W \Leftrightarrow \pi^r \psi_{\mathrm{P}}\big((1-\delta)p, (1-\delta)q_r\big)(1-\delta)(p-q_r) + (1-\pi^r)\psi_{\mathrm{P}}\big((1-\delta)p, (1-\delta)q_b\big)(1-\delta)(p-q_b) = 0.$$

Bordalo et al. (2013) argue that *homogeneity of degree zero* is a plausible property of a salience function and they assume that property in their analysis of salience in consumer choice. They define homogeneity of degree zero as follows: $\psi(\delta a_i, \delta b_i) = \psi(a_i, b_i)$ for all $\delta > 0$.

Under homogeneity of degree zero, eq. A10 reduces to:

(A11) $K \sim W \Leftrightarrow (1-\delta)[\pi^r\, \psi_{\mathrm{P}}(p, q_r)(p-q_r) + (1-\pi^r)\psi_{\mathrm{P}}(p, q_b)(p-q_b)] = 0.$

Note that eq. A7 is also the condition for $X \sim K$, and thus certainty betweenness holds under SWUP for basic pairs 1 through 7 if the probability salience function satisfies homogeneity of degree zero. A 'parameter-free' salience function, introduced by Bordalo et al. (2013), which satisfies ordering, diminishing sensitivity, and homogeneity of degree zero is shown below:

(A12) $\psi(a_i, b_i) := |a_i - b_i|/(|a_i| + |b_i|)$ if $a \neq 0$ or $b \neq 0$; and $\psi(0,0) := 0$.

In our estimations, we use another 'parameter-free' salience function introduced by Leland, Schneider and Wilcox (2017):

(A13) $\psi\big(a_i, b_i|(a,b)\big) := |a_i - b_i|/(|a_i| + |b_i| + \|(a,b)\|)$,

where $\|(a,b)\|$ is the Euclidean norm of the vector (a,b) that horizontally concatenates all like dimension vectors in a frame (i.e., all outcomes in a frame, or all probabilities in a frame, for both options in that frame). This salience function also satisfies ordering and diminishing sensitivity and a modified homogeneity of degree zero $\psi\big(\delta a_i, \delta b_i|(\delta a, \delta b)\big) = \psi\big(a_i, b_i|(a,b)\big)$. With this property, eq. A10 will still reduce to eq. A11 when $\delta = 0.5$. This restricted certainty betweenness is all we need for incentive compatibility of the indifference response (since the coin flip resolving indifference is a 50:50 mixture).

A3. Ranking Acts by Their Robustness to Ambiguity

All basic pairs from the experiment are shown below. We can rank how robust options A and B are to ambiguity by measuring the difference between the red and blue probability distributions for A and for B. The smaller the difference between the red and blue distributions, the greater the robustness to ambiguity. One standard approach to measuring differences between two probability distributions is to use an 'f-divergence'. Hellinger distance (Hellinger 1909; Sengar 2009) and total variation distance (Levin et al. 2009) are two common f-divergences. Let P and Q be discrete distributions with finite support. The Hellinger distance between them is:

$$HD(P,Q) = \frac{1}{\sqrt{2}}\left[\sum_{i=1}^{n}(\sqrt{p_i} - \sqrt{q_i})^2\right]^{1/2}.$$

The total variation distance is the maximum difference in probabilities that P and Q assign to the same event:

$$TVD(P, Q) = \max_i |p(i) - q(i)|.$$

Basic pair #	Option A								Option B							
	Red ticket				Blue ticket				Red ticket				Blue ticket			
1	$25	**0.50**	$0	0.50	$25	**0.50**	$0	0.50	$25	**0.75**	$0	0.25	$25	**0.25**	$0	0.75
2	$25	**0.50**	$0	0.50	$25	**0.50**	$0	0.50	$25	**1.00**	$0	0.00	$25	**0.00**	$0	1.00
3	$25	**0.67**	$0	0.33	$25	**0.67**	$0	0.33	$25	**1.00**	$0	0.00	$25	**0.33**	$0	0.67
4	$25	**0.33**	$0	0.67	$25	**0.33**	$0	0.67	$25	**0.67**	$0	0.33	$25	**0.00**	$0	1.00
5	$25	**0.50**	$0	0.50	$25	**0.50**	$0	0.50	$25	**1.00**	$0	0.00	$25	**0.25**	$0	0.75
6	$25	**0.50**	$0	0.50	$25	**0.50**	$0	0.50	$25	**1.00**	$0	0.00	$25	**0.33**	$0	0.67
7	$25	**0.33**	$0	0.67	$25	**0.33**	$0	0.67	$25	**0.50**	$0	0.50	$25	**0.00**	$0	1.00
8	$25	**0.67**	$0	0.33	$25	**0.33**	$0	0.67	$25	**1.00**	$0	0.00	$25	**0.00**	$0	1.00
9	$25	**0.67**	$0	0.33	$25	**0.50**	$0	0.50	$25	**1.00**	$0	0.00	$25	**0.33**	$0	0.67
10	$25	**0.50**	$0	0.50	$25	**0.33**	$0	0.67	$25	**1.00**	$0	0.00	$25	**0.00**	$0	1.00
11	$25	**0.50**	$0	0.50	$25	**0.33**	$0	0.67	$25	**0.75**	$0	0.25	$25	**0.00**	$0	1.00

Let the events be winning $25 and winning $0. The Hellinger distance and the total variation distance can be computed for options A and B above. The resulting values are shown in the tables below which reveal that they both imply the same ranking of 'robustness' to ambiguity. In each case, option A has a smaller distance between the red and blue distributions, indicating that it is more robust to ambiguity than option B in each pair.

Hellinger distance

Basic pair	A	B
1	0	0.37
2	0	1
3	0	0.65
4	0	0.65
5	0	0.71
6	0	0.65
7	0	0.54
8	0.24	1
9	0.12	0.65
10	0.12	1
11	0.12	0.71

Total Variation distance

Basic pair	A	B
1	0	0.5
2	0	1
3	0	0.67
4	0	0.67
5	0	0.75
6	0	0.67
7	0	0.5
8	0.333	1
9	0.167	0.67
10	0.167	1
11	0.167	0.75

A4. Proof of Proposition 1 in Section 3.3

Proposition 1: Let $\succ$ ($\sim$) denote strict preference (indifference) as determined by the Hurwicz-SWUP criterion in eq. 5. For the choice situations shown below in Figures A1 and A2 (where $\epsilon > 0$ and $z > 0$; basic pairs 1 to 4 are cases of these), with constant act Y and ambiguous act X:

(i) If $X \sim Y$ in the minimal frame then $X \succ Y$ in the transparent frame.

(ii) If $X \sim Y$ in the transparent frame then $Y \succ X$ in the minimal frame.

Proof: Set $u(z) = 1$ and $u(0) = 0$.

Proof of (i): Under eq. 5, $X \sim Y$ in the minimal frame in Figure A1 implies

$$\alpha\psi_{\mathrm{P}}(p+\epsilon, p)(\epsilon) + (1-\alpha)\psi_{\mathrm{P}}(p-\epsilon, p)(-\epsilon) = 0,$$

which implies $\alpha\psi_{\mathrm{P}}(p+\epsilon, p) = (1-\alpha)\psi_{\mathrm{P}}(p-\epsilon, p)$. By symmetry and diminishing absolute sensitivity of ψ_{P}, we have $\psi_{\mathrm{P}}(p+\epsilon, p) < \psi_{\mathrm{P}}(p-\epsilon, p)$. Hence $X \sim Y$ in the minimal frame implies that $\alpha > 0.5$.

For the transparent frame in Figure A2, under eq. 5, we have $X \succ Y$ if and only if $\alpha\psi_{\mathrm{X}}(x, 0)(1)(\epsilon) + (1-\alpha)\psi_{\mathrm{X}}(0, x)(-1)(\epsilon) < 0$.

By symmetry of ψ_{X}, the above inequality holds if and only if $\alpha > 0.5$. ■

Proof of (ii): Under eq. 5, $X \sim Y$ in the transparent frame if and only if $\alpha = 0.5$. Given $\alpha = 0.5$, diminishing sensitivity and symmetry of ψ_{P} imply $Y \succ X$ in the minimal frame. ■

Figure A1. Form of Basic Pairs 1-4 in Minimal Frames

	Red Ticket State				Blue Ticket State			
Y	z	p	0	$1-p$	z	p	0	$1-p$
X	z	$p+\epsilon$	0	$1-p-\epsilon$	z	$p-\epsilon$	0	$1-p+\epsilon$

Figure A2 Form of Basic Pairs 1-4 in Transparent Frames

	Red Ticket State						Blue Ticket State					
Y	z	p	0	ϵ	0	$1-p-\epsilon$	z	$p-\epsilon$	z	ϵ	0	$1-p$
X	z	p	z	ϵ	0	$1-p-\epsilon$	z	$p-\epsilon$	0	ϵ	0	$1-p$

A5 Experimental Materials

The experimental materials are available here:

http://www.chapman.edu/research-and-institutions/economic-science-institute/_files/WorkingPapers/schneider-leland-wilcox-ambiguity-framed-2016b.pdf

These materials include:

- The Instruction Booklet
- Ellsberg Experiment Booklet 1 (Choice Situations 1 – 20)
- Ellsberg Experiment Booklet 2 (Choice Situations 21 – 40)
- Ellsberg Experiment Booklet 3 (Choice Situations 41 – 60)