CHAPMAN UNIVERSITY

The Thinking Person's Mathematics
(Fall 2025)

Classroom: Doti Hall 105
Time: TuTh 11:30-12:45

> I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming – and a little mad. Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings. – Alfred North Whitehead

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Course Description: This course explores the role of mathematics in the life of the mind by focusing on fundamental parts of mathematics as a network of thought experiments: what is "counting"? How does counting relate to calculating? How does calculating relate to proving? The course also emphasizes how design affects mathematical exploration: for example, why do we insist on certain arithmetic laws, even in situations that appear to be far removed from ordinary arithmetic? This course can also serve as key in the CPSC curriculum as it provides the theoretical background needed for many upper-division courses including Data Structures (combinatorics, formal languages), Logic Design (Boolean algebras, number representation) and Integrated Circuit Design (automata theory, finite state minimization). Subject to approval the course satisfies Chapman's Quantitative Inquiry requirement.

Prerequisites: Math 101 or equivalent.

Type of Grade: Letter grade with Pass/No Pass option.

Number of Credits: 3 credits.

Textbook: This course has no required published textbook for purchase. Lecture notes and other learning materials will be made available to the students through Canvas.

Mathematics Program Learning Outcomes:

1. Graduates will be able to communicate mathematics orally and in writing. In particular, students will be able to use mathematical notation and terminology correctly in the context of communicating a mathematical argument orally or in writing.
2. Graduates will be able to read university level mathematical texts and professional literature and summarize key results in their own words.
3. Graduates will be able to prove basic results in mathematics.

Honors Program Learning Outcomes: Upon completing a course in the University Honors Program students will have:

1. Obtained a starting point for integrative exploration of the development of cultures and intellectual achievements through a variety of disciplinary and interdisciplinary perspectives;
2. Sharpened their ability to critically analyze and synthesize a broad range of knowledge through the study of primary texts and through engagement in active learning with fellow students, faculty, and texts (broadly understood);
3. Understood how to apply more integrative and interdisciplinary forms of understanding in the advancement of knowledge and in addressing complex challenges shaping the world;
4. Developed effective communication skills, specifically in the areas of written and oral exposition and analysis.

Course Learning Outcomes: By the end of the semester, we will be able to:

1. read, interpret, and use the vocabulary, notation and basic definitions of discrete mathematics;
2. show competence in understanding and discovering inductive proofs;
3. show mastery of basic reasoning with respect to sets and functions;
4. prove theorems regarding discrete structures.

Quantitative Inquiry Learning Outcome: Students will be able to create sophisticated arguments supported by quantitative evidence and can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate).

Time Allocation: A 3-credit course typically requires approximately 145 hours of your time to earn a passing grade of C or better. This includes 45 hours of classroom contact and approximately 100 hours of work outside of class.

Contents: This course is divided in three parts. This is a general list of topics that we will discuss.

Part I: Natural Numbers and Induction

- Counting is fundamental to all of Mathematics and Computation. Natural numbers provide us with a mental model of how counting works. Natural numbers come with two interrelated structures: one given by the arithmetical operations — addition ($+$) and multiplication ($\cdot$) — and the other given by the standard ordering — being smaller or equal than ($\leq$). We build up these structures from Peano's Axioms.
- The behavior of addition and multiplication give rise to the Laws of Arithmetic, and when considered in conjunction with the standard ordering, we obtain the Laws of Ordered Arithmetic. We prove all these laws from fundamental principles.
- Induction is a principle of Mathematics that allows us to establish properties of the collection of the natural numbers as a whole. We introduce the idea of induction and use it to prove many familiar and unfamiliar facts about Arithmetic.
- Putting items in some order, namely, "making lists," is also fundamental to all of Mathematics and Computer Science. Lists also come with several operations — prepend, append, concatenation, indexation, ... — , endowing the collections of lists with a rich structure. As we will see, lists are very closely related natural numbers. In particular, the general principle of Induction also pertains to lists. We use this principle to prove some useful facts about lists that are analogous to the Laws of Arithmetic.

Part II: Sets, Functions, Relations

- Functions are indispensable in Mathematics and Computation. In one form or another, they have played a role in every mathematics courses you've taken since late elementary school. In Computation, a programmer spends a lot of time constructing functions and reasoning about functions to perform specified tasks. In this section of the course, we take a close look at the concept of a function, and build up the needed mathematical infrastructure to do two main things:
  - Reason about functions: In this respect, our theory is also like a programming language. That is, a programmer does not only write code. A programmer also looks at existing code (from a code bank, for example) and tries to understand what it does and what its main properties are. We will distinguish several prominent types of functions: injections, surjections, and bijections.
  - Operate with functions: We will argue that the main operations between functions is composition, and we will investigate its properties. We will also obtain characterizations of the injections, surjections, and bijections according to their behavior with respect to composition.
- Functions require input and output. In Mathematics, the collections of data that serve as potential input and output are called sets. So in the first part of our investigation of functions, we also look at how to build sets and reason about them. In particular we will learn the principal properties of the main operations between sets, such as intersection, union, complementation, difference, Cartesian product, ...
- We will investigate how to capture the notion of the size of a set with the aid of functions, which will help us to compare the sizes of sets. We will rediscover Cantor's celebrated theorem that infinite is not a monolithic notion, but actually there is a constellation of infinities.
- Sometimes, it is useful to think about when two data are related (say, a given point may or may not be inside a given triangle) without explicitly bringing functions into the picture. We will introduce the idea of a relation. In particular, we will devote our attention to equivalence relations and the classifications associated.

Part III: Applications

- Once the structure of the natural numbers is in place, other collections of numbers can be build up from them, like the integers and the rational numbers, using equivalent relations in an appropriate way.
- Modular Arithmetic: The arithmetic you perform on a clock is not the same as standard arithmetic: 9 o'clock plus 5 hours is 2 o'clock, so $9 + 5 = 2$. Arithmetic on a dial behaves in some ways like standard arithmetic, but not in others. We investigate this and show how it related to our standard positional numeral systems of binary, hexadecimal, and decimal notation.
- We will end our course rediscovering prime numbers and proving fundamental facts about them. In particular, we will prove the Fundamental Theorem of Arithmetic, which says that prime numbers are, in some sense, the building blocks of all natural numbers. As an application, we will see how Modular Arithmetic and prime numbers are used as a fundamental part of cryptography.

Method of Evaluation: Grades will be based on the following assessments.

- **Homework Assignments (30%):** Written assignments will be posted (almost) every week, and will be collected on Thursday of the following week. *No late homework will be accepted.* A missing assignment will count as zero. The assignment with the lowest score will be dropped. You are

encouraged to work together, in small groups, as much as possible, but everyone should submit their own written answers, and clearly acknowledge on the first page of their assignments the colleagues with whom they worked.

The purpose of the homework assignments is manifold. Here are some of the highlights:

1. The assignments are intended to help you develop the habits of mind that characterize a mathematician. This includes to be puzzled a great deal of the time, but you can do so productively by learning from your honest attempts to understand and solve the problems, even if you have not solved them all to your complete satisfaction.
2. In addition, the assignments should serve as practice to learn how to communicate mathematical ideas in writing. Putting in black and white a solution to a problem is very much like storytelling or composing a painting. *"A painting must be painted and then looked at; a theorem must be printed and then read. The painter who thinks good pictures, and the mathematician who dreams beautiful theorems are dilettantes; an unseen work of art is incomplete."* (P. Halmos)
3. Finally, assignments should help inform you and me regarding your mathematical progress. It is your responsibility to do the work, but I strongly encourage you to seek my feedback during my scheduled students office hours or by email.

- **In-class Participation (30%):** Mathematics, as understood by mathematicians themselves, is highly collaborative and participatory. To experience mathematics as part of a one's intellectual life, these qualities must be taken seriously. The course will emphasis *understanding*, not mere competence in one sort of problem solving or another. So, regular active participation in discussion is imperative.
- **Midterm exam (25%):** During the sixth and ninth weeks of the semester, a two-part midterm will be given. The first part will be in the form of a take-home exam posted in the seventh week. The second part will take place in class on in the eight week after the take-home portion is due.
- **Final exam (25%):** This is a comprehensive take-home exam, encompassing all the material covered during the course, held during finals week. The exam is due on the last day of finals.

The reader will have notices something awry with these percentages. This may be a bug; it may be a feature. Join the class and find out.

Grading Scale: Letter grades will be no lower than the following cutoffs.

A = 93–100%	B− = 80–82.99%	D+ = 67–69.99%
A− = 90–92.99%	C+ = 77–79.99%	D = 63–66.99%
B+ = 87–89.99%	C = 73–76.99%	D− = 60–62.99%
B = 83–86.99%	C− = 70–72.99%	F = 0–59.99%

Communication: We will use Canvas as the main repository for this course. All the electronic announcements, assignments, due dates, supplementary material, ... will be posted there. There is a Canvas app for iPhone and Android and you can set it up to push notifications for your classes. Make sure that you are receiving notifications from Canvas through the app or by visiting the website frequently.

Submitted Work: The papers that you turn in must be neat. They should not look crammed, there must be a space between problems and on the margins for any possible comments. Your name and date must appear on the top of each sheet. And if you worked in a group, the names of your collaborators

should appear next to yours. The solutions to the problems should clearly indicate your reasoning process.

The best way to do homework is to work out your solution separately, then rewrite it neatly on the pages you will submit. Don't make the mistake of trying to work out a solution on the fly and then turning in that record of your attempts. It will almost surely be illegible.

You are strongly encouraged to submit typeset work using LaTeX. To learn how to use LaTeX, see the relevant posts on Canvas, or go to Overleaf, an online LaTeX service that offers free student accounts.

In-class Participation: Mathematics is a contact sport. You will be called on in class. Your contribution does not have to be flawless, but it should be thoughtful. A cogent explanation of where you are stuck can be just as valuable as a finely crafted response.

Attendance: You are expected to attend every class, as you have an obligation to contribute to the academic performance of all your colleagues by fully participating in the work of each class. If you miss a class meeting, you are responsible for knowing about any assignments or announcements made during the class.

Attendance to the midterm exam is compulsory; otherwise, a grade of zero will be recorded. Any excused, documented conflict with a test date must be reported to the instructor at least one week prior to the test date.

Classroom Policy: You will not be allowed to use electronic devices such as cell phones, music players, or computers during class in a way that is not relevant to the lecture. Texting during class is not acceptable. Calculators may be ridiculed (but not their owners).

Chapman University's Academic Integrity Policy: Chapman University is a community of scholars that emphasizes the mutual responsibility of all members to seek knowledge honestly and in good faith. Students are responsible for doing their own work and academic dishonesty of any kind will be subject to sanction by the instructor/administrator and referral to the University Academic Integrity Committee, which may impose additional sanctions including expulsion. Please see the full description of Chapman University's policy on Academic Integrity.

Health and Safety Protocols: Stay in touch with Chapman's health policies and guidelines, as they may change during the semester. The university continues to provide on-campus COVID testing at no cost to Chapman community members.

Students Office Hours: I have scheduled office hours on Tuesday 9:30-10:45; Thursday 2:30-4:00. These office hours will be held in person. Nonetheless, I will try to accommodate, via Zoom and by appointment, students who cannot attend in person. During these meetings you can ask questions, listen to your colleagues' questions, discuss covered material, discuss homework, ... These are relaxed, laid-back meetings, where everyone is free to participate as much or as little as they want.

Accommodations Procedure: In compliance with ADA guidelines, students who have any condition, either permanent or temporary, that might affect their ability to perform in this class are encouraged to contact the Disability Services Office. If you will need to utilize your approved accommodations in this class, please follow the proper notification procedure for informing your professor(s). This notification process must occur more than a week before any accommodation can be utilized. Please contact Disability Services at +1 (714) 516-4520 or visit the Disability Services website if you have questions regarding this procedure or for information or to make an appointment to discuss or request

potential accommodations based on documentation of your disability. Once formal approval of your need for an accommodation has been granted, you are encouraged to talk with your professor(s) about your accommodation options. The granting of any accommodation will not be retroactive and cannot jeopardize the academic standards or integrity of the course.

Religious Accommodation at Chapman University: Consistent with our commitment of creating an academic community that is respectful of and welcoming to persons of differing backgrounds, we believe that every reasonable effort should be made to allow members of the university community to fulfill their obligations to the university without jeopardizing the fulfillment of their sincerely held religious obligations. Please review the syllabus early in the semester and consult with your faculty member promptly regarding any possible conflicts with major religious holidays, being as specific as possible regarding when those holidays are scheduled in advance and where those holidays constitute the fulfillment of your sincerely held religious beliefs.

Student Support: Over the course of the semester, you may experience a range of challenges that interfere with your learning, such as problems with friend, family, and or significant other relationships; substance use; concerns about personal adequacy; feeling overwhelmed; or feeling sad or anxious without knowing why. These mental health concerns or stressful events may diminish your academic performance or reduce your ability to participate in daily activities. You can learn more about the resources available through Chapman University's Student Psychological Counseling Services.

Moreover, the Schmid Diversity, Equity, and Inclusion taskforce has put together a compilation of resources (financial, health, social, URM-specific, etc.) available to students at Chapman. Check it out on the DEI website.

Fostering a community of care that supports the success of students is essential to the values of Chapman University. Occasionally, you may come across a student whose personal behavior concerns or worries you, either for the student's well-being or yours. In these instances, you are encouraged to contact the Chapman University Student Concern Intervention Team who can respond to these concerns and offer assistance. While it is preferred that you include your contact information so this team can follow up with you, you can submit a report anonymously. 24-hour emergency help is also available through Public Safety at +1 (714) 997-6763.

Concerns: If at any time during the semester you wish to discuss class procedures, schedule, grades, or any class situation, please contact the instructor during regularly scheduled office hours or via email. Any concern that cannot be resolved directly with the instructor should be referred to Dr. Mihaela Vajiac, program director of mathematics.