Ambiguity Framed

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In his exposition of subjective expected utility theory, Savage (1954) proposed that the Allais paradox could be reduced if it were recast into a format which made the appeal of the independence axiom of expected utility theory more transparent. Recent studies consistently find support for this prediction. We consider a salience-based choice model which explains this frame-dependence of the Allais paradox and derive the novel prediction that the same type of presentation format which was found to reduce Allais-style violations of expected utility theory will also reduce Ellsberg-style violations of subjective expected utility theory since that format makes the appeal of Savage's "sure thing principle" more transparent. We design an experiment to test this prediction and find strong support for such frame dependence of ambiguity aversion in Ellsberg-style choices. In particular, we observe markedly less ambiguity-averse behavior in Savage's matrix format than in a more standard 'prospect' format. This finding poses a new challenge for the leading models of ambiguity aversion.

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1. Introduction

Expected utility (EU) theory (Von Neumann and Morgenstern, 1947) and subjective expected utility (SEU) theory (Savage, 1954) are widely recognized as the standard models of rational decision making under risk and uncertainty. Both models have also been applied as descriptive theories of actual behavior, although persistent empirical challenges were raised soon after the models were introduced. Allais (1953) devised pairs of choices, one involving a certain outcome and a risky prospect and the other a choice between two risky prospects where people frequently violate the independence axiom of EU. Ellsberg (1961) presented pairs of choices each involving a risky prospect (whose probabilities are given) and an uncertain prospect (whose probabilities are unknown) where people frequently violate the 'sure-thing' principle of SEU.

In his exposition of subjective expected utility, Savage (1954) digressed to address the Allais-type violations of the independence axiom. He conjectured that these violations might be reduced if the choice situations were reframed in a transparent format. Tests of this prediction, discussed below, have consistently found that the Allais paradox is susceptible to framing, with significantly fewer violations in Savage's proposed presentation format. Since the Ellsberg paradox also violates an independence condition, we ask whether applying Savage's presentation format to Ellsberg-style choices leads to fewer violations of SEU. To our knowledge, this question has not been investigated. Section 2 provides a motivating example and reviews a small existing literature. Section 3 introduces the model which inspired our experiment and derives the main prediction that we test. Section 4 describes our experimental design and protocol. Section 5 presents our results, and Section 6 concludes. Further econometric analysis is provided in Appendix A. A sufficient condition for the motivating model to be incentive compatible with our experimental design is presented in Appendix B. All materials from the experiment are accessible from a link in Appendix C.

2. Motivation

Consider the example of the Allais paradox discussed in Savage (1954) shown in the left panel of Figure 1 (where the payoffs are in thousands of dollars) presented in what we refer to as a *minimal* or *efficient frame.*[1] In this version, a decision maker chooses between lotteries p and q

[1] A minimal frame, as described in Leland and Schneider (2016), is a matrix presentation of choice alternatives which has the smallest dimension (e.g., fewest number of columns) necessary to represent those alternatives.

and then chooses between lotteries p' and q'. Lottery p offers $500,000 with certainty, whereas q offers a 10% chance of $2.5 million, an 89% chance of $500,000, and a 1% chance of $0. The independence axiom (and Savage's sure-thing principle) imply that a decision maker with strict preferences will choose either p and p' or q and q' (in accord with the decision maker's attitude toward risk). Yet Savage himself reports expressing a preference for p over q and for q' over p' (Savage, 1954), in violation of his own theory!

Figure 1. The Allais Paradox in Minimal and Transparent Frames

The Allais Paradox in Minimal Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p	500	0.10	500	0.89	500	0.01
q	2500	0.10	500	0.89	0	0.01

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
p′	500	0.11	0	0.89
q′	2500	0.10	0	0.90

The Allais Paradox in Transparent Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p	500	0.10	500	0.89	500	0.01
q	2500	0.10	500	0.89	0	0.01

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p′	500	0.10	0	0.89	500	0.01
q′	2500	0.10	0	0.89	0	0.01

Troubled by his own expressed preferences, Savage (1954) invites consideration of an alternative representation of the same choices similar to the presentation in the right panel of Figure 1. In this presentation, it is clear that p and q each offer an 89% chance of $500,000 and that p' and q' each offer an 89% chance of $0. Savage proposes that this change in framing may enhance the appeal of the independence axiom and produce more consistent choices.

We refer to the type of presentation in the right panel of Figure 1 as a *transparent frame* since it makes the normative appeal of the independence axiom transparent. In particular, a transparent frame isolates the common consequences of the lotteries under consideration and focuses attention on the differences between lotteries as prescribed by the independence axiom.

A number of recent studies (Leland, 2010; Bordalo et al., 2012; Incekara-Hafalir and Stecher, 2012; Birnbaum and Schmidt, 2015; Harman and Gonzalez, 2015) have investigated whether observed behavior is more consistent with Savage's theory when the Allais paradox choices are presented to subjects in transparent frames. All of these studies find support for Savage's conjecture. Incekara-Hafalir and Stecher (2012) conclude that "given a transparent presentation, expected utility theory performs surprisingly well."

The matrix presentation format for lotteries used by Savage has been formalized by Leland and Schneider (2016) who also develop a salience-based decision algorithm that operates over frames. We will show that a novel and general prediction of the model in Leland and Schneider (2016) is that ambiguity aversion is also susceptible to framing. In particular, the same type of frame that reduces Allais-style violations of EU is predicted to reduce Ellsberg-style violations of SEU. To illustrate, consider the pairs of choices in Figure 2 that were used in our experiment. The top pair is a choice between two state-dependent lotteries shown in minimal frames, where the lottery the decision maker plays depends on her choice (A or B) and the realization of an ambiguous state of the world (a 'red ticket' state or a 'blue ticket' state). The decision maker does not know the probability that the true state is red or blue. As in Ellsberg's classical paradox, one option (A) is risky (it yields the same lottery regardless of the state), whereas the other option (B) is ambiguous (it yields different lotteries in different states). The decision maker is also given a similar choice in which the lotteries assigned to the red and blue ticket states are reversed. This construction resembles that in Ellsberg's (1961) two-color paradox.

Figure 2. Ellsberg's Paradox in Minimal Frames (top) and Transparent Frames (bottom)

	You Draw a Red Ticket				You Draw a Blue Ticket			
	$	N/12	$	N/12	$	N/12	$	N/12
A	$25	6/12	$0	6/12	$25	6/12	$0	6/12
B	$25	9/12	$0	3/12	$25	3/12	$0	9/12

	You Draw a Red Ticket						You Draw a Blue Ticket					
	$	N/12	$	N/12	$	N/12	$	N/12	$	N/12	$	N/12
A	$25	6/12	$0	3/12	$0	3/12	$25	3/12	$25	3/12	$0	6/12
B	$25	6/12	$25	3/12	$0	3/12	$25	3/12	$0	3/12	$0	6/12

The SEU model predicts that a decision maker who strictly prefers A to B in Figure 2 will also strictly prefer B to A when the lotteries assigned to red and blue states are reversed—acting as if that agent assigns a subjective probability distribution over states. However, in similar types of choices, Ellsberg (1961) found that many people preferred A to B regardless of whether the assignment of lotteries to states is reversed. Since A offers a known probability of winning a prize, whereas the probability of winning in option B is ambiguous, the strict preference for A is termed *ambiguity aversion*.

The choices between A and B could, like the Allais lotteries, be presented in the 'transparent' frames shown in the bottom of Figure 2. In this 'Savage' presentation, for the choice between A and B, the common consequences in each state-contingent lottery are isolated, encouraging the decision maker to focus on the differences between A and B (the 3/12 chance of A paying $0 and B paying $25 in the red state and the 3/12 chance of A paying $25 and B paying $0 in the blue state). A decision maker who focuses only on these differences and assigns a uniform prior over states will then be indifferent between A and B, regardless of whether the assignment of lotteries to states is reversed. Thus, a transparent frame of the Ellsberg paradox is predicted to produce behavior closer to ambiguity neutrality. In the next section we show that such behavior is indeed predicted by the model of Leland and Schneider (2016) under fairly general conditions. However, such frame-dependence of ambiguity aversion is not consistent with any of the leading models of ambiguity aversion in the literature. We test this prediction in a new experiment and find strong support for the predicted framing effect.

There is but a small and very recent literature on the possibility that ambiguity attitudes are susceptible to framing effects. Chew et al. (2016) examine whether presentation of choice alternatives as text description versus payoff tables (so as to make the ambiguity inherent in the choices more or less explicit) influences the degree of ambiguity aversion observed. They find that for subjects who do not recognize ambiguity in some tasks, emphasizing ambiguity produces greater ambiguity aversion in others. However, subjects that recognized the ambiguity independent of presentation format tended to be more ambiguity-averse regardless of whether the ambiguity is emphasized. Trautmann and van der Kuijlen (2014) examine attitudes toward ambiguity for gains versus losses. They report results suggesting that ambiguity aversion varies according to whether the outcomes are gains or losses, as has been observed for attitudes toward risk. Finally, Voorhoeve et al., (2016) test the findings in Chew et al. (2016) and in Trautmann and van der Kuijlen (2014), and fail to find significant support for the hypotheses that emphasizing ambiguity, or reframing gains as losses, alters the prevalence of ambiguity aversion. With these mixed findings, we think there is room for more experimental work. Additionally, the next section provides a highly focused theoretical motivation for our new experiment.

3. A Model of Salience based Choice under Uncertainty

As in Leland and Schneider (2016), we employ a matrix representation of the attributes (i.e., payoffs and probabilities) of a pair of lotteries. A generic frame for simple Ellsberg-style choices which encompasses the basic pairs of alternatives used in our experiment is shown in Figure 3 in which there are two possible states – a "red ticket" state and a "blue ticket" state. The underlying state is unknown to the decision maker. Option A in Figure 2 offers lottery $\{x_1^r, p_1^r; \dots; x_{n_r}^r, p_{n_r}^r\}$ if the state is red and offers lottery $\{x_1^b, p_1^b; \dots; x_{n_b}^b, p_{n_b}^b\}$ if the state is blue. Likewise, Option B offers lottery $\{y_1^r, q_1^r; \dots; y_{n_r}^r, q_{n_r}^r\}$ if the state is red and lottery $\{y_1^b, q_1^b; \dots; y_{n_b}^b, q_{n_b}^b\}$ if the state is blue. All frames in the experiment were presented to be monotonically decreasing in outcomes for each state-contingent lottery. For the frame in Figure 3, this entails that $x_1^r \geq \cdots \geq x_{n_r}^r$ and $x_1^b \geq \cdots \geq x_{n_b}^b$ for Option A and analogous monotonicity for Option B. Note that the index $i \in \{1,2,\dots,n^\omega\}$ in Figure 3 denotes the location of the i^{th} column vector in the frame in state ω.

Figure 3. A Generic Frame under Ambiguity

	Red Ticket State								Blue Ticket State							
A	x_1^r	p_1^r	...	x_i^r	p_i^r	...	$x_{n_r}^r$	$p_{n_r}^r$	x_1^b	p_1^b	...	x_i^b	p_i^b	...	$x_{n_b}^b$	$p_{n_b}^b$
B	y_1^r	q_1^r	...	y_i^r	q_i^r	...	$y_{n_r}^r$	$q_{n_r}^r$	y_1^b	q_1^b	...	y_i^b	q_i^b	...	$y_{n_b}^b$	$q_{n_b}^b$

Given the notion of a frame as a matrix representation of state-contingent lotteries, we can model the behavior of a frame-sensitive decision maker by developing a computational decision algorithm which operates over frames. To do so, following Leland and Schneider (2016), we start with the SEU model of Anscombe and Aumann (1963).

Index the possible states of the world by $\omega \in \{1,2,\dots,m\}$. Denote ambiguous prospects by h and g, where h assigns lottery $h(\omega)$ with corresponding payoff and probability vectors $(\mathbf{x}^{\boldsymbol{\omega}}, \boldsymbol{p}^{\boldsymbol{\omega}})$ to each state. Likewise, g assigns lottery $g(\omega)$ with payoff and probability vectors $(\mathbf{y}^{\boldsymbol{\omega}}, \boldsymbol{q}^{\boldsymbol{\omega}})$ to each state. In the classic alternative-based evaluation model, there is a unique subjective probability distribution π over states (Anscombe and Aumann, 1963) such that h is chosen over g if and only if (1) holds:

(1) $\sum_{\omega}^{m} \sum_{i}^{n^{\omega}} \pi^{\omega} [p_i^{\omega} u(x_i^{\omega})] > \sum_{\omega}^{m} \sum_{i}^{n^{\omega}} \pi^{\omega} [q_i^{\omega} u(y_i^{\omega})]$

Note that (1) can be written equivalently as an attribute-based evaluation model in (2):

(2) $\sum_{\omega}^{m} \sum_{i}^{n^{\omega}} \pi^{\omega} [(p_i^{\omega} - q_i^{\omega})(u(x_i^{\omega}) + u(y_i^{\omega}))/2 \ + \ (u(x_i^{\omega}) - u(y_i^{\omega}))(p_i^{\omega} + q_i^{\omega})/2] > 0.$

Leland and Schneider (2016) note that this "attribute-based evaluation computes probability differences associated with outcomes weighted by the average utility of those outcomes plus utility differences of outcomes weighted by their average probability of occurrence." An agent who chooses according to (2) will make the same choices as an agent who chooses according to the SEU model in (1). But drawing on recent work which highlights the role of salience perception in decision making (e.g., Bordalo et al., 2012; Koszegi and Szeidl, 2013), suppose that when comparing lotteries, the decision maker systematically focuses on large differences in payoffs or probabilities and systematically overweights them as a consequence. To formalize this intuition, Leland and Schneider place weights $\psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})$ on probability differences and $\psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})$ on payoff differences, yielding a model in which h is strictly preferred to g if and only if inequality (3) holds:

(3) $\sum_{\omega}^{m} \sum_{i}^{n^{\omega}} \pi^{\omega} [\ \psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})(p_i^{\omega} - q_i^{\omega})(u(x_i^{\omega}) + u(y_i^{\omega}))/2$

$$+ \psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})(u(x_i^{\omega}) - u(y_i^{\omega}))(p_i^{\omega} + q_i^{\omega})/2] \ > \ 0.$$

Leland and Schneider (2016) refer to the model in (3) as "salience weighted utility over presentations" (SWUP). In (3), the weights $\psi_{\mathrm{P}}(p_i^{\omega}, q_i^{\omega})$ and $\psi_{\mathrm{X}}(x_i^{\omega}, y_i^{\omega})$ are "salience functions" satisfying two critical properties of salience perception noted in Bordalo et al. (2012; 2013):

Definition 1 (Salience Function): A *salience function* $\psi(w, z)$ is any (non-negative), symmetric and continuous function that satisfies the following two properties:

1. **Ordering:** If $[w', z'] \subset [w, z]$ then $\psi(w', z') < \psi(w, z)$.
2. **Diminishing Sensitivity:** for any $w, z, \epsilon > 0$, $\psi(w + \epsilon, z + \epsilon) < \psi(w, z)$.

The model in (3) can explain the Allais paradox framing effect conjectured by Savage. In the transparent frame in Figure 1, a decision maker who acts in accordance with (3) chooses p over q if and only if she chooses p' over q', consistent with the independence axiom. In contrast, the salience evaluations in the two choice pairs can differ under minimal frames, enabling the model to accommodate the Allais paradox. The model in (3) not only explains the Allais framing effect but also predicts a novel framing effect in the context of Ellsberg's paradox. We can now apply the SWUP model to demonstrate this prediction.

3.1 The Ellsberg Paradox in Minimal and Transparent Frames

We illustrate the SWUP model with basic pair 1 from our experiment which is shown in minimal frames in Figure 4. Normalize $U(\$25) = 1$, $U(\$0) = 0$, and let π^r denote the subjective probability that the true state is red. Then inequality (3) predicts that A is chosen over B if

$$\pi^r \psi_{\mathrm{P}}(0.5{,}0.75)(-0.25) + (1 - \pi^r)\psi_{\mathrm{P}}(0.5, 0.25)(0.25) > 0.$$

As observed by Leland and Schneider (2016), symmetry and diminishing sensitivity of ψ_{P} imply that $\psi_{\mathrm{P}}(0.5, 0.25) > \psi_{\mathrm{P}}(0.5{,}0.75)$. Thus, under a uniform prior, the decision maker whose behavior is characterized by (3) chooses risky lottery A over ambiguous option B, and likewise chooses A′ over B′ for *any* salience function ψ_{P}. Hence, SWUP predicts ambiguity aversion in minimal frames. In the minimal frames of Figure 4, all payoff differences within each column vector are zero, so that behavior under SWUP depends solely on the subjective prior over states and the probability salience function.

Figure 4. The Ellsberg Paradox in Minimal Frames

	Red Ticket State				Blue Ticket State			
A	\$25	0.50	\$0	0.50	\$25	0.50	\$0	0.50
B	\$25	0.75	\$0	0.25	\$25	0.25	\$0	0.75

	Red Ticket State				Blue Ticket State			
A′	\$25	0.50	\$0	0.50	\$25	0.50	\$0	0.50
B′	\$25	0.25	\$0	0.75	\$25	0.75	\$0	0.25

Next, consider transparent frames of the same two pairs, shown in Figure 5. Notice that here it is the probability differences within each column vector that are zero, so that behavior is determined solely by the subjective prior and the payoff salience function. In particular, inequality (3) now predicts that A is chosen over B if

$$\pi^r \psi_X(0{,}25)(-25) + (1 - \pi^r)\psi_X(25{,}0)(25) > 0.$$

Clearly, under a uniform prior over states and by symmetry of ψ_X, the decision maker is predicted to be indifferent between A and B (and is likewise predicted to be indifferent between A′ and B′). Thus, SWUP predicts ambiguity aversion in minimal frames and ambiguity-neutrality in transparent frames under very general conditions (for any utility function, and any salience functions)[2]. We designed and conducted an experiment to test the comparative statics prediction of SWUP that subjects will be closer to ambiguity neutrality in transparent frames than in minimal frames, which we describe in the following sections.

Figure 5. The Ellsberg Paradox in Transparent Frames

	Red Ticket State						Blue Ticket State					
A	\$25	0.50	\$0	0.25	\$0	0.25	\$25	0.25	\$25	0.25	\$0	0.50
B	\$25	0.50	\$25	0.25	\$0	0.25	\$25	0.25	\$0	0.25	\$0	0.50

	Red Ticket State						Blue Ticket State					
A′	\$25	0.25	\$25	0.25	\$0	0.50	\$25	0.50	\$0	0.25	\$0	0.25
B′	\$25	0.25	\$0	0.25	\$0	0.50	\$25	0.50	\$25	0.25	\$0	0.25

[2] SWUP can also be generalized to endow each agent with a parameter $\vartheta \in [0,1]$ which is the probability the agent naturally re-frames a transparent presentation into a minimal one. This is particularly plausible if people naturally think in minimal frames. In this sense, ϑ indexes the strength of the framing effect for that agent (agents with $\vartheta = 1$ are frame insensitive and agents with $\vartheta = 0$ conform to SEU in transparent frames but exhibit ambiguity aversion in minimal frames). Under this generalization, SWUP accommodates a reduction in ambiguity aversion without necessarily implying a shift all the way to ambiguity neutrality in transparent frames. The estimations presented in Appendices A suggest that the bulk of observed framing effects are due to a widespread weakening of ambiguity aversion, rather than its wholesale disappearance.

4. Design of Experiment

Our experiment consists of $j = 1, 2, \dots, 11$ "basic pairs" of options, where each pair involved a choice between a more ambiguous and a less ambiguous prospect. After subjects made their choices, the uncertainty surrounding each ambiguous prospect was resolved in two stages. In the first stage, the state (either a 'red ticket' state or a 'blue ticket' state) was realized and, conditional on that state, the decision maker then received an objective lottery to be played in the second stage. The state is determined by a draw at the end of the experiment from an opaque bag containing an unknown combination of 10 lottery tickets, where each ticket is either red or blue. There are repeated choice trials of all eleven choice pairs, with variations of presentation, to create a total of $s = 1, 2, \dots, 60$ choice situations encountered by each subject. Table 1 summarizes all variations of the basic pairs, making up the sixty situations in the experiment.

Table 1. Summary of Experimental Design Pair Variations

Option pairs: States and state-contingent lotteries									Number of trials of each pair					
	Option A				Option B				Minimal frame				Transparent frame	
Basic Pair (j)	Red Ticket State		Blue Ticket State		Red Ticket State		Blue Ticket State		Assignment of Lotteries to States as Shown; top option is:		Assignment of Lotteries to States Reversed; top option is:		Assignment of Lotteries to States as Shown; top option is:	Assignment of Lotteries to States Reversed; top option is:
	$25	$0	$25	$0	$25	$0	$25	$0	A	B	A	B	A	A
1	1/2	1/2	1/2	1/2	3/4	1/4	1/4	3/4	2	1	2	1	1	1
2	1/2	1/2	1/2	1/2	1	0	0	1	1	0	1	0	1	1
3	2/3	1/3	2/3	1/3	1	0	1/3	2/3	2	1	2	1	1	1
4	1/3	2/3	1/3	2/3	2/3	1/3	0	1	2	0	2	0	1	1
5	1/2	1/2	1/2	1/2	1	0	1/4	3/4	2	1	2	1	0	0
6	1/2	1/2	1/2	1/2	1	0	1/3	2/3	3	0	3	0	0	0
7	1/3	2/3	1/3	2/3	1/2	2/3	0	1	2	1	2	1	0	0
8	2/3	1/3	1/3	2/3	1	0	0	1	2	0	2	0	0	0
9	2/3	1/3	1/2	1/2	1	0	1/3	2/3	1	0	1	0	0	0
10	1/2	1/2	1/3	2/3	1	0	0	1	2	1	2	1	0	0
11	1/2	1/2	1/3	2/3	3/4	1/4	0	1	1	1	1	1	0	0

Notes: The fractions below each outcome are probabilities of that outcome. The true state (red or blue) is determined by the draw of a ticket from an opaque bag of ten tickets, some or all of which may be either red or blue. Every trial of every pair has a corresponding trial in which the assignment of lotteries to ticket colors is reversed, all else held constant. In most trials, option A is the top row of the table displayed to subjects (as shown in Figure 5 for basic pair 1) but this is reversed in twelve trials. The probabilities within the state-contingent lotteries are resolved by the roll of a twelve-sided die.

For every choice situation with the assignment of lotteries to states as shown in Table 1, there was a corresponding choice situation with this assignment of lotteries to states reversed. This counter-balancing serves two purposes. First, it helps neutralize any suspicion a subject might have that the contents of their ticket bag is ‘rigged’ to minimize payoffs from the experiment. Second, it also permits a direct test of SEU in a setup similar to Ellsberg’s two-color paradox, where the counter-balancing of lotteries to states is necessary to infer whether a subject acts as if she assigns coherent probabilities to the red ticket and blue ticket states. For Basic Pair 1, for example, this implies that if an SEU agent prefers B to A when the preferred lottery in Option B (the 75% chance of winning $25) is assigned to the red ticket state, the agent is acting as-if the probability that the state is red is greater than 0.50. The same agent should then prefer A to B when the preferred lottery in Option B is assigned to the blue ticket state.

In Table 1, basic pairs $j = 1, 2, 3$ and 4 are Ellsberg-style choices in that (i) they involve a choice between a risky lottery (which yields the same lottery regardless of the state) and an ambiguous lottery (which assigns different lotteries to different states) and (ii) both options in each of these pairs have the same expected payoff if the decision maker assigns a coherent uniform prior over states. These four basic pairs are our focus: 26 of the 60 situations s are trials of these pairs (18 minimal frame trials and 8 parallel frame trials). Choice pairs 5 through 11 are only presented in minimal frames: these provide extra information needed for estimation of models[3] (as in our Appendices A) and act as spacing trials between repeated trials of the central pairs 1 to 4. As shown in Table 1, there were two possible outcomes for each subject: They could receive either $25 or $0. Restricting payouts to two possible outcomes, as in Ellsberg’s paradoxes, permits analysis of ambiguity attitudes that is not contaminated by risk attitudes.

Figure 2 (from Section 2) shows Basic Pair 1 in minimal and transparent frames, exactly as these were presented to subjects in the experiment. As shown in the figure, both minimal and transparent frames were monotonic in that payoffs decreased (weakly) monotonically from left to right. The frames were also standardized so that minimal and transparent frames were presented in the same table format with the column “N/12” denoting the number of die rolls from a twelve-sided die yielding the payoff in the column to the left. Subjects were informed that the die rolls

[3] Transparent frames “align” probabilities in a manner such that by definition, $\psi_{\mathrm{P}} \equiv 0$ for all transparent frame trials. Therefore, such trials produce no useful information for any kind of estimation of ψ_{P}. For this reason, the experimental design includes many more minimal frame trials than transparent frame trials. Estimations of the SWUP model, based on all of this data, will appear in another paper.

corresponding to each payoff increased from left to right. For instance, in Option B in Basic Pair 1, any die roll between 1 and 9 paid \$25 and any die roll between 10 and 12 paid \$0 if a red ticket was drawn. Likewise, any die roll between 1 and 3 paid \$25 and any die roll between 4 and 12 paid \$0 if a blue ticket was drawn.

For each choice situation, subjects had the opportunity of selecting "I prefer Option A," "I prefer Option B," or "I am indifferent between Option A and Option B." If a subject selected "I am indifferent" in a particular choice situation and that choice situation was selected for payment, the choice between Option A and Option B was resolved by the toss of a fair coin (which subjects were informed of in the instructions).

Especially in Basic Pairs 1, 2, 3 and 4, we wish to interpret the indifference response, which is a 50:50 randomization between options A and B, as actual indifference between them. To do so, we need the "certainty betweenness" property described by Grant and Polak (2013). We can state this property as follows. Let Ω denote the set of states ω and let X denote the set of outcomes. An objective lottery is a known probability distribution p on X. Denote the set of objective lotteries by $\mathcal{P}(X)$. A subjective lottery or act, h, is a mapping $h\colon \Omega \rightarrow \mathcal{P}(X)$ which assigns an objective lottery to each state. Denote the set of acts by $\mathcal{H}$. A constant act assigns the same objective lottery to every state. Essentially, certainty betweenness assumes that indifference between an objective lottery and a subjective lottery implies indifference between the objective lottery and any probabilistic mixture of the objective lottery and the subjective lottery. Formally, the certainty betweenness axiom says the following:

Axiom (Certainty Betweenness): For any $h \in \mathcal{H}$, any constant act $\kappa \in \mathcal{H}$, and any $\alpha \in (0,1)$: $h \sim \kappa \ \Rightarrow \alpha h + (1-\alpha)\kappa \sim \kappa$.

In the presence of their other axioms, Certainty Betweenness is implied by the weak certainty independence axiom of Gilboa and Schmeidler (1989) assumed in their multiple priors model. The subset of Grant and Polak's monotone mean-dispersion preferences which satisfy certainty betweenness and Gilboa and Schmeidler's (1989) uncertainty aversion axiom is the class of multiple priors preferences (Grant and Polak (2013), Corollary 3).

Certainty betweenness can be applied to basic pairs 1 through 7 in Table 1 since they involve a choice between an ambiguous act (option B) and a constant act (option A). Since our focus is on basic pairs 1 through 4, we will treat certainty betweenness as a maintained hypothesis throughout our study. In our data analyses, we are therefore restricting attention to theories of ambiguity aversion which satisfies certainty betweenness. In particular, in Appendix A we employ the mean-standard deviation preferences described in Grant and Kajii (2007) and Grant and Polak (2013). Since the standard deviation dispersion function is non-negative, convex, symmetric, and satisfies certainty betweenness, these preferences have a corresponding representation in the vector expected utility model (Siniscalchi, 2009), the invariant biseparable representation (Ghirardato et al., 2004), and the multiple prior representation (Gilboa and Schmeidler, 1989) provided that the mean-standard deviation preferences are monotone—a hypothesis we do not reject in our Appendix A analysis. In Appendix B we demonstrate that SWUP satisfies certainty betweenness if the salience functions exhibit homogeneity of degree 0, a property that Bordalo et al. (2013) argue is plausible for a salience function and which they invoke in their analysis of salience effects in consumer choice. Since the framing effect between minimal and transparent frames is predicted under general conditions by SWUP (for any salience function) it also holds for the class of salience functions exhibiting homogeneity of degree 0.

4.1 Materials and Payment

The sixty choice situations were distributed across three booklets of twenty situations each, with one situation displayed on each page. These booklets are available in a link in Appendix C, which also includes the experimental instructions. The experiment was conducted over a total of five sessions: The first session had 24 subjects; the second, third and fourth sessions each had 14 subjects; and the fifth session had 13 subjects. The order of the three booklets was counter-balanced across sessions: In the first session, subjects received booklets 1, then 2, and then 3 (the booklet numbering as presented in the link in Appendix C); In the second and third sessions, subjects received booklet 3 first, followed by booklet 1 and then booklet 2; and in the fourth and fifth sessions, subjects received booklet 2 first, followed by booklet 3 and then booklet 1.

From the beginning to the end of each experimental session, each subject had an opaque bag hanging in the corner of his or her carrel. Subjects were truthfully told that each bag contained an unknown mixture of ten red and/or blue raffle tickets, and that the mixture could differ between bags. Subjects were not allowed to look in or draw from the bag at their carrels until the end of the experiment.[4] After all subjects had made all sixty choices, payments to subjects were determined as follows: For each subject, a numbered card was drawn (with replacement) from a deck of 60 cards, numbered from 1 to 60. The number drawn determined which of the 60 choice situations would count for payment for that subject. Each subject would then have the opportunity to draw a ticket from his or her bag and roll a twelve-sided die. The subject's payment was determined by the state (red or blue), the number rolled, the choice situation selected for payment, and the option (A, B, or indifferent) selected by the subject in that choice situation. This payment was either $25 or $0, which was added to a flat $15 participation fee. Average earnings from the experiment (including the participation fee) were $26.01.

4.2 Subjects and Protocol

Seventy-nine[5] undergraduate students at a private Western university were recruited to participate in a study on economic decision making requiring less than two hours of their time[6]. Subjects were read the experimental instructions aloud while they followed along in their own copies of the instruction booklet. After explaining the nature of the choices in the experiment, subjects were quizzed for their understanding of the types of decisions they would be making. Two attendants checked subjects' answers and explained answers in the event of errors. Then subjects were quizzed once more, and any remaining errors (very rare at that point) were corrected and explained. An attendant then read a final overview of the events that would take place during the session, and subjects were handed their first (of three) booklets containing the choice situations. Subjects proceeded through the first booklet at their own pace. When all subjects had completed their first booklet, they were given a short booklet of unrelated and unpaid filler tasks.[7] Once all subjects completed this, they were handed their second booklet of choice situations and, once all subjects completed it, they were given another booklet of unrelated and unpaid filler tasks. Upon completing those tasks, subjects were given their third

[4] In our estimations in Appendices A, we assume that any prior probabilities subjects place on the red and blue ticket states are constant across the choice situations. Our placement of the bags with the subjects, from the start to the finish of their session, is meant to make this estimation assumption most plausible. If this design feature does enhance subjects' perception of the bag's composition as fixed across trials, this also makes the interpretation of "ambiguity-averse choices" clearer (given other features of the design mentioned earlier).

[5] The planned sample was 80 subjects. One subject failed to show in the final session.

[6] Each of the five experimental sessions lasted approximately 90 minutes.

[7] Repetition of the choice situations across the three booklets is masked somewhat by the insertion of filler tasks between booklets, which encourages forgetting of earlier situations and choices made in them.

and final booklet of choice situations. Payments were determined and distributed once all subjects had completed this final booklet.

5. Results

Our experimental design varies the assignment of ticket colors to objective lotteries to better test SEU; additionally, for standard experimental reasons, we also vary whether the more ambiguous act is in the top or bottom row of the choice table displayed to subjects, and the order in which the choice situations are presented to subjects. We begin by asking whether any of these variations have any significant effect on observed choices. Estimations meant to examine this matter appear in Appendix A.1. We find no significant evidence of any such "nuisance variance" so we proceed with our analysis, ignoring ticket color assignment, top-bottom position, and task order variations.

In Section 5.1 we plot the raw data which enables us to compare deviations from ambiguity neutrality in minimal and transparent frames. Section 5.2 presents a simple interpretive framework (not tied to a particular theory) to organize aggregated choice data across pairs 1, 2, 3 and 4. Within-subject results are discussed further in Section 5.3. On occasion we also refer to the more theory-driven estimation results that appear in Appendix A.2.

5.1 Comparing Deviations from Ambiguity Neutrality in Minimal and Transparent Frames

To motivate our measure of the framing effect, we first introduce some notation. Define the following function:

$$c_{lj}^{e} = \begin{cases} 1 & if\ subject\ e\ chooses\ the\ risky\ alternative\ in\ trial\ l\ of\ pair\ j \\ 0.5 & if\ subject\ e\ chooses\ indifference\ in\ trial\ l\ of\ pair\ j \\ 0 & if\ subject\ e\ chooses\ the\ ambiguous\ alternative\ in\ trial\ l\ of\ pair\ j \end{cases}$$

Let M_j denote the set of minimal frame trials l of pair j, and let T_j denote the set of transparent frame trials l of pair j. Then, $\bar{c}_T^e := \sum_{j=1}^{4} \sum_{l \in T_j} c_{lj}^e / 8$ is the proportion of (eight) transparent frame trials in which subject e makes an ambiguity-averse choice, and $\bar{c}_M^e := \sum_{j=1}^{4} \sum_{l \in M_j} c_{lj}^e / 18$ is the proportion of (eighteen) minimal frame trials in which subject e makes an ambiguity-averse choice. Taking $\bar{c}_M^e = \bar{c}_T^e = 0.5$ to be ambiguity neutrality, define the following primary dependent measure of the framing effect for each subject e:

$$F^e = |\bar{c}_M^e - 0.5| - |\bar{c}_T^e - 0.5| > 0.$$

When $F^e > 0$, subject e's behavior is closer to ambiguity neutrality in transparent frames than in minimal frames (the subject deviates more from ambiguity neutrality in minimal frames). Note that $F^e \leq 0$ would be contrary to our predictions. Of the 79 subjects in our experiment, we observe 46 subjects with $F^e > 0$, 9 with $F^e = 0$, and 24 with $F^e < 0$. In addition, the magnitude of F^e values is frequently larger for positive values than for negative values. For instance, we observe 7 subjects with $F^e < -0.10$ but 27 subjects with $F^e > 0.10$. We observe just one subject with $F^e < -0.20$, but 18 subjects with $F^e > 0.20$. Table 2 displays the value of F^e for each of the 79 subjects in our experiment.

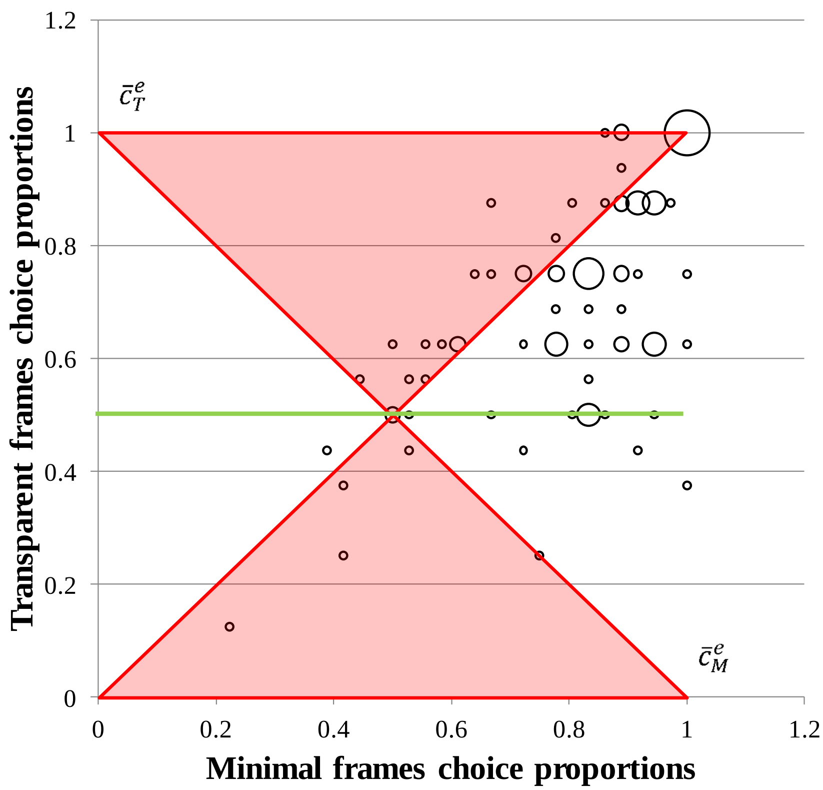
Table 2. Distribution of F^e Across All 79 Subjects

Cumulative	Frequency	F^e	Cumulative	Frequency	F^e
1	1	-0.208	49	4	0.083
2	1	-0.167	50	1	0.090
3	1	-0.139	52	2	0.097
4	1	-0.125	54	2	0.139
7	3	-0.111	55	1	0.146
8	1	-0.097	58	3	0.153
9	1	-0.083	59	1	0.160
11	2	-0.069	61	2	0.167
12	1	-0.049	62	1	0.201
14	2	-0.042	63	1	0.208
17	3	-0.035	64	1	0.250
19	2	-0.028	66	2	0.264
22	3	-0.014	67	1	0.271
24	2	-0.007	68	1	0.306
33	9	0.000	71	3	0.319
35	2	0.014	74	3	0.333
38	3	0.028	75	1	0.354
41	3	0.042	76	1	0.361
42	1	0.049	78	2	0.375
45	3	0.069	79	1	0.444

The "Cumulative" columns display the cumulative number of subjects exhibiting the corresponding value or a lower value of F^a. The "Frequency" column displays the number of subjects corresponding to each observed value of F^a.

Figure 6 graphs $\bar{c}_M^e$ and $\bar{c}_T^e$ on the horizontal and vertical axes, respectively. The shaded regions in the figure are the set of points inconsistent with the hypothesis that $F^e > 0$. Note that a large majority of points are in the unshaded regions, and the points that do fall in the shaded regions are close to the boundary. The graph thus provides strong initial support that most subjects exhibited the predicted framing effect. We also conducted a variety of two-tailed statistical tests against the null hypothesis of no effect ($F^e = 0$) at the aggregate level and consistently find strong support for the predicted framing effect (t-test: $p < 0.0001$; sign test: $p = 0.0115$; signed ranks test: $p < 0.0001$).

Figure 6. Ambiguity-averse choice proportions in two types of frames: Distribution across 79 subjects



Notes. Bubble sizes denote the number of subjects at each location. The smallest bubbles are a single subject, while the largest bubble (at the upper right in the figure) is six subjects.

5.2 An Aggregate Interpretation of the Data: A World with Two Types of Agents

We consider a simple descriptive framework for aggregate-level interpretation of our data, confining our attention here to the basic pairs 1, 2, 3 and 4 that were presented in both minimal and transparent frames. We view this framework as a 'lens' for interpreting our results, but not as a formal test of particular hypotheses. Our approach here does not commit to any ambiguity model but is based on a behavioral definition of ambiguity aversion and ambiguity neutrality in our experiment. The primary assumption of this framework is that our sampled population is composed of just two types of agents – those who are ambiguity-averse (defined as agents who always choose the risky lottery over the ambiguous one) and those who are ambiguity-neutral (those who either report indifference or randomize equally between choosing the risky and the ambiguous lottery).

The observed distribution of responses for the four basic pairs employed in both minimal and transparent frames are shown in Table 3, below. Recall that there are more responses in minimal frames since these pairs were repeated more often during the course of the experiment.

Table 3. Aggregate Distribution of Responses in Basic Pairs 1 – 4.

	Risky	Ambiguous	Indifference	Total
Minimal Frames	1072	255	95	1422
Transparent Frames	393	158	81	632

Denote the type of frame under consideration by $f \in \{m, t\}$, where m is a minimal frame and t is a transparent frame. Let θ_f denote the proportion of ambiguity-neutral agents in the population given type f frames for choice situations. In a world with only ambiguity-averse and ambiguity neutral agents, the proportion of ambiguity-averse agents will then be $1 - \theta_f$ given type f frames. Let ξ_f denote the probability that ambiguity-neutral subjects report indifference given type f frames and, when they do not, assume they randomize equally between choosing the risky or ambiguous option. This implies that the probabilities that ambiguity-neutral agents choose either the risky option or the ambiguous option given type f frames are equal to $(1 - \xi_f)/2$.

Let r_f, a_f, and i_f denote the number of observed choices of the risky option, the ambiguous option and the indifference option given type f frames. Denote the total number of choices made across all four basic pairs in frames of type f by $N_f \coloneqq r_f + a_f + i_f$. Note that under this framework, the choice of a risky option could be generated by either an ambiguity-averse agent or an ambiguity neutral agent. That is,

$$r_f = (1 - \theta_f)N_f + \theta_f \left(\frac{1 - \xi_f}{2}\right) N_f.$$

Also note that in this setup, choices of the ambiguous lottery could only be generated by ambiguity-neutral subjects according to the formula:

$$a_f = \theta_f \left(\frac{1 - \xi_f}{2}\right) N_f.$$

In addition, the number of indifference choices are given by the formula:

$$i_f = \theta_f \cdot \xi_f \cdot N_f.$$

From Table 3, we have $r_m = 1072$, $a_m = 255$, and $i_m = 95$ for minimal frames and $r_t = 393$, $a_t = 158$, and $i_t = 81$ for transparent frames. For both minimal and transparent frames, there are unique values for θ_f and ξ_f which exactly fit the distribution of observed choices.[8] In particular, it is straightforward to verify that $\theta_m = \left(\frac{605}{1422}\right) \approx 0.425$, $\xi_m = \left(\frac{95}{605}\right) \approx 0.157$, $\theta_t = \left(\frac{397}{632}\right) \approx 0.628$, and $\xi_t = \left(\frac{81}{397}\right) \approx 0.204$. These distributions of ambiguity-averse and ambiguity-neutral agents in the population (implied by this framework) are presented in Table 4. The framework implies that a fifth of our sampled population (62.8% − 42.5%) switches from ambiguity aversion to ambiguity neutrality when framing changes from minimal to transparent.

Table 4. Derived Distributions of Ambiguity-Averse and Ambiguity-Neutral Agents

	% Ambiguity-Averse	% Ambiguity-Neutral
Minimal Fames	57.5	42.5
Transparent Frames	37.2	62.8

[8] Note that while this simple model fits the observed data exactly, it would not do so if the proportion of ambiguity-seeking choices had been much higher (in particular, if $a_f > N_f/2$).

5.3 Index of Ambiguity Aversion

For each subject, we computed an ambiguity aversion index over the four basic pairs separately for transparent and minimal frames. For transparent frames, the ambiguity aversion index was constructed by computing the proportion of the four basic pairs for which a subject made consistent ambiguity-averse responses (choosing the risky lottery regardless of whether the red and blue states were switched). For minimal frames, since some of the basic pairs were repeated, the ambiguity aversion index was constructed by first computing, for each pair, the proportion of consistent ambiguity-averse responses in all possible combinations of paired choices for that pair. For example, Basic pair 1 was repeated three times in minimal frames and the corresponding version with the red-blue states switched was also repeated three times, resulting in nine paired choice combinations. The index was constructed by first computing the proportion of consistent ambiguity-averse responses for each pair out of these nine pairs. Given this index value for each basic pair, we can construct the ambiguity-averse index for each subject in minimal frames by summing these index values over all four basic pairs.

We found that 76% of subjects (60 of 79) had higher ambiguity aversion indices in minimal frames than in transparent frames and an additional 11% had the same index values in minimal and transparent frames. In addition, the average ambiguity aversion index value (with possible values varying from 0 to 4) was 2.58 in minimal frames and 1.71 in transparent frames.

6. Conclusions

Motivated by a new model of ambiguity aversion and by the success of Savage's conjecture in predicting the frame-dependence of the Allais paradox for choice under risk, we tested for an influence of framing on Ellsberg's paradox in decisions under uncertainty. We observed a highly significant effect framing effect in the direction predicted by the SWUP model in Leland and Schneider (2016). One important question warranting further investigation concerns the precise "locus" of the treatment effect. The econometric analysis in Appendix A enables us to test two competing hypotheses about this locus. The hypothesis that motivated our analysis is that subjects *want to* behave in an ambiguity neutral manner and do so when shown choices in transparent frames because in that frame the common consequences of all options are clearly visible, making the normative appeal of the sure-thing principle transparent.

An alternative hypothesis, based on Chew et al. (2016), is that subjects *do not want* to behave in an ambiguity neutral manner (i.e., they are truly ambiguity-averse) and only do so in transparent frames because they have a harder time recognizing the presence of ambiguity in such frames. Put differently, it might be that our framing effect does not move preferences closer to ambiguity neutrality, but rather increases the noise inherent in the decisions of subjects who are still ambiguity-averse. In Appendix A, we estimate a theory-driven econometric model based on the mean-dispersion preferences of Grant and Polak (2013). The model enables us to examine the locus of the observed treatment effect due to switching between minimal and transparent frames. In particular, the treatment effect may be due to a change in a *preference parameter* across frames (a decrease in ambiguity aversion), or the effect may be due to a change in a *precision parameter* across frames (an increase in decision noisiness).

Table A.3 in Appendix A provides four tests of these hypotheses, where each test is based on a specification allowing for a different kind of subject heterogeneity. In all four cases, we observe a significant reduction in the preference parameter representing ambiguity aversion due to the switch from minimal to transparent frames. In each case the effect is quite noticeable (roughly a 30% reduction in the ambiguity aversion parameter). In contrast, we find a significant decrease in a precision parameter (an increase in decision noise) under just one of the four specifications (and it is not the best-fitting specification). For these reasons, we believe our data provides stronger evidence that our observed framing effect reflects a genuine reduction in ambiguity aversion rather than a decreased ability to recognize ambiguity in such frames.

As noted in Section 2, a variety of fairly recent studies have investigated whether the Allais paradox is susceptible to framing. All of these studies (Leland, 2010; Bordalo et al., 2012; Incekara-Hafalir and Stecher, 2012; Birnbaum and Schmidt, 2015; Harman and Gonzalez, 2015) find significantly fewer violations of the independence axiom of expected utility theory, when the lotteries are recast from minimal frames (i.e., the standard ‘prospect’ presentation format) to transparent frames (i.e., the Savage matrix format). While the Ellsberg paradox violates a similar independence postulate, no such experiment has been conducted for ambiguity attitudes. In the present experiment, we find that the same types of frames which reduce Allais-type violations of objective expected utility theory also reduce Ellsberg-type violations of subjective expected utility theory.

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Appendix A. Econometric Analysis

Our econometric analyses are summarized in this appendix. In Appendix A.1, we conduct some analyses to investigate the impact of the experimentally induced sources of variance (changing the ticket color, switching the rows in which options were displayed, and reversing the order in which the pairs were presented) to determine whether they systematically influenced choices. Finding no evidence for such 'nuisance variance,' we proceed to a theory-driven econometric model in Appendix A.2.

Appendix A.1. Estimation of Experimentally Induced Sources of Variance

In this section we conduct a simple econometric analysis which does not rely on any particular theory of ambiguity preferences, to check whether any of the experimental manipulations had an impact on choices across all eleven pairs in Table 1.

Recall from Section 3 that in each choice situation s = 1 to 60, subjects can choose either Option A or Option B from a pair shown in Table 1, or may report indifference (which is resolved by a coin flip). Denote these three alternatives in situation s by $sk, k \in \{A, B, \sim\}$ and let $v_\tau^{sk} \in \{v_\tau^{sA}, v_\tau^{sB}, v_\tau^{s\sim}\}$ denote their values according to some theory, τ. A Luce model (McFadden's conditional logit) of choice probabilities P_τ^{sk} is

$$P_\tau^{sA} = exp(\lambda v_\tau^{sA})/D_\tau, P_\tau^{s\sim} = exp(\lambda v_\tau^{s\sim})/D_\tau, \text{ and } P_\tau^{sB} = exp(\lambda v_\tau^{sB})/D_\tau, \text{ where}$$

$$D_\tau = exp(\lambda v_\tau^{sA}) + exp(\lambda v_\tau^{s\sim}) + exp(\lambda v_\tau^{sB}).$$

and λ is a scale parameter, sometimes called "precision" or "sensitivity" (as $\lambda \to \infty$ the decision maker chooses the highest value alternative with certainty, and as $\lambda \to 0$ the decision maker chooses each of the three alternatives with a one-third probability). Dividing all terms by $exp(\lambda v_\tau^{sB})$, the choice probabilities can be rewritten as:

$$P_\tau^{sA} = exp[\lambda(v_\tau^{sA} - v_\tau^{sB})]/D_\tau, \quad P_\tau^{s\sim} = exp[\lambda(v_\tau^{s\sim} - v_\tau^{sB})]/D_\tau, \text{ and } P_\tau^{sB} = 1/D_\tau, \text{ where}$$

$$D_\tau = exp[\lambda(v_\tau^{sA} - v_\tau^{sB})] + exp[\lambda(v_\tau^{s\sim} - v_\tau^{sB})] + 1.$$

In all of our estimations, we will assume that subjects' preferences obey the certainty betweenness axiom (and the experimental design was predicated on this too, as noted in Section 4). In basic pairs 1 through 7, where option A is an objective lottery while option B is ambiguous, this assumption by itself, and the experimental design, imply that the value of the indifference response is $(v_\tau^{sA} + v_\tau^{sB})/2$. However, in pairs 8 through 11, options A and B are both ambiguous, so in those pairs there is a diversification motive giving the indifference response an increase in value. In our first estimation we will assume we can approximately capture that diversification utility by a quantity common to pairs 8 through 11. Let the parameter δ denote this quantity for pairs 8 through 11 (δ is zero for all other pairs). The diversification motive implies that we expect positive estimates of δ in our first estimation (in the Appendix A.2.1 estimations, the diversification motive is modeled in an explicit theoretical way without δ).

The Luce choice axiom may break down when some alternatives are more similar to each other than to other alternatives in a decision set. Because the indifference response is a mixture of options A and B, it might be deemed more similar to either of the options than they are similar to each other. Therefore, we also posit another quantity ϕ that will be added to the indifference value in all of our estimations, which we expect is negative (making indifference responses less probable than they would be if the choice axiom applied with no modifications). It is possible that ϕ could be positive if the coin flip makes the indifference response distinctive (or for other reasons), but our estimates of ϕ are always significantly negative in our data.

The preceding discussion entails that in our first estimation, we assume that $v_\tau^{s\sim} = (v_\tau^{sA} + v_\tau^{sB})/2 \ + \phi + \delta$ (with δ = 0 for pairs 1 through 7). This in turn implies that $v_\tau^{s\sim} - v_\tau^{sB} = \ (v_\tau^{sA} - v_\tau^{sB})/2 \ + \phi + \delta$. Defining $\Delta v_\tau^{s} \equiv v_\tau^{sA} - v_\tau^{sB}$, this modified conditional logit model becomes

$$P_\tau^{sA} = exp(\Delta v_\tau^s)/D_\tau,\ P_\tau^{s\sim} = exp[\lambda(\Delta v_\tau^s/2 \ + \phi + \delta)]/D_\tau, \text{ and } P_\tau^{sB} = 1/D_\tau,$$
$$\text{where } D_\tau \ = \ exp(\lambda \Delta v_\tau^s) + exp[\lambda(\Delta v_\tau^s/2 \ + \phi + \delta)] \ + \ 1.$$

The above formulation is appealing in that Δv^s has a straightforward interpretation as the difference between the values of Option A and Option B. The estimation in this section simply makes Δv^s a linear function of pair indicators and all the other experimentally induced sources of variance in responses—along with a random effect for subject-specific heterogeneity. The results of the first estimation are summarized in Table A.1.

Table A.1. Analysis of Effects of Experimentally Induced Sources of Variance

Meaning of estimated parameters	Estimates	Std. Error	p-value
Variance of subject-specific deviations from pair indicators	1.0	0.32	m[a]
A. effect of transparent frames (pairs 1, 2, 3, and 4)	−0.73	0.15	< 0.0001
B. effect of switching ambiguous states	0.074	0.12	0.55
C. effect of switching top and bottom row assignment	−0.073	0.13	0.57
D1. order effect—first twenty choice situations	−0.048	0.081	0.56
D3. order effect—last twenty choice situations	0.010	0.081	0.94
pair 1 indicator effect	1.8	0.84	0.039
pair 2 indicator effect	2.0	0.90	0.03
pair 3 indicator effect	1.7	0.83	0.045
pair 4 indicator effect	1.7	0.89	0.065
pair 5 indicator effect	−0.36	0.87	0.68
pair 6 indicator effect	−1.9	0.86	0.029
pair 7 indicator effect	3.3	0.82	0.0002
pair 8 indicator effect	2.3	0.95	0.017
pair 9 indicator effect	−0.0031	0.81	0.99
pair 10 indicator effect	0.98	0.87	0.27
pair 11 indicator effect	3.4	0.88	0.0003
Indifference parameter ϕ (all pairs)	−1.5	0.25	< 0.0001
Indifference parameter δ (pairs 8, 9, 10, and 11)	−0.38	0.23	0.11

Notes: [a]The "m" means that a p-value would be misleading in this case, since the natural null hypothesis (that the parameter equals zero) lies on the boundary of the parameter's allowable space (in this instance, the parameter is a variance).

The model estimated is close to a very standard one: A multinomial logit over three alternatives in each situation. The dependent variable is three-valued ($c \in \{1,0.5,0\}$) where $c = 1$ is the choice of Option A, $c = 0$ is the choice of option B and $c = 0.5$ is the indifference response. However, there are some special additions to that basic model: These include the two "indifference parameters" ϕ and δ at the bottom of Table A.1 where $\phi < 0$ means that indifference responses are less common than an unmodified conditional logit predicts, and the diversification motive in pairs 8 through 11 implies an expectation that $\delta > 0$ (but this is not borne out by the estimation). Second, to control for the correlation between the choices of each individual subject, a random effects specification (a normal distribution of subject-specific deviations from the pair indicator estimates) is included which helps to reduce Type I error. The estimated variance of these effects is presented in the first line in Table A.1. We next provide some interpretation of the other results in Table A.1:

Row A. This row presents the estimated deviation from the pair indicators (restricted to basic pairs 1 through 4) due to transparent versus minimal framing. It is negative and highly significant: Transparent frames reduced ambiguity aversion as predicted.

Row B. Every pair was presented in two ways, with either the red ticket or the blue ticket being the better state in the relatively ambiguous option. The insignificance of this effect says that there is no mean effect on ambiguous choices of this manipulation, suggesting, on average, subjects may have equal priors of the red and blue ticket states (see Appendix A.2 for further discussion).

Row C. In the presentations of the choice situations it is usually true that the top row of each presentation is option A while the bottom row is option B, but in twelve of the sixty choice situations this was reversed. The insignificance of this effect says that there is no mean effect on ambiguous choices of this manipulation, which suggests that we have no empirically important "response set" issue in our experiment.

Rows D1 and D3. We grouped our sixty choice situations into three booklets of twenty situations each and systematically varied the order in which subjects encountered the three booklets, so that each booklet was either the first, second or third booklet subjects encountered. The insignificance of these two effects suggests that we have no appreciable order effects.

All the effects described above, except the framing effect, are parameterized as deviations from the estimated pair indicator effects. So estimated pair-specific indicator effects in the table are interpreted as $\Delta v_{\tau}^{s} \equiv v_{\tau}^{sA} - v_{\tau}^{sB}$ under minimal framing. A significantly positive (negative) value of a pair intercept means that, on average, subjects prefer Option A (Option B) in that pair when presented in a minimal frame. The only pairs (5, 6 and 9) with negative estimates are pairs where Option B, which is usually the relatively ambiguous option, has an appreciably higher subjective expected value under the assumption of equal prior probabilities assigned to states.

Pair 9 is particularly interesting in that it is the only pair in which the pair-specific effect is not significantly different from zero. We can interpret that to mean that "the average subject" is indifferent between the two options in pair 9 under minimal framing. In pair 9, under the assumption of equal prior probabilities of the red and blue ticket draws, the relatively more ambiguous option's subjective expected value exceeds that of the relatively less ambiguous option by about $2.08. We can think of this as one measure of an ambiguity premium in this particular pair (under minimal framing).

Appendix A.2. A Theory-driven Empirical Model

In this appendix, we analyze our data under the lens of a theory-driven empirical model of ambiguity attitudes. We first present an overview of our approach and then proceed with a more detailed analysis. Let $\mathcal{M} = \{s \mid \text{situation } s \text{ is minimal framed}\}$, let $\mathcal{T} = \{s \mid \text{situation } s \text{ is transparent framed}\}$, and note that $s \in \mathcal{S} = \{1,2,\ldots,60\} \equiv \mathcal{M} \cup \mathcal{T}$. Also, let

(A.1) $$L_{\tau(s)}(c^s) = 1(c^s = 1)P_\tau^{sA} + 1(c^s = 0)P_\tau^{sB} + 1(c^s = 0.5)P_\tau^{s\sim}$$

be the likelihood of observation c^s, given a conditional logit model of choice probabilities as introduced generally in Section 4, eq. (1), and given a particular theory $\tau(s)$ governing choice in situation s. (Recall that a theory τ is a specification of v_τ^{sk} for each choice situation s, in the conditional logit model). The likelihood in eq. (A.1) is actually conditioned on all of the parameters that govern the probabilities P_τ^{sk}, but these are suppressed for readability.

The framework allows two different theories to govern choices, with just one theory governing choice in each situation s. In the theory-driven empirical model developed here, we choose subjective expected utility or SEU (indexed as theory $\tau = \alpha$) as an ambiguity-neutral theory, and a special case of Grant and Polak's (2013) "mean dispersion preferences" or MD (indexed as theory $\tau = \beta$) as a theory permitting ambiguity aversion and other ambiguity attitudes, depending on its special parameter as introduced below. We imagine four potential types of subjects in our subject population:

(1) $\alpha\alpha$ type: $\tau(s) = \alpha \ \forall \ s$;

(2) $\beta\alpha$ type: $\tau(s) = \beta \ \forall \ s \in \mathcal{M}, \tau(s) = \alpha \ \forall \ s \in \mathcal{T}$;

(3) $\alpha\beta$ type: $\tau(s) = \alpha \ \forall \ s \in \mathcal{M}, \tau(s) = \beta \ \forall \ s \in \mathcal{T}$; and

(4) $\beta\beta$ type: $\tau(s) = \beta \ \forall \ s$.

There are four possible likelihoods for our observations, depending on these types:

(1) Type $\alpha\alpha$ subjects act as SEU agents regardless of frame: We expect them to be rare.

(2) Type $\beta\alpha$ subjects are one potentially important locus for our treatment effect: These subjects act as MD agents in minimal frames and SEU agents in transparent frames.

(3) We expect the type $\alpha\beta$ to be very rare: These are subjects who act as SEU agents in minimal frames and MD agents in transparent frames, the opposite of our hypotheses.

(4) Type $\beta\beta$ subjects act as MD agents regardless of frame. They are the second potentially important locus for our treatment effect since we may also allow the degree of ambiguity aversion to be frame-dependent: It could happen that some subjects are ambiguity-averse overall, but less so in transparent frames than in minimal frames.

To complete the big picture of the framework, we let $\theta_{\tau\tau'} \in \{\theta_{\alpha\alpha}, \theta_{\beta\alpha}, \theta_{\alpha\beta}, \theta_{\beta\beta}\}$ be the proportions of our subject population that are of each type noted above. Then our complete likelihood function for a randomly selected subject is:

(A.2) $$L = \theta_{\alpha\alpha} \prod_s L_\alpha(c^s) + \theta_{\alpha\beta} \prod_{s\in\mathcal{M}} L_\alpha(c^s) \prod_{s\in\mathcal{P}} L_\beta(c^s) + \theta_{\beta\alpha} \prod_{s\in\mathcal{M}} L_\beta(c^s) \prod_{s\in\mathcal{P}} L_\alpha(c^s) + \theta_{\beta\beta} \prod_s L_\beta(c^s).$$

To employ this model, we need to specify the v_α^{sk} and v_β^{sk} formulas for SEU and MD, respectively. This is done in the following section.

A.2.1 A theory-driven empirical framework: Details of the two theory specifications

As noted in Section 3, since indifference responses were resolved by the toss of a coin, we have implicitly assumed the certainty betweenness property formalized in Grant and Polak (2013). We thus restrict our analysis to models of ambiguity attitudes which satisfy certainty betweenness. In particular, we adopt a type of mean-dispersion preferences from Grant and Polak (2013). The mean-dispersion class of preferences characterizes ambiguity attitudes which can be expressed in the form:

$$\mu(h, \pi) - \rho(d)$$

where $\mu(h, \pi)$ is the mean utility of act h with respect to a vector probability distribution π, and ρ is a dispersion function which aggregates the vector d of state-by-state deviations from the mean utility (i.e., d has representative element $d^\omega \coloneqq U\big(h(\omega)\big) - \mu(h, \pi)$, where $U\big(h(\omega)\big)$ is the expected utility of h in state ω).

We consider the particular case where the dispersion function is the standard deviation: That is, $\rho(d) \coloneqq \gamma \left[\sum_\omega \pi^\omega \left(U\big(h(\omega)\big) - \mu(h, \pi)\right)^2\right]^{0.5}$. These preferences are monotone over the domain $0 < \gamma^2 < \min_{\omega\in\bar{\Omega}}(\pi^\omega/(1 - \pi^\omega))$, where $\bar{\Omega} \coloneqq \{\omega \in \Omega{:}\, \pi^\omega > 0\}$. Over this domain, the dispersion function is also non-negative, convex, linearly homogeneous (which follows from certainty betweenness) and symmetric. Therefore, these preferences also have a vector expected utility representation (Siniscalchi, 2009), an invariant biseparable representation (Ghirardato et al., 2004) and a multiple priors representation (Gilboa and Schmeidler, 1989) (see Grant and Polak, 2013 for a discussion of these preferences and their properties). In this respect, our analysis is robust to some other well-known characterizations of ambiguity attitudes.

Let $u(\$25) = 1$ and $u(\$0) = 0$. Let r and b denote the red and blue ticket states, respectively. Also let p_r^{sk} and p_b^{sk} denote the objective (twelve-sided die roll) probabilities of receiving \$25, conditional on drawing a red or blue ticket, respectively, in options $sk \in \{sA, sB\}$. Then the subject's two possible objective expected utilities in the options sk, conditional on the ticket color drawn from the ticket bag, are simply p_r^{sk} and p_b^{sk}.

Assume further that π_τ denotes a subject's subjective probability of drawing a red ticket in situations where her choice behavior is governed by theory τ. We take π_τ to be constant across all options in those situations, and the experimental design encourages subjects to have that view since their individual ticket bags remain with them at all times during an experimental session. We might even argue that the design encourages subjects to view π_τ as constant regardless of the theory τ, but the econometric framework will allow it to vary with the theory. Then write μ_τ^{sk}, the subjective mean utility in alternative sk, given π_τ, as

$$\mu_\tau^{sk} = \pi_\tau p_r^{sk} + (1 - \pi_\tau) p_b^{sk}.$$

For SEU, that is for $\tau = \alpha$, we simply have $v_\alpha^{sk} = \mu_\alpha^{sk} = \pi_\alpha p_r^{sk} + (1 - \pi_\alpha) p_b^{sk}$. For MD, that is for $\tau = \beta$, we have (from the mean-standard deviation preferences in Grant and Polak 2013),

$$v_\beta^{sk} = \mu_\beta^{sk} - \gamma_\beta \rho_\beta^{sk}, \text{ where}$$

$$\rho_\beta^{sk} = \sqrt{\pi_\beta (p_r^k - \mu_\beta^{sk})^2 + (1 - \pi_\beta)(p_b^k - \mu_\beta^{sk})^2} = \left| p_r^k - p_b^k \right| \sqrt{\pi_\beta (1 - \pi_\beta)}.$$

The probabilistic model is almost the same one used previously for the estimation reported in Table 2, except that we now can drop the diversification motive parameter δ for indifference responses since this will be represented explicitly in $\rho_\beta^{s\sim}$, the dispersion term for the indifference response. The linear combination of the values of options sA and sB suggests a value of the indifference response, $v_\tau^{s\sim}$, equal to $(v_\tau^{sA} + v_\tau^{sB})/2$. When SEU governs behavior ($\tau = \alpha$) this is always correct; and because MD preferences satisfy certainty betweenness it will also be correct for pairs 1 through 7 when MD governs behavior ($\tau = \beta$). But in general (and specifically, in pairs 8 through 11), this is not quite right when MD governs behavior. In general, we have

$$(v_\beta^{sA} + v_\beta^{sB})/2 = \frac{1}{2}\left(SEU_\beta^{sA} + SEU_\beta^{sB}\right) - \gamma_\beta \frac{1}{2}\left(\left|p_r^{sA} - p_b^{sA}\right| + \left|p_r^{sB} - p_b^{sB}\right|\right)\sqrt{\pi_\beta(1-\pi_\beta)}.$$

However, when MD governs behavior, the actual value of the indifference response is

$$v_\tau^{s\sim} = \frac{1}{2}\left(SEU_\beta^{sA} + SEU_\beta^{sB}\right) - \gamma_\beta \frac{1}{2}\left|(p_r^{sA} - p_b^{sA}) + (p_r^{sB} - p_b^{sB})\right|\sqrt{\pi_\beta(1-\pi_\beta)}.$$

In terms of MD preferences, then, the former parameter δ was representing the difference

$$\gamma_\beta \frac{1}{2}\left[\left|p_r^{sA} - p_b^{sA}\right| + \left|p_r^{sB} - p_b^{sB}\right| - \left|(p_r^{sA} - p_b^{sA}) + (p_r^{sB} - p_b^{sB})\right|\right]\sqrt{\pi_\beta(1-\pi_\beta)} \geq 0,$$

which is the diversification motive for an indifference response in situation s. Notice that this is zero whenever $p_r^k - p_b^k = 0$ for either $k = sA$ or $k = sB$ (as is the case in pairs 1 through 7 for option A), so the construction of $v_\tau^{s\sim}$ above also satisfies certainty betweenness where it should.

With explicit expressions for the v_τ^{sk}, as given above for sk equal to sA, sB, or $s\sim$, the new modified conditional logit model for each theory τ will be

$$P_\tau^{sA} = exp(\lambda_\tau v_\tau^{sA})/D_\tau,\ P_\tau^{s\sim} = exp[\lambda_\tau(v_\tau^{s\sim} + \phi_\tau)]/D_\tau, \text{ and } P_\tau^{sB} = exp(\lambda_\tau v_\tau^{sB})/D_\tau, \text{ where}$$

$$D_\tau = exp(\lambda_\tau v_\tau^{sA}) + exp[\lambda_\tau(v_\tau^{s\sim} + \phi_\tau)] + exp(\lambda_\tau v_\tau^{sB}).$$

Notice that we still include the parameter ϕ_τ to account for the indifference response being less (or more) common than is predicted by the unmodified conditional logit, and allow it to depend on the theory governing behavior.

The framework outlined above is a finite mixture model with four types of agents. This model has ten parameters, summarized as follows:

- π_α and π_β (subjective probabilities assigned to red tickets, under theories α and β);
- λ_α and λ_β, precision parameters of subjects, for theories α and β;
- γ_β, disutility of mean utility dispersion when theory β governs behavior;
- ϕ_α and ϕ_β indifference response probability modifiers, for theories α and β; and
- $\theta_{\alpha\alpha}$, $\theta_{\beta\alpha}$, and $\theta_{\beta\beta}$, population proportions of three of the four subject types ($\theta_{\alpha\beta} = 1 - \theta_{\alpha\alpha} - \theta_{\beta\alpha} - \theta_{\beta\beta}$, and so is not another independent parameter).

Table A.2 shows results from this estimation. First, we see that neither π_α nor π_β are significantly different from 0.5, which indicates that, at the aggregate level, subjects have a uniform prior over the ticket colors; and both π_α and π_β are estimated with high precision. This implies that $\pi_\beta/(1-\pi_\beta) \approx 1$, and any conventionally-sized confidence interval for the estimate of γ_β is well within the interior of [0,1]. As mentioned previously, Grant and Polak's (2013) work shows that when $\gamma_\beta^2 < 1 \approx \pi_\beta/(1-\pi_\beta)$ and there are just two states, the MD preference model will be monotone. Since $\gamma_\beta$is significantly positive, ambiguity aversion is the norm when MD governs behavior. The parameter ϕ_τ is negative for both SEU and MD: indifference responses are less common than expected from an unmodified conditional logit model. The estimates for $\theta_{\alpha\alpha}$ and $\theta_{\beta\beta}$ imply that 17% of subjects are ambiguity-neutral (SEU agents) in both minimal and transparent frames and that 55% of subjects are MD agents in both types of frames.

The estimate $\theta_{\beta\alpha}$ implies that 27% of subjects switch from ambiguity aversion to ambiguity neutrality when framing changes from minimal to transparent, with only about 1% of subjects switching in the opposite direction. Given the estimated standard error of $\theta_{\beta\alpha}$, this does not differ significantly from the 20% increase in ambiguity-neutral behavior from transparent framing inferred from the simple descriptive framework discussed in Section 5.1.

Table A.2. Parameter Estimates for Mixture Model

Parameter	Estimates	Std. Error	p-value
π_α	0.49	0.0075	0.30[a]
π_β	0.51	0.0036	0.11[a]
λ_α	24	4.6	m[b]
λ_β	18	1.6	m[b]
γ_β	0.32	0.021	< 0.0001[c]
ϕ_α	−0.031	0.0079	0.0002[c]
ϕ_β	−0.12	0.014	< 0.0001[c]
$\theta_{\alpha\alpha}$	0.17	0.047	m[b]
$\theta_{\beta\alpha}$	0.27	0.09	m[b]
$\theta_{\beta\beta}$	0.55	0.098	m[b]

Notes:
[a]Against the null $\pi_\tau = 0.5$, two-tailed.
[b]m means "misleading." The problem is that the natural values for nulls are boundary points of the parameter spaces and *t*-statistics against boundary points are not statistically sound.
[c]Against the zero null, two-tailed.

A.2.2 Alternative sources for the treatment effect

Aside from switching between MD and SEU preferences when moving from minimal to transparent frames, there are two other plausible ways in which frames might yield the treatment effect, both having to do with our most common type according to the estimates above—the $\beta\beta$ type. One possibility is that many subjects are always MD types, but their ambiguity aversion is lessened under transparent frames. We examine this by letting there be two values of the γ_β parameter, $\gamma_{\beta m}$ and $\gamma_{\beta t}$, ambiguity attitudes given minimal and transparent frames, respectively. For estimation and hypothesis-testing, we parameterize this differently, as

$$\gamma_\beta = \gamma_{\beta m} - 1(s \in \mathcal{T})\Delta\gamma_{\beta t}, \text{ where } \Delta\gamma_{\beta t} \equiv \gamma_{\beta t} - \gamma_{\beta m},$$

and estimate $\gamma_{\beta m}$ and $\Delta\gamma_{\beta t}$. Then the sign and significance (or lack of it) of the estimate of $\Delta\gamma_{\beta t}$ tells us whether this possibility has empirical support.

As discussed in Sections 7 and 8, there is a second possibility suggested to us by the Chew et al. (2016) working paper titled "You Need to Recognize Ambiguity to Avoid It." Under this possibility, although transparent frames make the common components of each option transparent, such frames may dampen subjects' ability to "recognize ambiguity" and hence to avoid it. In terms of model parameters, this would be manifested in the precision parameter λ_β taking on lower values under transparent framing—that is, noisier decision making given transparent framing. This possibility is a quite different kind of explanation as to why transparent framing might reduce observed ambiguity aversion.

We investigate this latter possibility by letting there be two values of the λ_β parameter, $\lambda_{\beta m}$ and $\lambda_{\beta t}$, precision parameters given minimal and transparent frames, respectively. Again, for estimation and hypothesis-testing purposes, we parameterize this differently, as

$$\lambda_\beta = \lambda_{\beta m} - 1(s \in \mathcal{T})\Delta\lambda_{\beta t}, \text{ where } \Delta\lambda_{\beta t} \equiv \lambda_{\beta t} - \lambda_{\beta m},$$

and estimate $\lambda_{\beta m}$ and $\Delta\lambda_{\beta t}$. Then the significance (or lack of it) of the estimate of $\Delta\lambda_{\beta t}$ tells us whether this possibility has empirical support.

Table A.3 shows our estimates after we add these two ways in which the change to transparent frames might affect behavior. In Table A.3, subjective probabilities continue to be tightly estimated as indistinguishable from uniform priors for both SEU and MD agents. The two standard errors confidence interval for the coefficient of aversion to dispersion, $\gamma_{\beta m}$, is still well inside the interior of [0,1], suggesting that MD preferences are both monotone and ambiguity-averse. Given the p-value for $\Delta\lambda_{\beta t}$, there is no evidence that transparent frames make choices noisier when the MD theory governs behavior. The p-value for $\Delta\gamma_{\beta t}$ provides weak evidence that ambiguity aversion decreases with transparent framing when the MD theory governs behavior. The proportion of subjects now estimated to switch from ambiguity aversion in minimal frames to ambiguity neutrality in transparent frames is about 14%, and within a standard error of the 20% estimate from the simple saturated model in the text.

Table A.3. Parameter Estimates when γ_β and λ_β may be frame-dependent

Parameter	Estimates	Std. Error	p-value
π_α	0.49	0.0078	0.39[a]
π_β	0.5	0.0033	0.19[a]
λ_α	25	5.4	m[b]
$\lambda_{\beta m}$	19	1.6	m[b]
$\Delta\lambda_{\beta t}$	1.7	8.0	0.83[c]
$\gamma_{\beta m}$	0.32	0.020	< 0.0001[c]
$\Delta\gamma_{\beta t}$	−0.15	0.083	0.067[c]
ϕ_α	−0.023	0.0068	0.0013[c]
ϕ_β	−0.11	0.013	< 0.0001[c]
$\theta_{\alpha\alpha}$	0.16	0.046	m[b]
$\theta_{\beta\alpha}$	0.14	0.066	m[b]
$\theta_{\beta\beta}$	0.70	0.077	m[b]

Notes:
[a]Against the null $\pi_\tau = 0.5$, two-tailed.
[b]m means "misleading." The problem is that the natural values for nulls are boundary points of the parameter spaces and *t*-statistics against boundary points are not statistically sound.
[c]Against the zero null, two-tailed.

A.2.3 Accounting for Other Sources of Heterogeneity

Thus far, the only heterogeneity we have allowed for is the finite mixture of the four discrete subject types—through the $\theta_{\tau\tau'}$ parameters. It is worthwhile to add in some allowance for continuously distributed heterogeneity of the subjects within each of the four subject classes—mainly to see whether the results above are robust to such heterogeneity. Really, any of the parameters could have that kind of heterogeneity, and the framework can be generalized to include this. However, given that we have just 79 subjects, we think adding in continuously distributed heterogeneity of more than one parameter at a time pushes the between-subjects variational information we have too hard. Therefore, we have introduced heterogeneity of a single parameter, one parameter at a time, to check for changes in the estimates of our three key parameters $\theta_{\beta\alpha}$, $\Delta\gamma_{\beta p}$, and $\Delta\lambda_{\beta p}$, and to see which most likely drives the treatment effect.

Generally, let φ stand in for any parameter in our model, and suppose we knew that this parameter followed some distribution $F(\varphi|\eta)$ within each of our subject types (each viewed as a subpopulation of the whole subject population), where η is the vector of parameters (e.g. the location and scale of φ viewed as a random variable) governing the distribution F. Then we can modify eq. (A.1) to make the dependence of the likelihoods on φ explicit.

$$\text{(A.3)} \qquad L_{\tau(s)}(c^s|\varphi) = 1(c^s = 1)P_\tau^{sA}(\varphi) + 1(c^s = 0)P_\tau^{sB}(\varphi) + 1(c^s = 0.5)P_\tau^{s\sim}(\varphi)$$

We may then integrate out explicit dependence on φ, substituting for it the dependence on the parameters η of φ's distribution $F(\varphi|\eta)$, to obtain a generalization of eq. (A.2):

$$\text{(A.4)} \qquad L(\eta) = \theta_{\alpha\alpha} \int \prod_s L_\alpha(c^s|\varphi)\, dF(\varphi|\eta) +$$

$$\theta_{\alpha\beta} \int \prod_{s\in\mathcal{M}} L_\alpha(c^s|\varphi) \prod_{s\in\mathcal{P}} L_\beta(c^s|\varphi)\, dF(\varphi|\eta) +$$

$$\theta_{\beta\alpha} \int \prod_{s\in\mathcal{M}} L_\beta(c^s|\varphi) \prod_{s\in\mathcal{P}} L_\alpha(c^s|\varphi)\, dF(\varphi|\eta) +$$

$$\theta_{\beta\beta} \int \prod_s L_\beta(c^s|\varphi)\, dF(\varphi|\eta).$$

We need to pick a distribution $F(\varphi|\eta)$ to proceed further. We will use a normal distribution everywhere, sometimes converted into a lognormal distribution where we need to restrict the parameter in question to non-negative values (the precision parameters λ for instance). In our estimations, the required integrations are performed numerically by Gaussian quadrature.

Indifference responses suggest one kind of marked heterogeneity amongst our subjects. Near half of the subjects (37 of 79) never use the indifference response, but a fifth of the subjects (16 of 79) use it in more than 6 of the 60 situations. For this reason, one estimation introduces heterogeneity in the parameters ϕ_α and ϕ_β. Estimates of π_α and π_β in Tables A.1 and A.2, are very close to 0.5, but it is possible that there is variance of these priors across subjects. Especially in the case of π_α, we would like to know whether that was significantly in evidence. Therefore, we include an estimation with heterogeneity in π_α in subpopulations where it is present[9]. We also include an estimation with heterogeneity of 'precision' parameters λ since ignoring it (when it is present) is known to result in biased estimation of other parameters in other settings (Wilcox 2006). Finally, it seems plausible, even likely, that ambiguity aversion varies a good deal across the subjects who exhibit it, so we also include an estimation with heterogeneity in γ_β in subpopulations where it is present. Table A.4 shows the results of these four estimations.

From Table A.4, we see that the strongest evidence for the locus of our treatment effect is in the γ_β parameter: In particular, $\Delta\gamma_{\beta t}$ is consistently significant at $p < 0.01$ across all four of these estimations. In addition, the effect size for $\Delta\gamma_{\beta t}$ is large, relative to the size of $\gamma_{\beta m}$, consistently indicating between a 30% to a 50% reduction in ambiguity aversion (as inferred by the ratio of estimates $\Delta\gamma_{\beta t}/\gamma_{\beta m}$) when switching from minimal to transparent frames. The second possible locus for the treatment effect is that subjects act as MD agents in minimal frames and as SEU agents in transparent frames. We estimate that this subpopulation comprises between 7% and 14% of all subjects, depending on which parameter in Table A.4 is varied to account for heterogeneity. In contrast, we estimate that roughly 70% to 90% of subjects are MD agents in both minimal and transparent frames, but are markedly less ambiguity-averse in transparent frames. Thus, it appears that transparent frames significantly reduced ambiguity aversion for a very large proportion of subjects, but a relatively small fraction of the population shifted all the way to ambiguity neutrality. The third possible locus of the treatment effect, a change in the precision parameters given by $\Delta\lambda_{\beta t}$, is insignificant in three of four estimations, but is significant at the 0.01 level in the estimation with heterogeneity of the γ_β parameter. In the best-fitting estimation—that with heterogeneity of the indifference parameters ϕ_τ—there is no evidence that the precision parameters are driving the treatment effect.

[9] Adding in heterogeneity of π_β is not sensible since this is mathematically very similar to adding multiple priors to the mean-dispersion theory. Since our specification of the mean-dispersion preferences is a mathematical re-writing of multiple priors preferences in the neighborhood of our model estimates, adding heterogeneity of π_β results in a statistical model that is very poorly identified.

Table A.4. Parameter Estimates with heterogeneity within the four subpopulations

Parameter	Estimate (Std. Error and p-value below in parenthesis) in eq. (5) framework model allowing for heterogeneity of...			
	π_α	λ parameters	γ_β parameters	ϕ parameters
π_α	0.49 (0.025, p = 0.58)	0.50 (0.0013, p = 0.46)	0.46 (0.058, p = 0.44)	0.49 (0.014, p = 0.42)
π_β	0.50 (0.0034, p =0.27)	0.50 (0.0030, p = 0.28)	0.51 (0.0035, p = 0.11)	0.51 (0.0034, p = 0.09)
λ_α	28 (7.4, p = m)	32 (1.7, p = m)	36 (26, p = m)	24 (6.4, p = m)
$\lambda_{\beta m}$	19 (1.5, p = m)	18 (1.7, p = m)	21 (1.7, p = m)	20 (1.6, p = m)
$\Delta\lambda_{\beta t}$	1.5 (3.8, p = 0.69)	–3.2 (1.9, p = 0.10)	–4.1 (1.6, p = 0.01)	–1.1 (4.3, p = 0.80)
$\gamma_{\beta m}$	0.31 (0.027, p < 0.01)	0.30 (0.026, p < 0.01)	0.29 (0.040, p < 0.01)	0.32 (0.021, p < 0.01)
$\Delta\gamma_{\beta t}$	–0.15 (0.036, p < 0.01)	–0.12 (0.031, p < 0.01)	–0.11 (0.022, p < 0.01)	–0.13 (0.049, p < 0.01)
ϕ_α	–0.0098 (0.026, p = 0.71)	0.013 (0.0056, p = 0.03)	0.021 (0.0053, p < 0.01)	–0.020 (0.011, p = 0.07)
ϕ_β	–0.12 (0.019, p < 0.01)	–0.16 (0.024, p < 0.01)	–0.099 (0.015, p < 0.01)	–0.18 (0.021, p < 0.01)
$\theta_{\alpha\alpha}$	0.14 (0.059, p = m)	0.063 (0.031, p = m)	0.050 (0.027, p = m)	0.17 (0.046, p = m)
$\theta_{\beta\alpha}$	0.14 (0.068, p = m)	0.077 (0.043, p = m)	0.071 (0.044, p = m)	0.11 (0.050, p = m)
$\theta_{\beta\beta}$	0.72 (0.073, p = m)	0.86 (0.051, p = m)	0.88 (0.054, p = m)	0.72 (0.063, p = m)
σ_{π_α}	0.053 (0.029, p = m)	—	—	—
$\sigma_{ln(\lambda)}$	—	0.45 (0.031, p = m)	—	—
σ_{γ_β}	—	—	0.14 (0.018, p = m)	—
σ_ϕ	—	—	—	0.13 (0.017, p = m)
Log Likelihood	−3068.45	−2934.87	−2924.93	−2873.56

Notes: "p = m" means that a p-value is misleading. The problem is that the natural values for nulls are boundary points of the parameter spaces and t-statistics against boundary points are not statistically sound.

Appendix B: A Sufficient Condition for SWUP to Satisfy Certainty Betweenness

Consider the following frame involving a choice between an objective lottery, κ, and subjective lottery, h**.** This setup encompasses basic pairs 1 through 4 from our experiment as special cases.

	Red Ticket				Blue Ticket			
κ	\$25	p	\$0	$1-p$	\$25	p	\$0	$1-p$
h	\$25	q_r	\$0	$1-q_r$	\$25	q_b	\$0	$1-q_b$

Let r and b denote the red and blue ticket states, respectively. Normalize $U(25) = 1$, and $U(0) = 0$. Under SWUP, $h \sim \kappa \Leftrightarrow \pi^r \psi_{\mathrm{P}}(p, q_r)(p - q_r) + (1 - \pi^r)\psi_{\mathrm{P}}(p, q_b)(p - q_b) = 0$. Lottery κ and the mixture $g \coloneqq \alpha\kappa + (1 - \alpha)h$ are shown below in the red and blue ticket states:

	Red Ticket							
$\kappa(r)$	\$25	αp	\$0	$\alpha(1-p)$	\$25	$(1-\alpha)p$	\$0	$(1-\alpha)(1-p)$
$g(r)$	\$25	αp	\$0	$\alpha(1-p)$	\$25	$(1-\alpha)q_r$	\$0	$(1-\alpha)(1-q_r)$

	Blue Ticket							
$\kappa(b)$	\$25	αp	\$0	$\alpha(1-p)$	\$25	$(1-\alpha)p$	\$0	$(1-\alpha)(1-p)$
$g(b)$	\$25	αp	\$0	$\alpha(1-p)$	\$25	$(1-\alpha)q_b$	\$0	$(1-\alpha)(1-q_b)$

For basic pairs 1 through 4, certainty betweenness implies condition (B.1) for SWUP:

$$\text{(B.1)} \quad \alpha\kappa + (1-\alpha)h \sim \kappa \Leftrightarrow \pi^r \psi_{\mathrm{P}}\big((1-\alpha)p, (1-\alpha)q_r\big)(1-\alpha)(p - q_r)$$
$$+(1-\pi^r)\psi_{\mathrm{P}}\big((1-\alpha)p, (1-\alpha)q_b\big)(1-\alpha)(p - q_b) = 0.$$

Bordalo et al. (2013) argue that *homogeneity of degree zero* is a plausible property of a salience function and they assume that property in their analysis of salience in consumer choice. They define homogeneity of degree zero as follows: $\psi(\alpha x, \alpha y) = \psi(x, y)$ for all $\alpha > 0$.

Under homogeneity of degree zero, (B.1) reduces to (B.2):

$$\text{(B.2)} \; \alpha\kappa + (1-\alpha)h \sim \kappa \Leftrightarrow (1-\alpha)[\pi^r\, \psi_{\mathrm{P}}(p, q_r)(p - q_r) + (1-\pi^r)\psi_{\mathrm{P}}(p, q_b)(p - q_b)] = 0$$

Note that (B.2) reduces to the condition for $h \sim \kappa$, and thus certainty betweenness holds under SWUP for basic pairs 1 through 4 if the probability salience function satisfies homogeneity of degree zero. A 'parameter-free' salience function, introduced by Bordalo et al. (2013), which satisfies ordering, diminishing sensitivity, and homogeneity of degree zero is shown below:

$$\psi(x, y) \coloneqq \frac{|x - y|}{|x| + |y|}. \text{when it is not the case that } \mathrm{x} = \mathrm{y} = 0 \text{ , and } \psi(0,0) \coloneqq 0.$$

Appendix C (Supplemental Information)

The experimental materials are available here:

http://www.chapman.edu/research-and-institutions/economic-science-institute/_files/WorkingPapers/schneider-leland-wilcox-ambiguity-framed-2016b.pdf

These materials include:

- The Instruction Booklet
- Ellsberg Experiment Booklet 1 (Choice Situations 1 – 20)
- Ellsberg Experiment Booklet 2 (Choice Situations 21 – 40)
- Ellsberg Experiment Booklet 3 (Choice Situations 41 – 60)