Systemic Portfolio Diversification

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Financial Interconnectedness

- The classical asset allocation paradigm for an individual investor prescribes *diversification* across assets.
- The 2007–2009 global financial crisis highlighted the dangers of an interconnected financial system.
- Two main channels of financial contagion:
  - counterparty risk
  - **portfolio commonality**

Price Mediated or Counterparty Contagion?

- Counterparty network studies assume asset prices fixed at their book values: balance sheets only take hits at default events.
- Adrian and Shin (2008): *If the domino model of financial contagion is the relevant one for our world, then defaults on subprime mortgages would have had limited impact.*
- Empirical evidence suggests that financial institutions react to asset price changes by actively managing their balance sheets.
- **Price mediated propagation**: forced sales of illiquid assets may depress prices, and prompt financial distress at other banks with similar holdings.
- Greenwood, Landier and Thesmar (2015): measure of the vulnerability of a system of leverage-targeting banks. Overlapping portfolios and fire sale spillovers exacerbate banks' losses.

Research Question

- How do institutions *ex ante* structure their balance sheets when they account for the systemic impact of other large institutions?

Systemic risk triggered by fire-sales:

- Market events drive large negative price movements
- Excessive correlation due to common holdings may be destabilizing

- Benefits of diversification may be lost when most needed.
- Financial institutions close out positions in response to price drops.
- Sell-offs affect several institutions simultaneously and exacerbate liquidation costs.
- **Should we be concerned about a different (systemic) kind of diversification?**

Building Blocks

Financial Constraints

- A financial institution is forced to liquidate assets on a short notice to raise immediacy (margin calls, mutual funds' redemptions, regulatory capital requirements...).

Price Impact

- Sell-offs have a knock-down effect on prices.

The Model

- One period timeline
- Economy with N banks and K assets
- Initial asset prices normalized to 1\$
- Bank i's balance sheet:

  d_i debt,
  e_i equity,
  $w_i := d_i + e_i$ asset value,
  $\lambda_i := d_i/e_i$ leverage ratio,
  $\pi_{i,k}$ weight of asset k in bank i's portfolio

The Model

- Suppose each asset k is subject to a return shock Z_k
- Let $Z = (Z_1, \ldots, Z_K)$ be the vector of return shocks
- Bank i's return is $R_i := \pi_i \cdot Z = \sum_k \pi_{i,k} Z_k$

Assumption 1

Leverage threshold: Bank i liquidates assets if its leverage threshold $\lambda_{M,i}$ is breached.

- Bank i liquidates the minimum amount necessary to restore its leverage at the threshold:

$$\lambda_{M,i} w_i \left(R_i + \ell_i\right)^- ,$$

where $\ell_i := \frac{\lambda_{M,i} - \lambda_i}{(1+\lambda_i)\lambda_{M,i}}$ is the *distance to liquidation*.

The Model

Assumption 2

Exposures remain (roughly) fixed: Banks liquidate (or purchase) assets proportionally to their initial allocations.

- After being hit by te market shock Z, bank i trades an amount $\lambda_{M,i} w_i \left(R_i + \ell_i\right)^{-} \pi_{i,k}$ of asset k.

Assumption 3

Linear Price Impact: The cost of fire sales, i.e., the execution price, is linear in quantities.

- An aggregate trade q_k of asset k is executed at the price $1 + \gamma_k q_k$ per asset share.

The Model

- Market shocks Z_k are i.i.d. random variables.
- All assets have the same returns
- **Control variables**: banks choose their asset allocation weights π_i.
- **Objective function**: banks maximize expected portfolio returns.

Model Parameters

- w: size of the banks
- ℓ: riskiness of the banks
- γ: illiquidity of the assets

Model Limitations

- We ignore the possibility of default.
    - If $R_i \leq -\frac{1}{\lambda_i}$, the bank's equity is negative.

- We assume only one round of deleveraging.
    - Due to price impact, banks may engage in several rounds of deleveraging (Capponi and Larsson (2015)).

Equilibrium Asset Holdings

Each bank maximizes an objective function given by its expected portfolio return, i.e.,

$$\mathrm{PR}_i(\pi_i, \pi_{-i}) := E[\pi_i^T Z - \mathrm{cost}_i(\pi_i, \pi_{-i}, Z)].$$

Total liquidation costs of bank i:

$$cost_i(\pi_i, \pi_{-i}) := E\Bigg[\underbrace{\lambda_{M,i} w_i \left(\pi_i \cdot Z + \ell_i\right)^- \pi_i^T}_{\text{assets liquidated by bank } i} \mathrm{Diag}[\gamma] \underbrace{\sum_{j=1}^{N} \pi_j \lambda_{M,j} w_j \left(\pi_j \cdot Z + \ell_j\right)^-}_{\text{total quantities traded}} \Bigg].$$

Equilibrium Asset Holdings

Nash equilibrium

Let $X := \{x \in [0,1]^K \ : \ \sum_{k=1}^K x_k = 1\}$ be the set of admissible strategies. A (pure strategy) Nash equilibrium is a strategy $\{\pi_i^*\}_{1 \le i \le N} \subset X$ such that for every $1 \le i \le N$ we have

$$\mathrm{PR}_i(\pi_i^*, \pi_{-i}^*) \ge \mathrm{PR}_i(\pi_i, \pi_{-i}^*) \qquad \text{for all } \pi_i \in X.$$

Because assets' returns are identically distributed, the optimization problem of bank i is equivalent to minimizing $cost_i(\pi_i^*, \pi_{-i}^*)$.

Potential Game

- Assume $N = 2$, $K = 2$.
- Best response strategy of bank 1 is

$$\pi^*_{1,1} = \text{argmin}_{\pi_{1,1}} \Big\{ \lambda^2_{M,1} E\big[w_1^2 (\pi_1 \cdot Z + \ell_1)^2 (\pi^2_{1,1}\gamma_1 + (1-\pi_{1,1})^2\gamma_2)\mathbf{1}_{L_1}\big] +$$

$$\lambda_{M,1}\lambda_{M,2} E\big[w_1 w_2 (\pi_1 \cdot Z + \ell_1)(\pi_2 \cdot Z + \ell_2)(\pi_{1,1}\pi_{1,2}\gamma_1 + (1-\pi_{1,1})(1-\pi_{1,2})\gamma_2)$$

$$= \text{argmin}_{\pi_{1,1}} \Big\{ \cdots + \lambda^2_{M,2} E\big[w_2^2 (\pi_2 \cdot Z + \ell_2)^2 (\pi^2_{2,1}\gamma_1 + (1-\pi_{2,1})^2\gamma_2)\mathbf{1}_{L_2}\big] \Big\}.$$

Both banks minimize the same function!

Existence and Uniqueness

Theorem

Assume Z_k has a continuous probability density function. Then there exists a Nash equilibrium.

Single Bank Benchmark

- Consider the portfolio held by a bank when it disregards the impact of other banks.
- Bank seeks diversification to reduce likelihood of liquidation.
- Bank seeks a larger position in the more liquid asset to reduce realized liquidation costs.

Proposition

Let $N = 1$, $K = 2$, and $\gamma_1 < \gamma_2$. Then

- $\pi^S_{1,1} \in (\frac{1}{2}, \frac{\gamma_2}{\gamma_1+\gamma_2})$, *where* $(\pi^S_{1,1}, 1 - \pi^S_{1,1})$ *minimizes the bank's expected liquidation costs.*
- $\pi^S_{1,1}(\ell)$ *is decreasing in* ℓ.

Identical Assets/Banks

- If there is no heterogeneity in the system (across assets or across agents), then in equilibrium all banks hold the same portfolio.
- In the presence of other identical banks, assets become more "expensive", but the banks' relative preferences do not change.
- The system behaves as a single representative bank.

Proposition

- *If $\gamma_1 = \gamma_2$, then $\pi_{i,1} = 50\%$ for all i.*
- *Let $\bar{\pi}$ be the optimal allocation in asset 1 of a bank with distance to liquidation $\bar{\ell}$, when $N = 1$.*
  If $\ell_i = \bar{\ell}$ for all i, then $\pi_{i,1} = \bar{\pi}$ for all i.

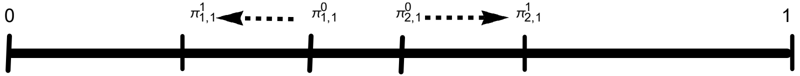
Introducing Heterogeneity

Proposition

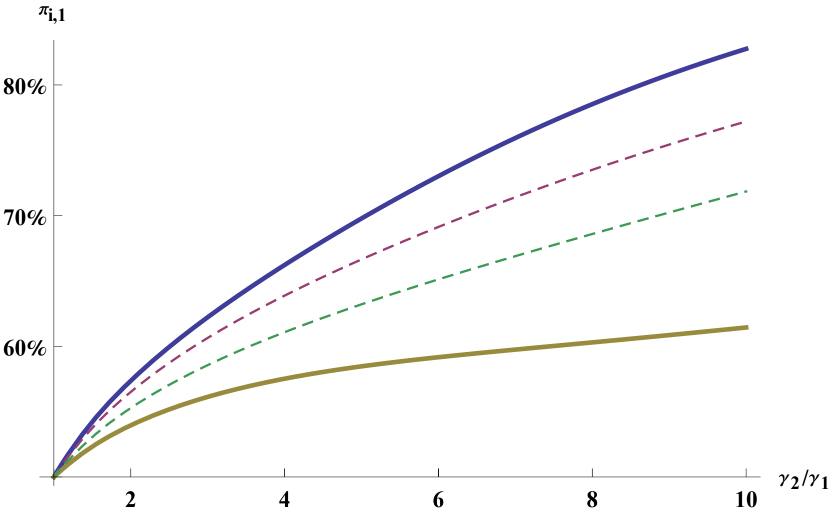
Assume $N = 2$, $\gamma_1 < \gamma_2$ and $\ell_1 > \ell_2$.

- $|\pi^*_{1,1} - \pi^*_{2,1}| > |\pi^S_{1,1} - \pi^S_{2,1}|$, *where* $\pi^S_{i,1}$ *is the bank i's optimal asset 1 allocation in the single agent case.*

- Let f_i be the best response function of bank i, $i = 1, 2$.
- Let $\pi^0_{1,1}$ be the optimal allocation of bank 1, if bank 2 has the same leverage ratio.
- Recursively, $\pi^n_{1,1} := f_1(\pi^{n-1}_{2,1})$, $\pi^n_{2,1} := f_2(\pi^{n-1}_{1,1})$
    - banks are more and more **diverse**, until an equilibrium is reached.



Comparative Statics



Increasing heterogeneity across assets.

Social Costs

- Are banks behaving as a benevolent social planner would like?
- If not, what are the social costs of this mechanism?

Social Planner

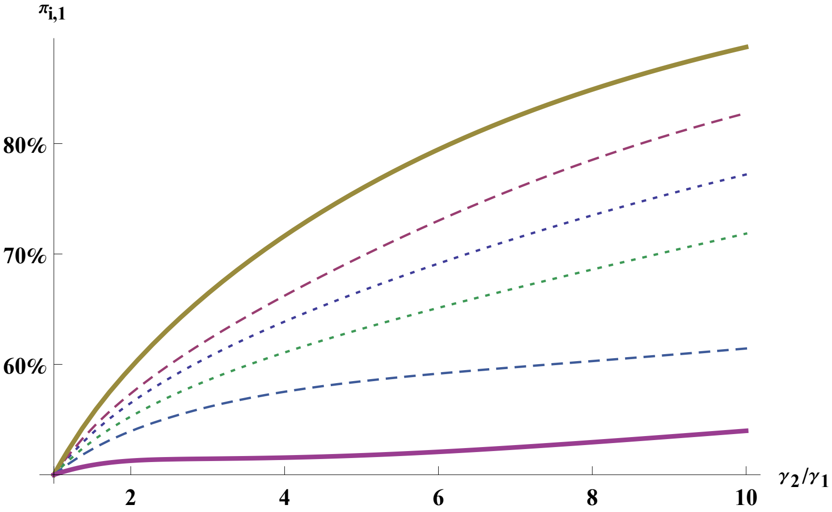
- Minimizes objective function: $TC(\pi_1, \cdots, \pi_N) := \sum_{i=1}^{N} cost_i(\pi_i, \pi_{-1})$.

Proposition

- *If $\ell_i = \bar{\ell}$ for all i, the minimizer π^{SP} of TC is the unique Nash equilibrium.*
- *Assume $N = 2$. If $\ell_1 \neq \ell_2$, then π^{SP} is not a Nash equilibrium. In particular, $|\pi_{1,1}^{SP} - \pi_{2,1}^{SP}| > |\pi_{1,1}^{*} - \pi_{2,1}^{*}|$.*

- In equilibrium, banks are not **diverse** enough!
- Each bank accounts for the price-impact of other banks on its execution costs, but neglects the externalities it imposes on the other banks.

Social Planner



Tax Systemic Risk

Proposition

If each bank i pays a tax equal to

$$T_i(\pi) := \sum_{j \neq i} M_{i,j}(\pi),$$

where $M_{i,j}(\pi_i, \pi_j) := \lambda_{M,i}\lambda_{M,j} w_i w_j E\left[(R_i + \ell_i)^- (R_j + \ell_j)^- \pi_i^T Diag[\gamma]\pi_j\right]$*, then the equilibrium allocation is equal to the social planner's optimum.*

- $M_{i,j}(\pi_i, \pi_j)$ are the externalities that bank i imposes on bank j.
- By internalizing these externalities, the objectives of the banks align with the social planner's objective.

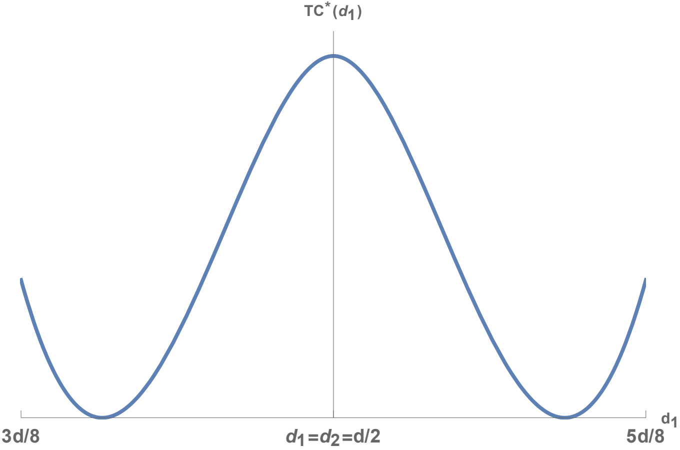
Is Higher Heterogeneity Socially Desirable?

- Yes....

Proposition

Assume the system has two banks and two assets with aggregate asset value w and debt d.
Assume $w_1 = w_2 = \frac{w}{2}$ and $d_2 = d - d_1$. Define $TC^*(d_1)$ as the total expected liquidation costs in equilibrium as function of d_1. Then $d/2$ is a local maximum for $TC^*(d_1)$.

Benefits of Heterogeneity



Total expected liquidation costs for different levels of leverage heterogeneity

Conclusions

- Systemic liquidation risk affects the banks' asset allocation decisions, in that they reduce their portfolio overlap.
- To achieve the socially optimal allocation, banks should reduce portfolio commonality even further.
- A tax makes banks internalize their contribution to systemic risk.
- Higher heterogeneity in the system reduces aggregate liquidation costs.

Banks' Portfolio Allocation in Practice

- Every quarter, banks file form FR Y-9C with the Federal Reserve, providing information on their balance sheet composition.
  This information is publicly available
- A bank active in a certain market will have specific information about this market, and may therefore infer additional information on the competitors' portfolio composition.

Banks’ Portfolio Allocation in Practice

Schedule HC-B—Securities

	Held-to-Maturity				Available-for-Sale			
	(Column A) Amortized Cost		(Column B) Fair Value		(Column C) Amortized Cost		(Column D) Fair Value	
Dollar Amounts in Thousands	BHCK	Amount	BHCK	Amount	BHCK	Amount	BHCK	Amount
1. U.S. Treasury securities	0211	0	0213	0	1286	27,526,000	1287	27,816,000
2. U.S. government agency and sponsored agency obligations (exclude mortgage-backed securities)[1]	HT50	0	HT51	0	HT52	0	HT53	0
3. Securities issued by states and political subdivisions in the U.S.	8496	4,831,000	8497	4,869,000	8498	36,659,000	8499	38,121,000
4. Mortgage-backed securities (MBS)								
a. Residential pass-through securities:								
(1) Guaranteed by GNMA	G300	5,898,000	G301	5,697,000	G302	19,323,000	G303	18,813,000
(2) Issued by FNMA and FHLMC	G304	17,847,000	G305	17,600,000	G306	40,291,000	G307	39,656,000
(3) Other pass-through securities	G308	0	G309	0	G310	0	G311	0
b. Other residential mortgage-backed securities (include CMOs, REMICs, and stripped MBS):								
(1) Issued or guaranteed by U.S. Government agencies or sponsored agencies[2]	G312	0	G313	0	G314	319,000	G315	322,000
(2) Collateralized by MBS issued or guaranteed by U.S. Government agencies or sponsored agencies[2]	G316	0	G317	0	G318	0	G319	0
(3) All other residential mortgage-backed securities	G320	0	G321	0	G322	9,035,000	G323	9,217,000
c. Commercial MBS:								

Excerpt from the latest FR Y-9C filing by JPMorgan.

Uniqueness

Technical Assumptions

- Z_k has continuous probability density function, increasing on $[-\infty, 0]$, and the random vector Z is spherically symmetric.
- ℓ_i is sufficiently small.

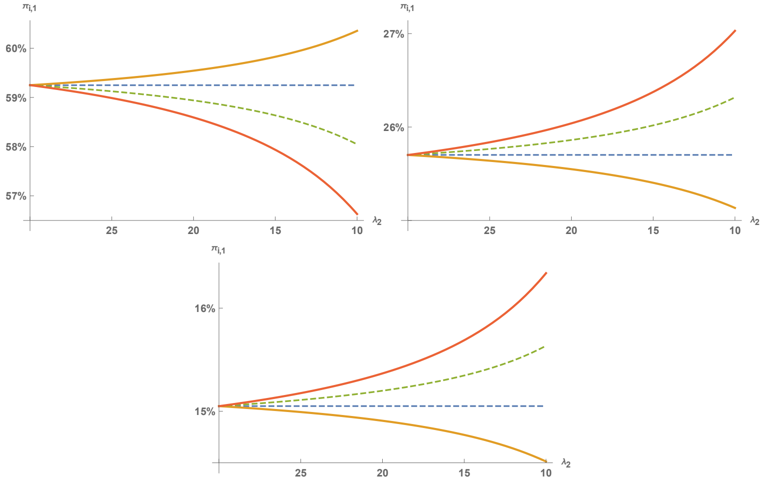
If $\gamma_1 = 0$ and $\gamma_i > 0$ for $i > 1$, then the unique Nash equilibrium is $\pi^*_{i,1} = 1$.

Theorem

Let $N = 2$ and $K = 2$. If assets and banks are "close enough", then there is a unique Nash equilibrium.

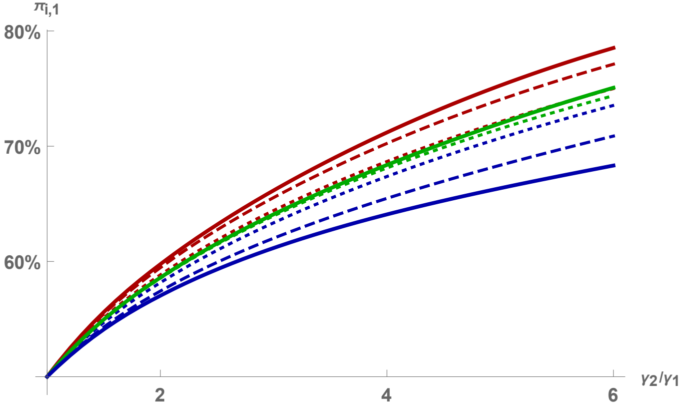
- Assume that the Nash equilibrium is unique.

Multiple Assets



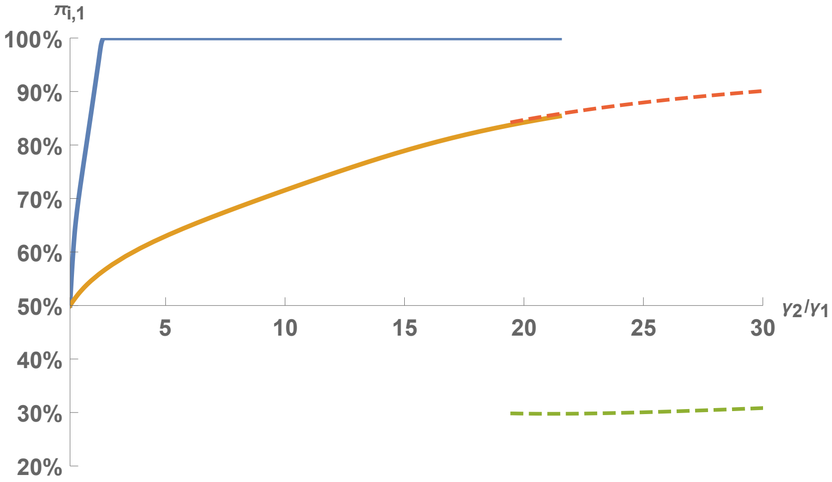
Banks reduce portfolio overlap in *each* asset.

Multiple Banks



Most (resp. least) leveraged bank increases its position in the most (resp. least) liquid asset even further.

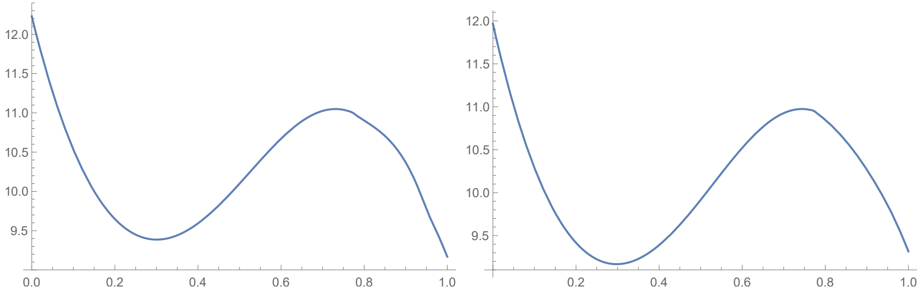
Multiple Equilibria



$$w_1 = 100 w_2.$$

Multiple Equilibria

For $\gamma_2 = 20.5\,\gamma_1$, $(\pi_{1,1}, \pi_{2,1}) = (84.63\%, 100\%)$ and $(85.09\%, 29.77\%)$ are both equilibria.



$\pi_{2,1}$ vs $cost_2$, for $\pi_{1,1} = 84.63\%$ (left) and $\pi_{1,1} = 85.09\%$ (right).

Future Extensions

- Profit-maximizing banks, assets heterogeneous in returns
- Dynamic model: decoupling asset allocations from liquidation strategy

Counterparty Risk Networks

- Treats financial system as a network and studies systemic consequences of initial shocks.
- Eisenberg and Noe (2001) develop an interbanking clearing framework to analyze propagations of losses originated from defaults.
- Related contributions include:
  - Amini et al (2016): resilience to contagion and asymptotic analysis
  - Glasserman and Young (2014): network spillovers versus direct shocks to firm's assets
  - Capponi, Chen, and Yao (2014): implications of liability concentration on the system's loss profile via majorization
  - Acemoglu, Ozdaglar and Tahbaz-Salehi (2015): dependence of contagion risk on network topology