Opacity: Insurance and Fragility*

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Abstract

What are the effects of banks holding opaque, complex assets? Should regulators require bank assets to be more transparent? I study these questions in a model of financial intermediation where opacity determines how long the realized value of an asset remains unknown. By allowing a bank to sell assets before the realization is known, opacity provides insurance to the bank's depositors. However, higher opacity also increases depositors' incentives to join a bank run. In choosing the level of opacity, therefore, a bank faces a trade-off between providing insurance and increasing fragility. If depositors can accurately observe the level of opacity, banks will choose the socially-efficient level. If depositors are unable to observe this choice, however, banks will have an incentive to become overly opaque and regulation to limit opacity will improve welfare.

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1 Introduction

A decade ago, the economy was in the midst of the global financial crisis. An important feature of this crisis was widespread *runs* in which depositors and other creditors withdrew funds from a variety of shadow banking arrangements.[1] One such arrangement was Asset-Backed Commercial Paper (ABCP) conduits, some of which invested funds into complex assets that were difficult to assess in a timely manner. This type of *opacity* is blamed for causing or at least exacerbating the global financial crisis. The Dodd-Frank Wall Street Reform and Consumer Protection Act was introduced in 2010 to "to promote the financial stability of the United States by improving accountability and transparency in the financial system." Subsequently, new rules were stipulated, for example, stronger prudential standards for financial firms that use derivatives and a prohibition on commercial banks from sponsoring and investing hedge funds. It is, however, said that banks have been historically and purposefully opaque. The opacity enables banks to issue *information insensitive* liabilities by keeping asset qualities unknown and isolating the valuation of liabilities from the risk of assets.[2] Doing so allows bank liabilities to be a stable median of exchange and a store of value. This role of opacity is an important feature not only of traditional commercial banks but also of shadow banks.[3] For example, an ABCP conduit issues information insensitive liabilities in the form of commercial paper backed by Asset Backed Securities, Mortgage Backed Securities or derivatives that may be highly complex and risky. This disparity between these two views raises a fundamental question: should the banking system be transparent or opaque?.

This paper addresses the question by constructing a version of the Diamond and Dybvig (1983) model of financial intermediation that illustrates the costs and benefits of opacity in a unified framework. In particular, I study an environment with financial markets and fundamental uncertainty as in Allen and Gale (1998) and with limited commitment as in Ennis and Keister (2009). I add the ability of a bank to make its assets opaque in the sense that it will take time to discern the true value of the assets. Until the true state is known, the bank's assets will trade

[1]See Gorton and Metrick (2012) for further details.

[2]See Gorton and Metrick (2012), Holmström (2015) and Dang, Gorton, Holmström, and Ordoñez (2017).

[3]Gorton, Lewellen, and Metrick (2012) show that the main provider of these information insensitive instruments has shifted from commercial banks issuing demand deposits to shadow banks.

in financial markets based on their expected payoff. By choosing the level of opacity, the bank determines how many of its depositors will be paid while its assets remain information insensitive in this sense. The bank's assets mature in the long-term and yield the realized return, which implies that the bank's repayments to its creditors in the long-term are necessarily contingent on the realized return. Opacity, therefore, can make the bank's liabilities information insensitive only in the short-term. This fact, in turn, affects depositors' decisions on when to withdraw. In this model, I show that opacity of a bank's assets is way of providing insurance to the bank's depositors. At the same time, however, it may worsen financial fragility. I use this model to derive the optimal level of opacity and discuss the conditions under which regulation that limits opacity is desirable.

Opacity here can, in practice, be interpreted as the complexity of the bank's asset. Derivatives and asset backed securities tend to be more complex and harder to assess and even financial firms themselves may have difficulty assessing their asset qualities.[4] For this reason, I assume *symmetric information*: neither the bank nor depositors and outside investors have information on the asset quality during the information insensitive period.[5] Complexity differs even among this class of assets by how it is structured, and hence I assume the choice of opacity is a continuous variable.

I begin my analysis by showing that opacity generates a risk-sharing opportunity in the spirit of the classic Hirshleifer (1971) effect. The asset price depends on the expected asset return until the realized return is known and, hence, opacity provides depositors with insurance against fundamental uncertainty. In other words, opacity allows the bank to transfer the asset-return risk from risk-averse depositors to risk-neutral investors. A higher level of opacity insures more depositors from the uncertainty in the short-term. The repayments made to depositors who wait to withdraw are made using matured assets and, hence, are still exposed to uncertainty. My first contribution is to discover a novel mechanism through this type of insurance raises the possibility of a self-fulfilling bank run. The insurance offered by opacity is available only to depositors who

[4]Ben Bernanke, the chair of Federal Reserve Board at that time, testified that "Financial firms sometimes found it quite difficult to fully assess their own net derivatives exposures... The associated uncertainties helped fuel losses of confidence that contributed importantly to the liquidity problems" in September 2010.

[5]There are papers supposing asymmetric information such that bank has more information. See , for instance, Bouvard, Chaigneau, and Motta (2015), Monnet and Quintin (2017) or Faria-e Castro, Martinez, and Philippon (2017).

withdraw before the asset returns are known. This limited distribution of insurance enhances depositors' incentive to withdraw early and, as a result, a bank run is more likely to occur.

My second contribution is to derive the optimal level of opacity. In choosing a level of opacity, the bank faces a trade-off between providing insurance and increasing the run susceptibility. The optimal level of opacity depends on the extent to which outside investors discount future consumption and on the volatility of asset returns. When the asset returns are more volatile, the insurance is more beneficial and a higher level of opacity is optimal. My third contribution is to show that when the choice of opacity is unobservable by depositors, regulating opacity can improve the allocation of resources and financial stability. Depositors may have difficulty evaluating the details of complex structures of derivatives or asset backed securities. I find that, in this case, the bank will choose the highest possible level of opacity. As a result, the associated expected utility of depositors will be lower and fragility will be higher than the optimal. Introducing a regulatory limit on opacity can then improve welfare. This analysis provides a novel justification for regulating opacity.

Related literature: My paper contributes to a growing literature on opacity and financial stability. My paper is the first to study how opacity itself makes depositors more likely to panic and to show that higher opacity is always worse for financial stability. Existing studies on opacity in theoretical models of bank runs or roll-over risk conclude that opacity enhances or has mixed effects on financial stability. Key differences in these papers are the assumption of asymmetric information and the focus on information-driven bank runs. My paper studies an environment with symmetric information and it is the opacity itself that increases the run susceptibility. Parlatore (2015) builds a global game model of bank runs based on Goldstein and Pauzner (2005) and shows that transparency increases the economy's vulnerability to bank runs. She interprets the precision of private signals about fundamentals as opacity. In her environment, transparency means precise information about the fundamental state, which enhances the strategic complementarity of depositors' withdrawal decisions. She shows that a lower precision reduces the risk of bank runs by removing possible coordination incentives. Chen and Hasan (2006, 2008) build a model with two banks in which their investment returns are random but correlated. Depositors receive a signal and condition their withdrawal decision on the quality of this signal. Their results show that transparency raises run incentives and destabilizes the banking system. Bouvard et al. (2015) and Ahnert and Nelson (2016) study the rollover behavior of a bank's creditors in a global game and show that opacity has mixed effects on financial stability.

My analysis also contributes to a growing literature on information and risk-sharing in financial intermediation. This paper is the first to discover that opacity itself increases financial fragility. My paper shares the idea that opacity provides insurance with Kaplan (2006) and Dang et al. (2017). Kaplan (2006) extends Diamond and Dybvig (1983) to include risky investment and compares two types of deposit contracts: the middle-period repayments are contingent (transparent) or non-contingent (opaque) on the realization. In contrast, I measure opacity by the time it takes to verify asset returns and, hence, the degree of opacity is continuous. He shows that non-contingent contract generates risk-sharing effects. The idea that less information can improve welfare originated in Hirshleifer (1971). A key assumption of Kaplan's paper is that bank runs are costlessly prevented by policy makers with commitment. My paper studies an environment with limited commitment and, hence, partial runs possibly occur. Dang et al. (2017) study effects of opacity on roll-over behavior in a model of financial intermediation. They show that a bank can provide a fixed amount of goods, safe liquidity, independently from the realized return of its assets if the returns are unobservable, or opaque. My paper shares the idea of Hirshleifer effects with these papers, but goes further in studying how this insurance affects not only the allocation of resources but also financial fragility. Also, I introduce financial markets where the bank can liquidate its projects as in Allen and Gale (1998), and the opacity affects the liquidation value. Kaplan (2006) and Dang et al. (2017) do not have mechanisms that opacity affects liquidation value of assets. I will show that this asset price is a source of the Hirshleifer effect but also a source of bank runs.

The idea that opacity enhances financial stability is often studied with risk-taking behavior of banks. Jungherr (2016) characterizes the optimal level of opacity in an environment where opacity reduces the risk of bank runs but encourages banks to take excess risk. He shows that, when asset returns are correlated, banks choose higher opacity than the socially optimal level in order to hide information about their portfolio. Cordella and Yeyati (1998), Hyytinen and Takalo (2002) and Moreno and Takalo (2016) also show that transparency may enhance the bank's risk-taking and increases the chance of a bank failure. Shapiro and Skeie (2015) study the optimal disclosure about bailout policies in resolving a bank. A higher willingness of bailouts reduces run incentives of depositors but leads to risker behavior of the bank. In my model, the bank does not have a portfolio choice and the bank's risk-taking is not the source of fragility.

Adverse selection is another and growing idea in studying opacity, together with runs. Monnet and Quintin (2017) and Faria-e Castro et al. (2017) model disclosure by combining the ideas of bank runs, competitive financial markets as in Allen and Gale (1998) and the Bayesian persuasion approach developed by Kamenica and Gentzkow (2011). My paper also combines the idea of bank runs and competitive financial markets but supposes symmetric information. They study how asymmetric information drives adverse selection in financial markets but disclosure negatively affects runs or roll-over risk, characterizing the optimal use of disclosure. My paper also introduces a financial market in which bank can trade its assets, but one of the key assumption of the paper is that the assets will be traded at a discounted pooling price.

Opacity is an popular idea in discussions of disclosing stress test results as well. Goldstein and Sapra (2014) review this literature and show that opacity is preferred in a majority of studies. Goldstein and Leitner (2015) emphasize the Hirshleifer effect to characterize the optimal information disclosure. Alvarez and Barlevy (2015) study a mandatory disclosure of bank's balance sheet in an environment where banks are inter-connected. They show that the mandatory disclosure of a bank's balance sheet may reassure not only its creditors but also other banks' creditors by reducing concerns of contagion, but it loses an opportunity of risk-sharing. These works suppose that a regulator or bank have more information about bank's assets, whereas my paper studies an environment of symmetric information.

The rest of paper is organized as follows: Section 2 introduces the model environment and the definition of equilibrium and financial fragility. Section 3 derives the equilibrium condition for a bank run and analyzes the effect of increasing opacity on financial fragility. Section 4 characterizes the optimal level of opacity subject to the trade-off between risk-sharing and fragility. I study the case where the choice of opacity is unobservable in Section 5 and then conclude.

2 The Model

The analysis is based on a version of Diamond and Dybvig (1983) augmented to include a choice regarding the transparency of a bank's asset. The model also includes financial markets to trade assets as in Allen and Gale (1998) and the limited commitment features of Ennis and Keister (2010). This section describes the model environment including agents, technologies, financial markets and information structure.

2.1 The environment

Depositors: There are three periods, labeled $t = 0, 1, 2$, and a continuum of depositors, indexed by $i \in [0, 1]$. Each depositor has preferences given by

$$u(c_1^i, c_2^i; \omega_i) = \frac{(c_1^i + \omega_i c_2^i)^{1-\gamma}}{1-\gamma} \tag{1}$$

where c_t^i expresses consumption of the good in period t. The coefficient of relative risk aversion γ is greater than 1. The parameter ω^i is a binominal random variable with support $\Omega \equiv \{0, 1\}$, which is realized in period 1 and privately observed by each depositor. If $\omega^i = 1$, depositor i is patient, while she is impatient if $\omega^i = 0$. Each depositor is chosen to be impatient with a known probability $\pi \in (0, 1)$, and the fraction of impatient depositors in each location is equal to π.

Technology: Each depositor is endowed with one unit of good at the beginning of period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the last period. A unit of the good invested in period 0, called a *project*, matures at period 2 and yields a random return R_j where $j \in J = \{b, g\}$. A project yields R_g with probability n_g and $R_b < R_g$ with probability $n_b = 1 - n_g$. The return realizes at the beginning of period 1.

Investors: The project can be securitized and traded in period 1 as *asset*, in a competitive asset market, and a large number of wealthy risk-neutral investors may purchase them. These investors have endowments in period 1 and preferences are given by

$$g(c_1^f, c_2^f; \rho) \equiv c_1^f + \rho c_2^f, \tag{2}$$

where c_t^f is the period-t consumption of investor f. The parameter $\rho < 1$ captures differences in preferences of investors relative to depositors. Investors' endowments are large enough that their preferences are never constrained. This setup implies that, given an expected return $\mathbb{E}R$ and information at hand, the asset is valued as $p = \rho\mathbb{E}R$ by these investors.

Financial intermediation: The investment technology is operated at a central location, in which depositors pool and invest resources together in period 0 to insure individual liquidity uncertainty. This intermediation technology can be interpreted as financial intermediary, or *bank*. At the beginning of period 1, each depositor learns her type and either contacts the bank to withdraw funds at period 1 or waits until period 2 to withdraw. Depositors are isolated from each others in period 1 and 2, and they cannot engage in trade. Upon withdrawal, a depositor must consume immediately what is given. Repayments follow a *sequential service constraint* as in Wallace (1988) and Peck and Shell (2003). Depositors who choose to withdraw in period 1 are assumed to arrive in random order and each repayment is made sequentially. Similarly to Peck and Shell (2003), when making her choice, a depositor does not know her position in the order of withdrawals.

Bank opacity: The bank can make its securitized assets opaque in the sense that it will take time to reveal the realization. A degree of opacity of its assets is determined in period 0, denoted by $\theta \in [0, \pi]$, and this degree of opacity is known to depositors. The opacity is measured by the time length to reveal the realization of asset returns in period 1, and the time is measured by a number of withdrawals. Therefore, before θ withdrawals, nobody knows the realization of R_j. After θ withdrawals, everybody know the realization.

2.2 Decentralized economy

The intermediation technology is operated by a large number of banks. Banks behave competitively and act to maximize the expected utility of their depositors at all times. In period 0, having deposits a bank securitizes its project with choosing a degree of opacity θ anticipating subsequent events, and this chosen degree of opacity is common knowledge. Actual amounts of payments will depend on a non-cooperative simultaneous-move game in which the bank chooses a repayment strategy and depositors choose a withdrawal strategy at the beginning of period 1. The bank anticipates an equilibrium path to be played in this game to determine the degree of opacity in period 0.

Limited commitment: Each can anticipate that a fraction π of its depositors will be impatient, but does not observe whether a given depositor is patient or impatient. Payments are, therefore, contingent not on a depositor's type but on the other available information at the time of the withdrawal. At π withdrawals, the bank can make inference about whether any patient depositor has withdrawn or not. A *bank run* is defined as withdrawals by a positive measure of patient depositors. The bank reacts to a run as it recognizes the run is underway and I assume the run stops at this point as in Ennis and Keister (2009), which can be interpreted as that bank's reaction restores confidence in the bank.[6] The way the bank reacts has lack of commitment and the bank allocates remaining consumption efficiently: The bank makes repayments to the remaining impatient depositors after π withdrawals in case of runs. This assumption of no-commitment is crucial to prohibit banks to prevent self-fulfilling runs by a time-inconsistent policy.[7]

Repayment plan: The bank sets a state-contingent repayment plan by the beginning of period 1. The repayment made by bank in period 1 can be summarized by a function

[6]This reaction can be, for example, a resolution with haircut. This assumption can be generalized by having more rounds of coordination failure. See Ennis and Keister (2010) for the details. Having multiple coordination failure, however, does not change main mechanisms in this model and results remain unchanged qualitatively.

[7]Diamond and Dybvig (1983) shows that by pre-committing to a payment schedule, e.g. deposit freeze, banks could prevent equilibrium bank runs. However, suspending payments and giving zero consumption to remaining impatient depositors after a fraction π of depositors have withdrawn are time-inconsistent in the spirit of Kydland and Prescott (1977). Ennis and Keister (2009), on the other hand, show that bank runs can occur in equilibrium if banks fail to commit to their initial payment schedules.

$$c : [0, \pi + \pi(1-\pi)] \times \{b, g\} \mapsto \mathbb{R}^2_+ \tag{3}$$

where the number $c_j(\mu)$ is the payment to μ-th depositor withdrawing at period 1 in state j and $\mu \in [0, \pi + \pi(1-\pi)]$. The opacity in sequential service implies $c_b(\mu) = c_g(\mu)$ for $\mu = [0, \theta]$. In case of runs, some patient depositors withdraw in the first π withdrawals. Although runs stop at the π-th withdrawal, there are still $\pi(1-\pi)$ impatient depositors who will need to withdraw in period 1. Remaining withdrawal is made to these impatient depositors, and payments made in period 1 will be at most a fraction $\pi + \pi(1-\pi)$ of depositors. The remaining patient depositors who have chosen to withdraw at period 1 are convinced to wait until period 2 at this point. In period 2, the payment to these remaining patient depositor will be made by equally dividing the matured projects. The repayment plan in period 1 is subject to feasibility constraints such that

$$\int_0^{\pi+\pi(1-\pi)} c_j(\mu) d\mu \leq \theta p_u + (\pi - \theta) p_j, \forall j, \tag{4}$$

where p_u is the discounted expected return of asset such that $p_u = n_b p_b + n_g p_g$. The asset trades at the price p_u before and at the price p_j after θ withdrawals.

Withdrawal plan: Depositors decide a contingent withdrawal plan at the same time when the bank makes decision. A depositor's withdrawal plan is conditioned on both her type and an extrinsic sunspot variable $s \in S = [0, 1]$ that are unobservable to bank.[8] Let y_i denote the withdrawal strategy for depositor i such that

$$y_i : \Omega \times S \mapsto \{0, 1\},$$

where $y_i(\omega_i, s) = 0$ corresponds to withdrawal in period 1 and $y_i(\omega_i, s) = 1$ corresponds to withdrawal in period 2. A bank run, therefore, occurs if $y_i(1, s) = 0$ for a positive measure of patient depositors. Let y denote the profile of withdrawal plans for all depositors.

Expected payoffs: Given θ, the strategies (c, y) determine a level of consumption that

[8] See, for example, the discussion in Diamond and Dybvig (1983), Cooper and Ross (1998) and Peck and Shell (2003).

each depositor receives at every possible cases as a function of her position of withdrawal order. Rewriting (1) so that (c_1^i, c_2^i) are a function of θ, the depositor i's preferences are contingent on both ω_i and θ such that $u(c_1^i, c_2^i; \omega_i, \theta)$. Let $v(c, (y_i, y_{-i}), \theta)$ denote the expected utility of depositor i as a function of her chosen strategy y_i, that is

$$v_i(c, y; \theta) = \mathbb{E}[u(c_1^i, c_2^i; \omega_i, \theta)], \tag{5}$$

where the expectation $\mathbb{E}$ is over ω_i and her position in the order of withdrawals.[9] The bank determines the repayment plan to maximize the expected utility of depositors:

$$U(c, y; \theta) = \int_0^1 v_i(c, y_i; \theta) di. \tag{6}$$

The bank chooses θ to maximize the depositors' expected utilities by anticipating its effects on the equilibrium to be played in the game.

2.3 Timeline

The timing is summarized in Figure 1. In period 0, depositors deposit their endowments with the bank, the bank chooses opacity, and the period ends. This choice of opacity immediately becomes public information. At the beginning of period 1, each depositor makes a contingent withdrawal plan whether to contact her bank for withdrawal at period 1 or wait on her type and sunspot state. At the same time, bank sets a contingent repayment plan on the realization of sunspot state and fundamental state. After they make contingent plans, the state of the world is realized. Depositors learn their type ω^i and the realized sunspot state, and make actions accordingly to the plan. Depositors who have chosen to withdraw at period 1 are randomly assigned positions in the queue at the bank. The bank begins redeeming deposits withdrawn by depositors from the front of the line sequentially. To make these repayments, the bank sells assets in the financial market. These assets are valued by investors according to their payoff. Agents observe the asset quality after the θ-th withdrawal, after which assets are vakued according to the realized fundamental.

[9] See Appendix for a explicit expression for (c_1^i, c_2^i).

Observing the realized fundamental state, the amount of repayment is modified accordingly to the repayment plan. At the π-th withdrawal, the bank infers the realization of sunspot state by whether an additional withdrawal occurs or not, and the amount of repayment become modified. The withdrawal by a patient depositor (bank run) stops at π withdrawals, and hence a further withdrawal can occur only by the remaining impatient depositors. At period 2, all projects mature and bank repays remaining depositors.

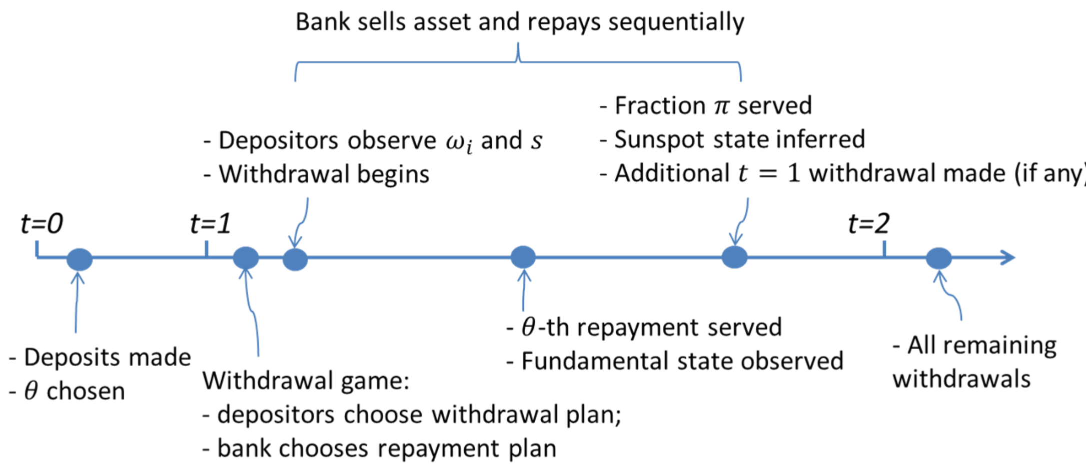

Figure 1: Timeline

2.4 Financial autarky

In absence of the banking system, a depositor places her endowment directly into a project. An individual depositor has no choice on opacity because opacity can only be created by a bank, which is more elaborately organized entity. An individual in autarky is, therefore, exposed to idiosyncratic risk. Goods are obtained in period 1 through asset trading and prices are determined at the financial market. Investors purchase assets in whatever quantity depositors desire at price $p_j = \rho R_j$. A depositor, therefore, consumes ρR_j if she is impatient and consumes R_j if she is patient. A depositor's expected utility in financial autarky will be defined as

$$\mathcal{W}^A = \Sigma_{j=g,b} n_j \{\pi u(p_j) + (1-\pi)u(R_j)\}. \tag{7}$$

2.5 The full information allocation

I first characterize the problem of a benevolent banking authority who can observe depositors' types and can control both the bank's opacity choice, the withdrawal decisions of depositors and the bank's payments. The authority, however, can not observe the realization of fundamental state until θ withdrawals and can not direct the choices of investors. The objective of this authority is to maximize the expected utilities of depositors. The authority chooses the level of opacity θ and how much and at which period each depositor consumes based upon their types subject to the sequential service constraint. I call the allocation characterized by this problem the *full information allocation*.

The full information allocation would give consumption to impatient depositors at period 1 and to patient depositors at period 2. Let c_1 denote the level of consumption given to θ impatient depositors, c_{1j} the level to the other impatient depositors in the fundamental state j, and c_{2j} the level to patient depositors in the fundamental state j. The authority chooses $(\theta, c_1, \{c_{1j}, c_{2j}\}_{j=b,g})$ to solve.

$$\max_{[\theta, c_1, \{c_{1j}, c_{2j}\}_{j=b,g}]} \theta u(c_1) + \Sigma_j n_j \Big[(\pi - \theta)u(c_{1j}) + (1-\pi)u(c_{2j})\Big] \tag{8}$$

subject to

$$\theta \frac{c_1}{p_u} + (\pi - \theta)\frac{c_{1j}}{p_j} + (1-\pi)\frac{c_{2j}}{R_j} = 1, \forall j. \tag{9}$$

where (p_u, p_g, p_b) represents the price of the asset sold in the market in different situations: p_u is the price before the fundamental state revealed and p_j is the price in state j after the state revealed. Before the fundamental state is revealed, the price will be determined at a discounted value of an expected return $p_u = n_b p_b + n_g p_g$. Notice that the optimal arrangement has the feature that θ impatient depositors receive c_1 and the remaining impatient depositors receive goods contingent on the fundamental state. All of patient depositors are exposed to the uncertainty on fundamental state.

Letting λ_h denote the multiplier on the constraint in the fundamental state j, the solution to this problem is characterized by the following first-order conditions.

$$\begin{aligned}
u(c_1) - n_g u(c_{1g}) - n_b u(c_{1b}) &\geq \frac{c_1}{p_u}(\lambda_g + \lambda_b) - \frac{c_{1g}}{p_g}\lambda_g - \frac{c_{1b}}{p_b}\lambda_b && (10)\\
\lambda_g + \lambda_b &= p_u u'(c_1) && (11)\\
\lambda_g &= n_g p_g u'(c_{1g}) = n_g R_g u'(c_{2g}) && (12)\\
\lambda_b &= n_b p_b u'(c_{1b}) = n_b R_b u'(c_{2b}). && (13)
\end{aligned}$$

The last two equations show that patient depositors are expected to consume more than impatient ones in each fundamental state j. It is, however, not immediately clear about relations among other variables. Combining these first-order conditions with the resource constraint, I establish the following proposition:

Proposition 1. The full information allocation involves full opacity $\theta = \pi$.

which means that opacity is set at the maximal and all impatient depositors receive c_1. This result reflects that the opacity provides depositors with insurance against the fundamental uncertainty and, therefore, more opacity always increases welfare. I will below examine its effect on financial fragility and characterize the optimal level of opacity under a trade-off between creating insurance opportunities and worsening financial fragility.

2.6 Withdrawal game

Depositors and the bank choose their withdrawal strategies and a repayment plan, respectively, at the same time in period 1. In this simultaneous move game, a depositor's strategy is y_i in maximizing $v(c, (y_i, y_{-i}); \theta)$ and the bank's strategy is c in maximizing $U(c, y; \theta)$. An equilibrium of this game is then defined as follows:

Definition 1. Given θ, an *equilibrium of the withdrawal game* is profile of strategies (c^*, y^*) such that

1. $v_i(c^*, (y_i^*(s), y_{-i}^*(s)); \theta) \geq v_i(c^*, (y_i(s), y_{-i}^*(s)); \theta)$ for all s, for all y_i, for all i

2. $U(c^*, y^*(s); \theta) \geq U(c, y^*(s); \theta)$ for all c

Notice that the bank takes the strategies of depositors as given and chooses a best response to these strategies, and a change in c does not influence the behavior of depositors. However, the payoffs of this game depend on θ. Let $\mathcal{Y}(\theta)$ denote the set of equilibria of the game associated with the choice θ such that

$$\mathcal{Y}(\theta) = \{(c^*, y^*) \mid \theta\}. \tag{14}$$

2.7 Banking problem

My interest is in how the interaction between depositors' withdrawal decisions and the bank's repayment plan depends on the level of opacity θ. The bank chooses the level of opacity in period 0 to maximize the expected utility of depositors. Notice that the function (6) depends on θ. The bank chooses θ by anticipating equilibrium outcomes in the withdrawal game and this choice of θ is immediately observable to depositors. The bank's problem is, therefore,

$$max_\theta \; U(\tilde{c}, \tilde{y}; \theta \mid (\tilde{c}, \tilde{y}) \in \mathcal{Y}(\theta)). \tag{15}$$

2.8 Discussion

The result in Proposition 1 has a common feature with Kaplan (2006) and Dang et al. (2017) such that uncertainty about asset returns provides an opportunity of risk-sharing. Impatient depositor receives a level of consumption c_1 that is independent of the realized return of her bank's asset. In other words, the opacity indeed assures that the value of short-term debt a bank produces is information insensitive. This result supports the view that the bank should enhance opacity. I will, however, show that opacity has a cost in financial fragility and the optimal level of opacity is not trivial. The opacity certainly provides the insurance by transferring risk from risk-averse depositors to risk-neutral investors, but transfers the risk only of the first θ depositors withdrawing in period 1. Therefore, the degree of opacity influences the expected payoff of depositors who withdraw in period 1.

3 Equilibrium in the withdrawal game

In this section, I study equilibrium in the withdrawal game given the bank's choice of opacity and show how opacity affects equilibrium. I am interested in the probability capturing the existence of an equilibrium in which bank run occurs. A standard way in the literature is studying the condition such that the following strategy profile is a part of equilibrium.[10] Denote $\hat{y}_i(\omega_i, s)$ be *q-strategy profile* such that

$$\hat{y}_i(\omega_i, s; q) = \begin{Bmatrix} \omega_i \\ 0 \end{Bmatrix} \text{ if } s \begin{Bmatrix} \geq \\ < \end{Bmatrix} q \text{ for some } q \in [0,1], \forall i. \tag{16}$$

In this strategy profile, impatient depositors withdraw at period 1 and patient depositors withdraw at period 2 if the realized sunspot state is $s < q$, but both types of depositors withdraw at period 1 if the sunspot state $s \geq q$. Notice that, since s is uniformly distributed on $[0,1]$, the value q is the probability of a run associated with this strategy profile. I first derive conditions under which a run equilibrium exists by (i) studying the bank's best response to this q-strategy profile and (ii) verifying whether this profile is part of equilibrium.

3.1 The best-response allocation

In period 1, the bank makes a repayment plan c, which pays c_1 to the first θ depositors and contingent amounts after θ withdrawals. At the θ-th withdrawal, the fundamental state is revealed to every agent and the bank switches to pay an amount c_{1j} to each of the following withdrawal until the π-th withdrawals in state j. Once π withdrawals have occurred, the bank will be able to infer the sunspot state by observing whether an additional withdrawal is requested at this point or not. At this point, all uncertainty has been resoled and the bank gives a common amount c_{1j}^R to the remaining impatient depositors in state j. Letting π_s be the remaining impatient depositors, the profile (16) generates $\pi_{s\geq q} = 0$ and $\pi_{s<q} = \pi(1-\pi)$. Additionally, each of the remaining patient depositors will receive a common amount c_{2j}^N if $s \geq q$ and c_{2j}^R if $s < q$ in state j. Given θ

[10] See, for example, Cooper and Ross (1998), Peck and Shell (2003) and Ennis and Keister (2010).

and q, I will below characterize these consumption levels $(c_1, \{c_{1j}, c_{1j}^N, c_{2j}^N, c_{2j}^R\}_{j=b,g})$ to solve:

$$\max_{[c_1, \{c_{1j}, c_{1j}^N, c_{2j}^N, c_{2j}^R\}_{j=b,g}]} \theta u(c_1) + \Sigma_j n_j \Big[(\pi - \theta)u(c_{1j}) + (1-q)(1-\pi)u(c_{2j}^N) \\ + q(1-\pi)[\pi u(c_{1j}^R) + (1-\pi)u(c_{2j}^R)]\Big] \tag{17}$$

subject to

$$(1-\pi)\frac{c_{2j}^N}{R_j} = 1 - \theta\frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1j}}{p_j}, \tag{18}$$

$$\pi(1-\pi)\frac{c_{1j}^R}{p_j} + (1-\pi)^2\frac{c_{2j}^R}{R_j} = 1 - \theta\frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1j}}{p_j}, \forall h. \tag{19}$$

Notice, in sunspot state $s < q$, that the bank must continue to serve the additional $\pi(1-\pi)$ depositors after π withdrawals at period 1. The right hand side of each constraint implies remaining resources at the π-th withdrawals. The solution satisfies the following first-order conditions.

$$u'(c_1) = n_g\frac{p_g}{p_u}u'(c_{1g}) + n_b\frac{p_b}{p_u}u'(c_{1b}) \tag{20}$$

$$u'(c_{1j}) = \frac{R_j}{p_j}\{(1-q)u'(c_{2j}^N) + qu'(c_{2j}^R)\} \tag{21}$$

$$u'(c_{1j}^R) = \frac{R_j}{p_j}u'(c_{2j}^R), \forall j. \tag{22}$$

Notice that $c_{1j}^\beta < c_{2j}^\beta$ always holds, but other relations especially between c_1 and c_{2j}^β depend on θ and q. The best-response allocation to profile (16) is summarized by the vector

$$\mathcal{A}(\theta, q) \equiv \{c_1, \{c_{1j}, c_{1j}^N, c_{2j}^N, c_{2j}^R\}_{j=b,g}\} \tag{23}$$

that solves the problem (17) and is characterized by conditions above. Finally, this best-response allocation has the following feature:

Lemma 1. *$U(c^*, \hat{y}(q); \theta)$ is decreasing in q.*

The intuition behind this lemma is that the bank anticipates that runs are more likely to occur and becomes more cautious as q increases. The bank allocates less consumption into period 1 and more consumption into period 2. This lemma implies that the bank's precautionary behavior has to distort the allocation.

3.2 Equilibrium bank runs

I now study whether the q-strategy profile is a part of an equilibrium in the withdrawal game given θ and thus the financial system is stable or fragile. The profile is sometimes a part of equilibrium, whether or not it is depends on q. I will find what values of q, given θ, make profile (16) to be an equilibrium. Let $\mathcal{Q}$ be a set of q such that

$$\mathcal{Q}(\theta) = \{q : \hat{y}(q) \in \mathcal{Y}(\theta)\}. \tag{24}$$

Since impatient depositors do not value any consumption at period 2, they strictly prefer to withdraw at period 1. I only have to consider the actions of patient depositors to find the set $\mathcal{Q}(\theta)$. Patient depositors receive c_{2j}^N or c_{2j}^R depending on the sunspot state s and the fundamental state j if she waits until period 2. She receives, however, c_1 or c_{1j} in any sunspot state in the fundamental state j if she withdraws in period 1. There can be, therefore, two conditions corresponding to each sunspot state to characterize the set $\mathcal{Q}(\theta)$. I first study the upper bound of the set such that

$$\bar{q} = argmax\{q \in \mathcal{Q}(\theta)\}, \tag{25}$$

by comparing expected payoffs by actions in the sunspot state $s < q$ as in Keister (2016) and Li (2017). The expected payoffs by withdrawing in period 1 or 2 are respectively

$$\mathbb{E}u(c_{1k}) = \frac{\theta}{\pi}u(c_1) + \left(1 - \frac{\theta}{\pi}\right)\Sigma_j n_j u(c_{1j}) \tag{26}$$

$$\mathbb{E}u(c_{2j}^N) = \Sigma_j n_j u(c_{2j}^N) \tag{27}$$

$$\mathbb{E}u(c_{2j}^R) = \Sigma_j n_j u(c_{2j}^R) \tag{28}$$

where k denotes her position in the order of withdrawals. From (20)-(22), it is straightforward to show that $\mathbb{E}u(c_{1k})$, $\mathbb{E}u(c_{2j}^N)$ and $\mathbb{E}u(c_{2j}^R)$ depend on q in the following way:

Lemma 2. The best-response allocation $\mathcal{A}(\theta, q)$ satisfies that:

1. $\mathbb{E}u(c_{1k})$ is monotonically decreasing in q,

2. $\mathbb{E}u(c_{2j}^N)$ and $\mathbb{E}u(c_{2j}^R)$ are monotonically increasing in q

The intuition behind this lemma is that the bank becomes more conservative as q increases and give less consumption to early withdrawals in $t = 1$. The threshold $\bar{q}$ satisfies $\mathbb{E}u(c_{1k}) = \mathbb{E}u(c_{2j}^R)$, and this lemma assures that there always exists an unique value of $\bar{q}$. When $q > \bar{q}$, withdrawing in period 1 is not a best response. If $\bar{q} = 1$, there is always an equilibrium in which a patient depositor certainly withdraw in period 1 in any sunspot state such that $y_i(\omega_i, s) = 0$ for all i for all s.

Proposition 2. There is an equilibrium in which bank run certainly occurs if and only if $\bar{q} = 1$.

I now turn my attention to the greatest lower bound of q such that

$$\underline{q} = argmin\{q \in \mathcal{Q}(\theta)\}, \tag{29}$$

by solving for q such that $\mathbb{E}u(c_{2j}^N) = \mathbb{E}u(c_{1k})$ holds. Lemma 2 assures that there always exists an unique value for $\underline{q}$ as well. This threshold value $\underline{q}$ is the minimum value of q in which patient depositors prefer to wait until period 2 when all other patient depositors wait until period 2. When q is small, the bank becomes aggressive to give consumption in period 1, and patient depositors may withdraw in period 1 whatever the sunspot state is. This threshold $\underline{q}$ has the following feature:

Proposition 3. There is an equilibrium in which no bank run occurs if and only if $\underline{q} = 0$.

This proposition means that a no-run strategy profile, such that $y_i(\omega_i, s) = \omega_i$ for all i for all s, is a part of equilibrium if and only if $\underline{q} = 0$.[11] Finally, $\underline{q}$ and $\bar{q}$ have the following feature.

Proposition 4. $\underline{q}\begin{Bmatrix} = \\ > \end{Bmatrix} 0$ if and only if $\bar{q}\begin{Bmatrix} < \\ = \end{Bmatrix} 1$.

[11] In Ennis and Keister (2010), Li (2017) and many others, the value of $\underline{q}$ is always at 0 and a no-run equilibrium always exists. See Subsection 3.4 for further discussion.

This proposition means that there are two cases: (i) when $\underline{q} = 0$, there always exists a no-run equilibrium and co-exists with equilibria in which runs occur with probability $q \leq \bar{q}$ and (ii) when $\underline{q} > 0$, there always exists an equilibrium in which runs certainly occur and co-exists with equilibria in which runs occur with probability $q > \underline{q}$. Since my interest is the susceptibility to runs, the following analysis focuses on $\bar{q}$ as a measure of fragility to study an impact of opacity on a banking panic.

3.3 The impact of opacity

I now ask the following question: how does an increase in the opacity affect financial fragility measured by $\bar{q}$? The first θ withdrawals made by selling risky assets at discounted expected values p_u, and hence these withdrawals are isolated from the fundamental uncertainty (Hirschleifer effects). By doing so, the bank transfers risks of the asset from risk-averse depositors to risk-neutral investors for the first θ withdrawals in period 1. It is worth emphasizing that a higher θ benefits those who withdraw after θ withdrawals in period 1 or withdraw in period 2. The bank can insure more depositors from the fundamental uncertainty through opacity, and hence the shadow price of giving consumption to these depositors decreases. As the opacity increases, the bank reduces the amount of goods c_1 and increase amounts of goods payments after θ. The opacity, therefore, benefits patient depositors as well even if they withdraw in period 2. However, these patient depositors are still exposed to the fundamental uncertainty. By withdrawing at period 1, a patient depositor can be isolated from the uncertainty if she arrives early enough. The possibility to get an insurance against the fundamental uncertainty depends on θ. Higher θ means that a patient depositor has a higher chance of arriving at the bank before the state is investigated and of getting the insurance. When θ is sufficiently high, joining a run becomes more profitable than waiting until period 2 although returns at period 1 are relatively smaller than period 2 by ρ. As θ further increases, the incentive to join a run becomes even stronger and financial fragility keep growing.

Proposition 5. $\bar{q}$ is monotonically increasing in θ.

This result is illustrated in Figure 2, which depicts $\bar{q}$ as a function of θ in the horizontal axis given

$(\gamma, \pi, n, R_g, R_b, \rho) = (2, 0.5, 0.5, 2, 1, 0.9)$. Notice that $\bar{q}$ remains at zero for a while as θ increases, which implies that the chance of obtaining the insurance against the fundamental uncertainty is sufficiently low for a patient depositor. Joining a run is, therefore, too risky for her because she has a high chance of receiving c_{1j}^R which is exposed to the fundamental uncertainty and is even smaller than c_{2j}^R. As θ increases, an attempt to obtain the insurance becomes more attractive and an incentive to join a run becomes higher.

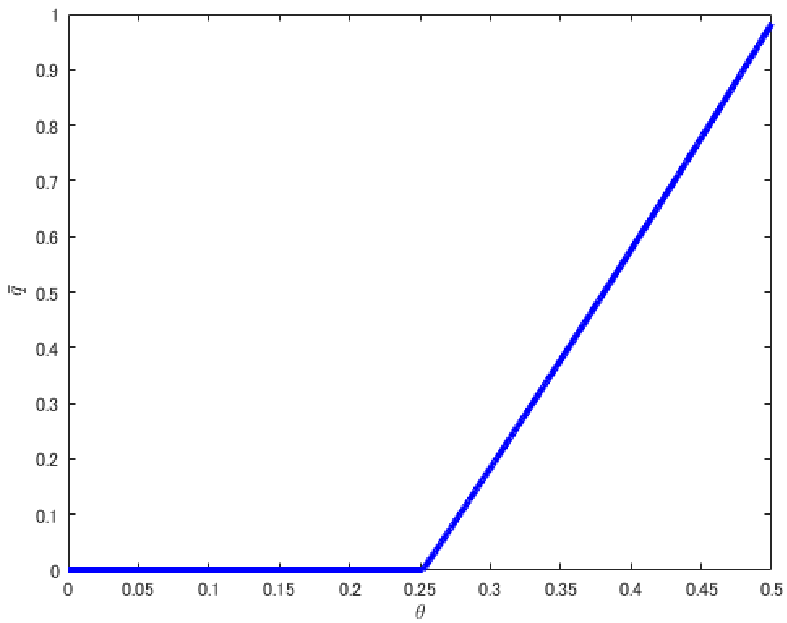

Figure 2: (Numerical example) $\bar{q}$ over θ

Proposition 6. Given θ, the financial fragility $\bar{q}$ rises when

- risk-aversion γ increases,
- the relativity of returns over periods ρ increases,
- the gap of returns between fundamental states $(R_g - R_b)$ increases,
- the fundamental state is more uncertain (when n is closer to $\frac{1}{2}$).

This proposition shows how key parameters govern benefits and costs of insurances. The relativity of returns, which captures what fraction the asset is discounted at period 1, show the cost of insurance, implying that a patient depositor has to give up a part of returns to obtain the insurance. It is straightforward that higher costs discourage her from joining a run, and hence effects of opacity on fragility is diminished. All other parameters determine the incentive to insure the fundamental uncertainty. When the asset return is highly volatile, or when a depositor is more risk-averse, the opacity further raises the incentive to join a run and worsens financial fragility even more.

3.4 Discussion

It is worth noting that a run can be rational even if an expected amount of goods repaid at period 2 is larger than the one of goods repaid before θ at period 1 such that $c_1 < \mathbb{E}c_{2j}^R$. When ρ is small, this inequality is more likely to hold because the price of asset is low. There exists cases such that $c_1 < \mathbb{E}c_{2j}^R$ but $\mathbb{E}u(c_{1k}) \geq \mathbb{E}u(c_{2j}^R)$, which is attributed to the insurance benefits. This is also the reason why the full information allocation is not incentive compatible in some cases, which is $\underline{q} > 0$. In Diamond and Dybvig (1983), Cooper and Ross (1998), Ennis and Keister (2010) and others, the full information allocation is always incentive compatible, which can be translated into $\underline{q} = 0$. Peck and Shell (2003) and Shell and Zhang (2018) introduce a preference parameter in such a way that patient depositors value short-term consumption more and show that the full information allocation is not implementable in some cases. These cases correspond to $\underline{q} > 0$.

The negative effect of opacity on financial fragility is not caused by the risk-sharing itself but by its distribution. Notice that depositors, who arrive before θ-th withdrawals, receive the insurance benefits. I have assumed that bank can not sell assets at a pooling price more than necessary to redeem demanded repayments. It is possible to generalize this assumption so that the bank could sell more assets than necessary to redeem demanded repayments before the state is verified. In an extreme case, the bank would sell all assets before any withdrawal occur. It seems, however, reasonable to assume that selling assets takes a time. It is hard to tell which of selling assets and redeeming repayments is quicker. Gorton and Metrick (2012) discuss that the recent financial crisis was a run on the repo market, and then repo transactions could be even faster than redemption at a counter deposit. Unless bank can sell all assets before θ withdrawals, some of bank's assets are still exposed to the uncertainty. And then, this generalization does not change my results qualitatively.

In relation to the literature, proposition 5 disputes their conclusion that opacity helps financial stability. This opposing result comes mainly from a difference of the assumption about the depositors' withdrawal decision. In their models, depositors make a withdrawal decision after they receive partial or full information about the realization of asset returns. In other words, bank runs in their models are information-driven. My model allows depositors to make a withdrawal decision before they learn the realization of the fundamental state. This environment enables me to study self-fulfilling bank runs together with the opacity that have been missed in the literature. This distinction between information-driven runs and self-fulfilling runs explains the difference in the results. Existing results focus on the information-driven runs and show that opacity helps financial stability by preventing depositors to identify unsound banks. I propose the new channel from the opacity to financial fragility: The opacity raises the susceptibility to self-fulfilling runs.

4 Optimal opacity

In this section, I study a bank's choice of the degree of opacity in period 0. When the bank chooses θ, it is creating a withdrawal game based on that level of opacity. The bank must form a belief about which equilibrium of the withdrawal game will be played for each possible value of θ. I suppose that the bank expects the worst equilibrium in the sense of welfare associated with that value of θ to be played. By doing so, I will find the solution in the banking problem choosing the degree of opacity.

4.1 Worst scenario

A bank faces uncertainty about the specification of the realized sunspot variable and hence the run susceptibility. I suppose that the bank minimizes losses in the worst case over the set $\mathcal{Q}(\theta)$. In other words, the bank chooses θ by $max_{\theta} min_{q \in \mathcal{Q}(\theta)}\ U(c^*, \hat{y}(q); \theta)$. It is straightforward to show that the worst scenario in the welfare sense is $\bar{q}$ and hence $\bar{q} \in argmin\ U(c^*, y^*; \theta)$. Recall that a run equilibrium exists if and only if $q \leq \bar{q}$ and that the expected utility of depositors $U(c, \hat{y}; \theta, q)$, evaluated at $\mathcal{A}(\theta, q)$, is decreasing in q.

Recall that the expected utility of depositors (6) is the function of (c, y) given θ. In this approach, (c, y), and hence the function U, are determined by $(\theta, \bar{q}(\theta))$. Defining $\mathcal{W}(\theta, \bar{q}(\theta))$ such that

$$\mathcal{W}(\theta, \bar{q}(\theta)) = U(c^*(\theta), \hat{y}(\bar{q}(\theta)); \theta). \tag{30}$$

The solution to this problem characterize an equilibrium in this overall economy together with the strategy profiles in the withdrawal game to be played upon the choice θ.

Definition 2. An *equilibrium with an observable degree of opacity* is 3-tuple (c^*, y^*, θ^*) such that

1. $(c^*, y^*) \in \mathcal{Y}(\theta^*)$

2. $\theta^* \in argmax \mathcal{W}(\theta, \bar{q}(\theta))$.

4.2 Banking problem

The bank chooses $\theta \in [0, \pi]$ to maximize the ex-ante welfare of depositors such that

$$max\ \mathcal{W}(\theta, \bar{q}(\theta)). \tag{31}$$

The first argument captures the direct effect of opacity, which is providing insurance. The second argument shows the indirect effect, which is a worse financial fragility. The optimal level of opacity will be determined under the trade-off between insurance and raising. I will below discuss characteristics of the optimal level of opacity.

4.3 Optimal opacity

I characterize the optimal level of θ. A higher level of opacity means that more depositors can get the insurance c_1 instead of c_{1j}, which is preferable to risk-averse depositors. It, however, increases financial fragility $\bar{q}$.

In determining the optimal opacity, the channel to transmit the cost of opacity into the welfare is $\bar{q}\theta$. In some cases, an increment θ does not increase $\bar{q}(\theta)$ and the highest opacity will be optimal. This case can occur, for example, when the project returns are largely discounted by investors such that ρ is very low, which implies that an increase of θ does not make a run attractive enough. In other cases, the optimal level of θ is at an interior solution $\hat{\theta}$ or at the other corner solution ($\theta = 0$).

Proposition 7. For some parameter values, $\theta^* < \pi$.

This result is illustrated in Figure 3, which depicts the welfare along θ given the same parameter set $(\gamma, \pi, n, R_g, R_b, \rho) = (2, 0.5, 0.5, 2, 1, 0.9)$ to Figure 2. The benefit of risk-sharing raises the welfare when θ is small, but eventually $\bar{q}$ begins to increase and leads to mis-allocation as θ increases. At some point, this cost of mis-allocation dominates the risk-sharing benefits and diminishes the welfare. The optimal level of opacity is pinned down by balancing this trade-off.

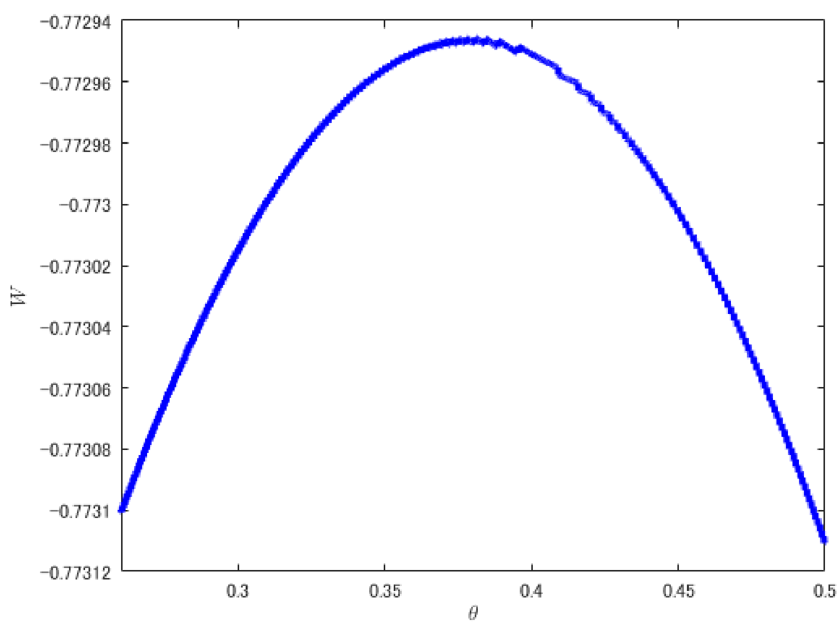

Figure 3: (Numerical example) Welfare

I now specify $\gamma = 2$ to see determinants of θ in a closed-form solution. The following two propositions show how the optimal level is pinned down in case of $\gamma = 2$.

Proposition 8. When $\gamma = 2$, the optimal level of θ is,

$$\theta^* = min\left\{\pi, \frac{\pi^{\frac{1}{2}}\rho^{\frac{1}{4}}(\Delta p_u)^{\frac{1}{2}}[\{\pi(1-\pi)+(1-\pi)^2\rho^{\frac{1}{2}}\}^{\frac{1}{2}} - \{\pi\rho^{\frac{-1}{2}}\}^{\frac{1}{2}}]}{1-(\Delta p_u)^{\frac{1}{2}}}\right\}, \tag{32}$$

where $\Delta \equiv n_g p_g^{-1} + n_b p_b^{-1}$.

Proposition 9. When $\gamma = 2$, the optimal opacity becomes larger when

- the discount rate ρ increases
- the gap of returns between fundamental states $(R_g - R_b)$ increases,
- the fundamental state is more uncertain (when n is closer to $\frac{1}{2}$).

Intuition for the last two bullet points is simple. When the asset returns are more uncertain, the bank has higher incentives to raise opacity to insure the fundamental uncertainty. Effects of ρ will explained through the incentive mechanism of depositors. A higher ρ reduces opportunity costs associated by runs and hence costs of bank runs, which raises the optimal level of opacity. The maximum opacity $\theta^* = \pi$ will be efficient, particularly when ρ is sufficiently large such that bank runs are much less costly even if it occurs.

4.4 Comparison with the autarky

This banking system provides insurances against uncertainty at the individual preferences and the fundamental state. Diamond and Dybvig (1983) study the former role of bank, and Cooper and Ross (1998) imply that, in some cases, the susceptibility to runs undermines the benefits of this role. My analysis has shown that the bank's second role strengthens benefits of the banking system but also costs of fragility. I now study if there exists a case in which the banking system is dominated by the financial autarky. On top of that the bank insures the individual liquidity shock, it now chooses θ to insure the fundamental uncertainty subject to a higher run susceptibility.

Proposition 10. $\mathcal{W}(\theta) > \mathcal{W}^A$ for any value of θ.

This proposition shows that the banking system always implements a better allocation than the financial autarky when the bank chooses opacity θ by the robust control view. If the susceptibility of runs is high, the bank chooses a lower opacity to reduce incentives of runs.

4.5 Discussion

These results do not deny the role of bank's short-term debt as safe liquidity, but show why *bank as secret keepers* is more susceptible to runs. This paper proposes that, in some cases, only very short-term debts should be information insensitive and that the other short-term debts should have the risk, which limits the amount of information insensitive liabilities produced by banks. The optimal amount will depend on a situation surrounding the bank, for example how uncertain and risky its project is (γ, n, R_g, R_b) and the discount rate of investors (ρ).

5 Regulating opacity

I have supposed that the level of opacity is observable to depositors. However, it may not be easy to know how opaque the bank's assets especially in case of structured assets like derivatives. Now suppose instead that depositors do not observe the level of opacity chosen by her bank. The depositors chooses, therefore, a withdrawal decision without any information on θ, and the bank's choice on θ does not directly affect the depositors' behavior.

5.1 Modified withdrawal game

In this environment, the bank's choice on θ becomes a part of withdrawal game. In this simultaneous-move game, the bank chooses (c, θ) at the same time to maximize $U(c, y, \theta)$. Depositors choose y_i as before to maximize $v_i(c, y, \theta)$. An equilibrium of this game is defined as follows:

Definition 3. An *equilibrium of the modified withdrawal game* is profile of strategies $(c^{**}, y^{**}, \theta^{**})$ such that

1. $v_i(c^{**}, (y_i^{**}(s), y^{**}-i(s)), \theta^{**}) \geq v_i(c^{**}, (y_i(s), y_{-i}^{**}(s)), \theta^{**})$ for all s, for all y_i, for all i

2. $U(c^{**}, y^{**}(s), \theta^{**}) \geq U(c, y^{**}(s), \theta)$ for all c and for all θ

5.2 The best response allocation

Similarly to Section 3.1, I consider the strategy profile (16) and study the best response of the bank. Given q, I will below characterize the optimal level of θ and these consumption levels $(\theta, c_1, \{c_{1j}, c_{1j}^N, c_{2j}^N, c_{2j}^R\}_{j=b,g})$ to solve:

$$\max_{[\theta, c_1, \{c_{1j}, c_{1j}^N, c_{2j}^N, c_{2j}^R\}_{j=b,g}]} \theta u(c_1) + \Sigma_j n_j \Big[(\pi - \theta) u(c_{1j}) + (1-q)(1-\pi) u(c_{2j}^N) \\ + q(1-\pi)[\pi u(c_{1j}^R) + (1-\pi) u(c_{2j}^R)] \Big] \quad (33)$$

subject to

$$(1-\pi)\frac{c_{2j}^N}{R_j} = 1 - \theta \frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1j}}{p_j}, \quad (34)$$

$$\pi(1-\pi)\frac{c_{1j}^R}{p_j} + (1-\pi)^2 \frac{c_{2j}^R}{R_j} = 1 - \theta \frac{c_1}{p_u} - (\pi - \theta)\frac{c_{1j}}{p_j}, \forall h. \quad (35)$$

Only difference from Section 3.1 is that θ is now a one of the choice variables. The solution to this problem is characterized by the first-order conditions (20)-(22) and the following first-order condition of θ:

$$u(c_1) - c_1 u'(c_1) \geq (1-n)\{u(c_{1g}) - c_{1g} u'(c_{1g})\} + n\{u(c_{1b}) - c_{1b} u'(c_{1b})\}. \quad (36)$$

By these conditions, I can establish the following proposition:

Proposition 11. Conditioned on $\hat{y}(q)\forall q$, the bank's conditional dominant strategy is $\theta = \pi$.

The solution has, therefore, the feature that θ is at the corner solution. The intuition behind this result is that the bank has an incentive to raise θ as much as possible because it generates risk-sharing opportunities without directly influencing the behavior of depositors. The solution of this problem, then, corresponds to $\mathcal{A}(\pi, q)$.

The strategy profile (16) is a part of equilibrium when $q \in \mathcal{Q}(\pi)$. Following Section 4, I consider the worst scenario over the possible q. As in Lemma 1, the best response allocation characterized above has a feature that $U(c^*, \hat{y}, \pi)$ is decreasing in q. Therefore, the worst scenario corresponds to $\bar{q}$. The expected utility of depositors is, therefore, $\mathcal{W}(\theta, \bar{q}(\pi))$.

5.3 Roles of policy

The regulator may improve the welfare by regulating opacity in the case that θ is unobservable. Suppose that the regulator places an upper bound on θ. Letting $\bar{\theta}$ be this upper bound, suppose that the government imposes a regulation that $\theta \leq \bar{\theta} = \theta^*$ and lets everyone know it, then depositors will expect their bank to choose θ^*. The outcome of this model is the same as the one in Section 4.

Proposition 12. When θ is unobservable, regulating $\theta \leq \theta^*$ improves the expected utility of depositors up to the same level to the case θ is observable.

This regulation would correspond in practice to a restriction on asset types or structures that bank can invest in. When the optimal level of opacity is lower, this regulation is more likely to improve welfare. Proposition 9 implies that that is when asset returns are less uncertain or when bank can liquidate their assets at higher prices relative to its (expected) fundamental values.

The regulation may be operated flexibly over business cycle, because the parameters that I discussed would vary over cycle. For example, if the gap of returns is higher (lower) in times of booms (recessions), the restriction should be counter-cyclically tightened and transparency should be more enhanced at the time of recessions.

6 Conclusion

I have presented a model of financial intermediation in which opacity, measured by a time to verify a fundamental state, not only creates risk-sharing opportunities but also raises incentives of joining a run. The benefit of risk-sharing is available only before the state is verified and not distributed to depositors withdrawing later. A higher opacity increases the probability that a patient depositor could arrive before the state is verified and thus raises expected payoffs by joining a run. The incentive becomes even stronger when asset returns are more uncertain or when bank can liquidate their assets at higher prices relative to its (expected) fundamental values. By having risk dominance as a equilibrium selection mechanism, I have characterized the optimal level of opacity under the trade-off between risk-sharing effects and runs. A higher opacity is efficient when asset returns are volatile or when liquidation values of assets are higher relative to its (expected) fundamental values. Finally, I discussed roles of policy by supposing that all bank experience a run with the same probability. My analyses explained why and how transparency should be enhanced by policies. A lack of policy intervention may lead to excessive opacity and raise the susceptibility to runs. I found that regulation on opacity would restrict the choice or opacity taxes would reduce the distortive incentives so that the constrained efficient allocation could be implemented.

These results show that Hirschleifer effects are accompanied by financial fragility, because the insurance benefits would not be distributed evenly to depositors. Dang et al. (2017) discuss that opacity is necessary for banks to produce *safe liquidity*, but my finding shows that depositors are more likely to panic by opacity. That is, the bank's function of creating money-like securities has a trade-off with financial fragility. In order to provide such securities stably, bank may create *risky liquidity* as well.

This series of results propose the optimal level of opacity and necessity of restricting a choice of opacity given an economic environment. I may interpret the banking system facing a higher volatile assets (or more risky) as a shadow bank and the one facing a lower volatile assets (or less risky) as a traditional commercial bank. My results, then, imply that a shadow bank should choose a higher opacity by sacrificing a higher run susceptibility, and that a traditional commercial bank should choose a lower opacity by enhancing stability. My analysis, furthermore, have shown that a commercial bank tends to choose inefficiently higher opacity worsening financial fragility. My discussion, then, provides rationales to restrict a traditional bank on choosing opacity but not to restrict a shadow bank. The view of Dang et al. (2017) supports both type of banks to have a higher opacity, but I may suggest that a traditional commercial bank should not have a high level of opacity and be restricted by the government.

I conclude by noting potentially promising directions for future researches. Firstly, government guarantees are a popular policy to prevent banking panics and may complement the provision of safe liquidity. However, guarantees in bad times may distort banks' incentives and also there could be various formulations of guarantees. It is, then, not clear how guarantees interplay with opacity and what scheme of guarantees is efficient. Secondly, investors may discount the uncertainty of asset returns. When they can not distinguish banks having good or bad assets, adverse selection problem may arise and asset prices may be severely discounted until the returns are verified. A lower pooling price reduces risk-sharing benefits but also incentives of joining a run. It is ambiguous which of these competing effects dominate the other. Furthermore, an associated pecuniary externality would call for roles of another policy. Studies on these extensions would be an interesting future research.

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A Expected individual consumption

I here denote the expected individual consumption explicitly. Suppose a measure Ψ of patient depositors follow run strategy such that

$$\Psi(y) = \int_0^1 \mathbb{1}_{(y_i(1,s<q)=0)} di.$$

If $y_i(\omega_i, s) = \omega_i$,

$$\begin{aligned}
c_1^i =&(1-q)\left(\frac{\theta}{\pi}u(c_1) + \left(1-\frac{\theta}{\pi}\right)\left(n_g u(c_{1g}^N) + n_b u(c_{1b}^N)\right)\right) \\
&+ q\left[\frac{\theta}{\pi+\Psi(1-\pi)}u(c_1) + \frac{\pi-\theta}{\pi+\Psi(1-\pi)}\left(n_g u(c_{1g}) + n_b u(c_{1b})\right)\right. \\
&\left. + \left(1-\frac{\pi}{\pi+\Psi(1-\pi)}\right)\left(n_g u(c_{1g}^R) + n_b u(c_{1b}^R)\right)\right], \\
c_2^i =&(1-q)\left(n_g u(c_{2g}^N) + n_b u(c_{2b}^N)\right) + q\left(n_g u(c_{2g}^R) + n_b u(c_{2b}^R)\right).
\end{aligned}$$

If $y_i(\omega_i, s) = \begin{Bmatrix} \omega_i \\ 0 \end{Bmatrix}$ if $s \begin{Bmatrix} \geq \\ < \end{Bmatrix} q$,

$$\begin{aligned}
c_1^i =&(1-q)\left(\frac{\theta}{\pi}u(c_1) + \left(1-\frac{\theta}{\pi}\right)\left(n_g u(c_{1g}^N) + n_b u(c_{1b}^N)\right)\right) \\
&+ q\left[\frac{\theta}{\pi+\Psi(1-\pi)}u(c_1) + \frac{\pi-\theta}{\pi+\Psi(1-\pi)}\left(n_g u(c_{1g}) + n_b u(c_{1b})\right)\right], \\
c_2^i =&(1-q)\left(n_g u(c_{2g}^N) + n_b u(c_{2b}^N)\right) + q\left(1-\frac{\pi}{\pi+\Psi(1-\pi)}\right)\left(n_g u(c_{2g}^R) + n_b u(c_{2b}^R)\right).
\end{aligned}$$

B Full solution to the bank's problem (Section 3.1)

I here summarize the full characterization of the solution to the banking problem. Through constraints (18)-(19) and the first-order conditions (20)-(22), it is straightforward to derive each consumption variables given θ as below.

$$
\begin{aligned}
c_1(\theta) =& \frac{p_u}{\theta + \eta(\frac{\Delta}{p_u^{1-\gamma}})^{\frac{1}{\gamma}}} && (37)\\
c_{1j}(\theta) =& \frac{p_j(1-\theta\frac{c_1}{p_u})}{\eta}, && (38)\\
c_{2j}^N(\theta) =& \frac{R_j(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\eta}, && (39)\\
c_{1j}^R(\theta) =& \frac{p_h(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}, && (40)\\
c_{2j}^R(\theta) =& \frac{p_h(1-\theta\frac{c_1}{p_u})(\frac{\Lambda^{\frac{1}{\gamma}}}{\eta})}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}\left(\frac{1}{\rho}\right)^{\frac{1}{\gamma}}, \forall h \in b, g && (41)
\end{aligned}
$$

where

$$
\begin{aligned}
\eta =& (\pi-\theta) + \Lambda^{\frac{1}{\gamma}} > 0,\\
\Delta =& n_g p_g^{1-\gamma} + n_b p_b^{1-\gamma} > 0,\\
\Lambda =& (1-q)\left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} + q\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} > 0.
\end{aligned}
$$

The last inequality implies the first inequality because its first term is positive ($\theta \in [0, \pi]$).

C Proofs for selected results

Proposition 1. Recall the set of constraints (9) and the associated first-order conditions (11)-(13). I combine them to reduce the maximization problem to the maximization problem of θ:

$$
max_{\theta\in[0,\theta]} \frac{1}{1-\gamma} p_u^{1-\gamma}\left(\theta + \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\left\{(\pi-\theta) + (1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}\right)^{\gamma}.
$$

If the solution to this problem has an interior solution, condition (10) binds. Otherwise, it is slack. Taking derivative of θ, I obtain

$$\underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\text{negative}}\underbrace{p_u^{1-\gamma}\left\{\theta\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)+\left\{\pi+(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\gamma-1}}_{\text{positive}}\underbrace{\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)}_{\text{negative}}>0$$

where

$$\left(\frac{\Delta}{p_u^{1-\gamma}}\right)=\left(\frac{n_g p_g^{1-\gamma}+n_b p_b^{1-\gamma}}{(n_g p_g+n_b p_b)^{1-\gamma}}\right)>1.$$

Therefore, the objective function is monotonically increasing in θ, and θ is at the corner solution.

□

Lemma 1. The bank's best response to $\hat{y}(\theta)$ is summarized in the vector $\mathcal{A}(\theta,q)$ in which each consumption variable is derived in Section B. Substituting (37)-(41) into the objective function (17), I can derive the objective function $U(c^*,\hat{y}(q);\theta)$ as a function of θ such that

$$U(c^*,\hat{y}(q);\theta)=\left(\frac{1}{1-\gamma}\right)p_u^{1-\gamma}\left\{\theta\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)+(\pi+\Lambda^{\frac{1}{\gamma}})\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\gamma} \tag{42}$$

where (η,Δ) follows the notation in Section B. Taking a derivative of θ,

$$\frac{\partial U(c^*,\hat{y}(q);\theta)}{\partial\theta}=\underbrace{\left(\frac{\gamma}{1-\gamma}\right)}_{\text{negative}}\underbrace{p_u^{1-\gamma}\left\{\theta\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)+(\pi+\Lambda^{\frac{1}{\gamma}})\right\}^{\gamma-1}}_{\text{positive}}\underbrace{\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)}_{\text{negative}}>0.$$

The second group becomes straightforward if it is written as

$$\theta\left(1-\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)+(\pi+\Lambda^{\frac{1}{\gamma}})=\theta+\eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}>0.$$

The third sign is implied by the risk-aversion such that

$$\left(\frac{\Delta}{p_u^{1-\gamma}}\right)=\left(\frac{n_g p_g^{1-\gamma}+n_b p_b^{1-\gamma}}{(n_g p_g+n_b p_b)^{1-\gamma}}\right)>1.$$

$\square$

Lemma 2. I first show that the expected payoff (26) is monotonically decreasing in q. Recall the expected payoff in period 1:

$$\begin{aligned}\mathbb{E}u(c_{1k}) =& \frac{\theta}{\pi}u(c_1) + \left(1 - \frac{\theta}{\pi}\right)\Sigma_j n_j u(c_{1j}) \\ =& \left(\frac{1}{1-\gamma}\right)\left\{\frac{\theta}{\pi}p_u^{1-\gamma}\left(\frac{1}{\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}\right)^{1-\gamma} + \left(1-\frac{\theta}{\pi}\right)\Delta\left(\frac{\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}{\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}\right)^{1-\gamma}\right\}\end{aligned}$$

Taking a derivative of θ,

$$\frac{\partial \mathbb{E}u(c_{1k})}{\partial q} = (-1)\underbrace{\left(\frac{1}{\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}\right)^{1-\gamma}\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\left\{\frac{\theta}{\pi}p_u^{1-\gamma} + \left(1-\frac{\theta}{\pi}\right)\Delta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}}_{\text{positive}}\underbrace{\frac{\partial \eta}{\partial q}}_{\text{positive}} < 0.$$

The positive sign of $\frac{\partial \eta}{\partial q}$ is because

$$\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\} > \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}. \tag{43}$$

Similarly, I next show that the expected payoff (28) is monotonically increasing in q. Letting $A = \frac{1}{\pi(1-\pi)+(1-\pi)^2(\frac{1}{\rho})^{\frac{1-\gamma}{\gamma}}}(\frac{1}{\rho})^{\frac{1}{\gamma}}$,

$$\begin{aligned}\frac{\partial \mathbb{E}u(c_{2j}^R)}{\partial q} \quad = \quad & \Delta A^{1-\gamma}\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1-\gamma}{\gamma}}\left(\frac{1}{\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}\right)^{-\gamma}\Lambda^{\frac{1-2\gamma}{\gamma}} \cdot \\ & \left\{\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\Delta\frac{\partial \eta}{\partial q} + \left(\frac{1}{\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}}\right)\left(\frac{1}{\gamma}\right)\frac{\partial \Lambda}{\partial q}\right\} > 0.\end{aligned}$$

where all of the terms are positive. $\square$

Proposition 2. When $\bar{q} = 1$, for any value of $qin[0,1]$, $\mathbb{E}u(c_{1k}) \geq \mathbb{E}u(c_{2j}^R)$ holds. Notice that q-

strategy profile with $q = 1$ is equivalent to a strategy profile $y_i(\omega_i; s) = 0\ \forall s,\ \forall i$ that is a certain run strategy profile. Therefore, there exists an equilibrium in which bank run certainly occurs when $\bar{q} = 1$. When $\bar{q} < 1$, $\mathbb{E}u(c_{1k}) \geq \mathbb{E}u(c_{2j}^R)$ holds for $q \leq \bar{q} < 1$ by definition. Then, q-strategy profile with $q = 1$ is not a part of equilibrium and hence there does not exist an equilibrium in which runs certainty occur. □

Proposition 3. When $\underline{q} = 0$, for any value of $qin[0,1]$, $\mathbb{E}u(c_{1k}) \leq \mathbb{E}u(c_{2j}^N)$ holds. Notice that q-strategy profile with $q = 0$ is equivalent to a strategy profile $y_i(\omega_i; s) = \omega_i\ \forall s,\ \forall i$ that is a no-run strategy profile. Therefore, there exists an equilibrium in which no bank run occurs when $\underline{q} = 0$. When $\underline{q} > 0$, $\mathbb{E}u(c_{1k}) \leq \mathbb{E}u(c_{2j}^N)$ holds for $q \geq \underline{q} > 0$ by definition. Then, q-strategy profile with $q = 0$ is not a part of equilibrium and hence there does not exist an equilibrium in which no run occurs. □

Proposition 4. I first find $\bar{q}$ such that $\mathbb{E}u(c_{1k}) = \mathbb{E}u(c_{2j}^R)$ holds. Equating $\mathbb{E}u(c_{1k})$ and $\mathbb{E}u(c_{2j}^R)$,

$$\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} \left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}} \rho = \Lambda.$$

By substituting Λ and solving for q,

$$\bar{q} = \frac{\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} \left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}} \rho - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}} \tag{44}$$

Similarly, I derive $\underline{q}$ such that $\mathbb{E}u(c_{1k}) = \mathbb{E}u(c_{2j}^N)$ holds. Equating $\mathbb{E}u(c_{1k})$ and $\mathbb{E}u(c_{2j}^N)$,

$$\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} (1-\pi)^{\gamma} \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}} \rho = \Lambda. \tag{45}$$

By substituting Λ and solving for q,

$$\underline{q} = \frac{\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} (1-\pi)^{\gamma} \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}} \rho^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}$$

Conditions $\bar{q} \begin{Bmatrix} \leq \\ > \end{Bmatrix} 1$ and $\underline{q} \begin{Bmatrix} \geq \\ < \end{Bmatrix} 0$ reduce to the same condition

$$\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}} \rho \begin{Bmatrix} \geq \\ < \end{Bmatrix} 1. \tag{46}$$

$\square$

Proposition 5. I below find $\frac{\partial \bar{q}}{\partial \theta}$ by differentiating (44) w.r.t θ. Notice that the denominator of (44) is positive and θ appears only in the first term of the numerator. Differentiating $\bar{q}$ by θ gives:

$$\frac{\partial \bar{q}}{\partial \theta} = \underbrace{\frac{\gamma}{1-\gamma}}_{\text{negative}} \underbrace{\left(\frac{1}{\pi}\right)\left(1 - \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{1}{1-\gamma}} \chi,}_{\text{positive}} \tag{47}$$

where $\chi = \frac{\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}}\rho}{\left\{\pi(1-\pi)+(1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}} > 0.$ $\square$

Proposition 6. Recall the explicit form of $\bar{q}(\theta)$:

$$\bar{q} = \frac{\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} \left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{-1}{1-\gamma}}\rho - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}$$

When $(R_g - R_b)$ increases, n is closer to $\frac{1}{2}$, or γ increases, the relevant term is only the first term of the numerator. Each of these changes raises a benefit of risk-sharing as $\frac{\Delta}{p_u^{1-\gamma}}$ decreases, which pushes $\bar{q}$. Parameters ρ and γ have effects on the risk-sharing between good and bad fundamental states and the risk-sharing between period 1 and period 2, and hence they appear every terms. Suppose γ or ρ increase, then effects on $\bar{q}$ depend on $\left\{\frac{\theta}{\pi} + \left(1 - \frac{\theta}{\pi}\right)\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right\}^{\frac{\gamma}{1-\gamma}} p_u^{1-\gamma}\Delta^{\frac{-1}{1-\gamma}}\rho$. Notice that this term also increases as γ or ρ increase, and thus $\bar{q}$ increases. $\square$

Proposition 7. Recall the expected utility (42), I take the derivative of θ to solve for θ^*:

$$\frac{\partial U(c^*, \hat{y}(\bar{q}(\theta)); \theta)}{\partial \theta} = \frac{\gamma p_u^{1-\gamma}}{1-\gamma}\left(\theta + \eta\left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1}\left\{1 + \left(\frac{\Delta}{p_u^{1-\gamma}}\right)^{\frac{1}{\gamma}}\frac{\partial \eta}{\partial \theta}\right\}.$$

The optimal level of opacity is, then, $\theta^* = min\{\pi, \hat{\theta}\}$ such that

$$-\gamma x(\hat{\theta})^{\frac{2}{1-\gamma}} + Cx(\hat{\theta})^{\frac{\gamma}{1-\gamma}} + 1 = 0 \tag{48}$$

where

$$\begin{aligned}
x(\hat{\theta}) &= \Big(\frac{\Delta}{P_u^{1-\gamma}}\Big)^{\frac{1}{\gamma}} + \Big(\frac{\hat{\theta}}{\pi}\Big)\Big(1 - \Big(\frac{\Delta}{P_u^{1-\gamma}}\Big)^{\frac{1}{\gamma}}\Big) && (49)\\
\Delta &= n_g p_g^{1-\gamma} + n_b p_b^{1-\gamma} && (50)\\
C &= p_u^{\frac{1-\gamma}{\gamma}} \Delta^{\frac{-1}{\gamma(1-\gamma)}} \rho^{\frac{1}{\gamma}} \Big\{\pi(1-\pi) + (1-\pi)^2\Big(\frac{1}{\rho}\Big)^{\frac{1-\gamma}{\gamma}}\Big\}\Big(\frac{\Delta}{P_u^{1-\gamma}}\Big)^{\frac{1}{\gamma}} && (51)
\end{aligned}$$

Notice that this solution does not have a closed-form solution. Given a parameter set $(\gamma, \pi, n, R_g, R_b, \rho) = (2, 0.5, 0.5, 2, 1, 0.9)$, $\theta^* < \pi$ as shown in Figure 3. When $\gamma = 2$, this solution has a closed-form and it is illustrated in Proposition 8. □

Proposition 8. Suppose $\gamma = 2$. Then, the expected utility (42) can be written as:

$$U(c^*, \hat{y}(\bar{q}(\theta)); \theta) = \frac{p_u^{-1}}{1-\gamma}\Bigg(\theta(1-a_0) + \Bigg(a_0 + \Bigg(a_0 + (1-a_0)\frac{\theta}{\pi}\Bigg)^{-1} a_1 + a_0\pi\Bigg)^2\Bigg),$$

where

$$\begin{aligned}
a_0 &= (\Delta p_u)^{\frac{1}{2}}\\
a_1 &= \Delta p_u \rho^{\frac{1}{2}}\{\pi(1-\pi) + (1-\pi)^2\rho^{\frac{1}{2}}\}.
\end{aligned}$$

Taking a derivative, I derive θ^* such that $\frac{\partial U(c^*, \hat{y}(\bar{q}(\theta)); \theta)}{\partial\theta} = 0$ as below.

$$\begin{aligned}
0 &= (1-a_0)\Bigg\{\pi\Bigg(a_0 + (1-a_0)\frac{\theta^*}{\pi}\Bigg) + a_1\Bigg(a_0 + (1-a_0)\frac{\theta^*}{\pi}\Bigg)^{-1}\Bigg\}\Bigg\{1 - \frac{a_1}{\pi}\Bigg(a_0 + (1-a_0)\frac{\theta^*}{\pi}\Bigg)^{-2}\Bigg\}\\
\Rightarrow \quad \theta^* &= \frac{\pi^{\frac{1}{2}}\rho^{\frac{1}{4}}(\Delta p_u)^{\frac{1}{2}}[\{\pi(1-\pi) + (1-\pi)^2\rho^{\frac{1}{2}}\}^{\frac{1}{2}} - \{\pi\rho^{\frac{-1}{2}}\}^{\frac{1}{2}}]}{1 - (\Delta p_u)^{\frac{1}{2}}}
\end{aligned}$$

□

Proposition 9. I prove this result by taking a derivative of each component:.

- the discount rate ρ:

  Since $\Delta p_u = \big(n_g\frac{1}{p_g} + n_b\frac{1}{p_b}\big)\big(n_g p_g + n_b p_b\big) = \big(n_g\frac{1}{R_g} + n_b\frac{1}{R_b}\big)\big(n_g R_g + n_b R_b\big)$, ρ is relevant only in the numerator. Each term in the numerator increases as ρ increases and hence θ^* is increasing in ρ.

- the difference of returns over state $(R_g - R_b)$:

  These parameters affect θ^* through Δp_u. An increase of the difference of returns raises $\Delta p_u = \big(n_g\frac{1}{p_g} + n_b\frac{1}{p_b}\big)\big(n_g p_g + n_b p_b\big)$, and hence θ^* increases.

- the probability of asset being good n: Similarly to the case $(R_g - R_b)$, a change of n affects Δp_u. When n becomes closer to $\frac{1}{2}$, this term Δp_u increases as the fundamental uncertainty is more uncertain. Correspondingly, the optimal level of opacity θ^* increases.

□

Proposition 10. I begin this proof by finding the threshold value of q such that

$$\tilde{q} \text{ such that } \mathcal{W}(0, \tilde{q}) = \mathcal{W}^A. \tag{52}$$

Recall $\mathcal{W}(0, \tilde{q}) = U(c^*, \hat{y}(q); \theta)$ and the expected utility (42). Equating (42) and (7) and solving for q, I obtain

$$\tilde{q} = \frac{\left\{\left(\pi + (1-\pi)\left(\frac{1}{\rho}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}} - \pi\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}{\left\{\pi(1-\pi) + (1-\pi)^2\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} - \left\{(1-\pi)\left(\frac{1}{\rho}\right)^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma}}.$$

Because $\rho < 1$, $\bar{q}(0) < \tilde{q}$ holds for any parameter sets. By Lemma 1,

$$\mathcal{W}(0, \bar{q}(0)) > \mathcal{W}(0, \tilde{q}) = \mathcal{W}^A.$$

Therefore, $\mathcal{W}(\theta, \bar{q}(\theta)) \geq \mathcal{W}(0, \bar{q}(0))$. □

Proposition 11. I first characterize the solution to the modified banking problem. The objective function (33) and the set of constraints (34)-(35) remain unchanged from the bank's problem in

Section 4, but here is one more choice variable θ. The solution is characterized by the resource constraint (34)-(35), the first-order conditions (20)-(22) and (36). Combining these equation, I characterize optimal consumption levels by θ and I formulate the optimization problem as a function of θ:

$$U(c^{**}(\theta), \hat{y}(q), \theta) = max_{\theta \in [0,\pi]} \frac{p_u^{1-\gamma}}{1-\gamma} \left(\theta + \eta \left(\frac{\Delta}{p_u^{1-\gamma}} \right)^{\frac{1}{\gamma}} \right)^{\gamma}.$$

If the first-order condition (36) binds, this problem has an interior solution. Otherwise, it has a corner solution for θ. I differentiate this objective function by θ:

$$\frac{\partial U(c^{**}(\theta), \hat{y}(q), \theta)}{\partial \theta} = \underbrace{\frac{\gamma}{1-\gamma}}_{\text{negative}} \underbrace{p_u^{1-\gamma} \left(\theta + \eta \left(\frac{\Delta}{p_u^{1-\gamma}} \right)^{\frac{1}{\gamma}} \right)^{\gamma-1}}_{\text{positive}} \underbrace{\left(1 - \left(\frac{\Delta}{p_u^{1-\gamma}} \right)^{\frac{1}{\gamma}} \right)}_{\text{negative}}, \tag{53}$$

where the sign of the second last and last group are implied by Proof of Lemma 1. Therefore, $U(c^{**}(\theta), \hat{y}(q), \theta)$ is monotonically increasing in θ given $\hat{y}(q) \forall q$. The optimal level of θ will be at the maximum level $\theta^{**} = \pi$. □

Proposition 12. By Proof of Proposition 10, θ^{**} in this environment is a corner solution. By limiting the bank's choice set from $[0, \pi]$ to $[0, \theta^*]$, θ^{**} is still a corner solution but at the same level to θ^*. Since the objective function and the set of constraints are the same to Section 4, the expected utility of depositors are equivalent to the θ observable case with $\theta = \pi$. Then, $\mathcal{W}(\theta^*, \bar{q}(\theta^*)) \geq \mathcal{W}(\pi, \bar{q}(\pi))$. □