Honors Course
Euclid's Geometry:
An Elementary Mathematical Theory from a (Relatively) Superior Point of View

Tuesday, Thursday 2:30-3:45
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Course Description Some basic theorems of Euclid’s Geometry are part of common knowledge. They are taught to any kids, starting from the first grades of elementary schools, then somehow proved in middle and high schools, and pervasively applied, or appealed to while teaching any branch of mathematics: not only pure geometry, but also trigonometry, algebra, calculus, and even arithmetic. An obvious example is the Pythagorean Theorem, presented and proved (or merely justified) in various different forms, but always denoted with this same name.

This makes usual to think that Euclid’s Geometry is a collection of straightforward truths, and is part, as such, of the most basic piece of knowledge shared by virtually all cultured humans. This is false, however, at least if an appropriate mathematical meaning is ascribed to the term ‘Euclid’s Geometry’. What is true is that Euclid’s Geometry is an elementary mathematical theory. The difference is crucial. On the one side, a mathematical theory is quite different from a collection of truths; it is rather a system of theorems proved in agreement to a codified set of rules and/or constraints. On the other side, its being elementary is a no way the same as its (only) including straightforward truths, and this even if theorems are taken to be truths (an assumption which is, indeed, far from obvious); it is rather its providing a ground for a number of other mathematical theories, and its not being required, in turn, to be grounded on any other one.

But there is more, since providing a ground for a number of other mathematical theories and not being required, in turn, to be grounded on any other one is quite different from being straightforward. As any mathematical theory, also the most elementary ones (in this proper sense), and so certainly also Euclid’s geometry, require a number of assumptions, whose legitimacy comes much less from their straightforwardness (or intrinsic evidence), than on their expressive power and deductive fruitfulness. Among these assumptions, some are there to fix the modalities of proofs, better the intellectual moves admitted, by appealing to which a proof can be conducted.

Again, though elementary (in the mentioned sense), any relevant mathematical theory, and certainly, Euclid’s Geometry among them, can be considered from a superior point of view (the expression is due to Federigo Enriques, one of the main mathematicians of 19th and 20th centuries—born in 1871 add dead in 1946—also greatly concerned with mathematical education and teaching). This means that this theory can, on the one side, be inserted with a larger net of theories and articulated with them, by studying their mutual interactions and making their reciprocal dependence vary in a number of ways, and, on the other side, reformulated in different, and often quite sophisticated ways, by appealing to more subtle methods of definition and proof. A quite important, and mathematically very influential way to make it has been suggested, in between the end of 19th and the beginning of 20th centuries, by David Hilbert, one of the most productive and authoritative mathematicians of all times.
Content. The purpose of the class is to make a student aware of the nature of Euclid’s Geometry, as shortly described above, and to provide some examples of the possibility of considering it from a superior point of view, by insisting, in particular, on Hilbert’s work. This will require to enter a number of details, so showing how even the most apparently very well-known among its theorems and their proofs present relevant features that remain unnoticed when these theorems are seen in isolation and considered as straightforward truths.

To this purpose, the teachers will go through the first, and possibly other books of Euclid’s Elements, and some parts of Hilbert’s Foundation of Geometry, by explaining both their quite sophisticated structure and some relevant aspects of them.

Course Learning Outcomes. The aim is increasing not only and not mainly the mathematical culture and knowledge of students, but also and overall they capacity of abstract and critical thinking, and their sense of historicity and evolution of mathematics, and, more generally, of science and knowledge.

Instructional strategies. The instructor will begin any lecture with a series of questions students may have on what we discussed or read during the previous class. This should set the stage for the next topic. If students don’t have questions, the instructor will ask questions, and we should all strive for a setting where we can openly discuss ideas and engage each other in the learning process. If the instructor will assign homework, they will also be discussed the following week. After that, the instructor will offer the lecture, being always open to requirements of clarifications, questions, and discussion.

Prerequisites. Though involving some technical issues, the class does not require any specific prerequisite, though students are expected to be able to follow simple logical arguments. What is absolutely necessary is the willingness to look at apparently obvious questions, and to have an appreciation for the intermingling of logical and historical development of ideas.

Textbook. We do not require any textbook, but both Euclid’s Elements (in Heath’s English translation: Euclid, The Thirteen Books of the Elements, Dover Publication, various editions and reprints) and some parts of Hilbert’s Foundation of Geometry (in Unger’s translation, revised by P. Bernays: D. Hilbert, Foundation of Geometry, Open Court Publishing, various editions and reprints) will be object of study. The teachers will also make use of an important recent book — R. Hartshorne, Geometry: Euclid and Beyond, Springer, 2000 —, and distribute notes to the students, and refer them to online resources.

Grading. The teachers will regularly assign homework (once a week) and there will be both a mid-term and a final exam. The final grade will be computed as follows:

Class participation 20%
Homework 30%
Mid-term 20%
Final exam 30%

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community of scholars which emphasizes the mutual responsibility of all members to seek knowledge honestly and in good faith. Students are responsible for doing their own work, and academic dishonesty of any kind will not be tolerated anywhere in the university.

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