A Neuronal Theory of Human Economic Choice

John Dickhaut

Vernon Smith
Economic Science Institute, Chapman University

Baohua Xin
Rotman School of Management, University of Toronto

Aldo Rustichini
Department of Economics, University of Minnesota

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Abstract

We develop and test a model that provides a unified account of the neural processes underlying behavior in the classical economic choice task. The model portrays brain processes engaged in evaluating information in the experimental stimuli. This portrayal produces a consistent account of several important features of the decision process in different environments (e.g., when the probability is specified or not): these features include the choices made, the time to decide, the error rate in choice, and the patterns of brain activation.

Complex information describing two economic options is represented on the retinas of the subject as a collection of photons. A collection of photons is converted (via a sequence of neuronal processes) to a measure indicating which option has the higher utility magnitude. Data are processed by the brain until evidence sufficient to favor one option is reached.

The model predicts that the further two stimuli are from each other in utility space, the faster the reaction time will be, fewer errors in choice will be made, and less brain activation will be required to make the choice; the model also predicts that choices with ambiguity can be made quicker and will require less brain activation in the horizontal intraparietal sulcus than for choices with risk. Also, we demonstrate how, ceteris paribus, with a larger certainty option in the choice, there is more brain activation, and furthermore, with less experience on the part of the subject making choices, there is more activation.

1. Introduction

We study the classical choice task, simultaneously collecting information on actual choices, reaction time and brain activation to arrive at a unified account of a decision maker's choice process. Theories of economic choice have long ignored the fundamental role that effort can play in the resolution of the choice process, though it has been hypothesized that choice itself requires some form of effort allocation to resolve options in choice. We propose a random walk model of decision process with endogenous barriers as a function of effort that has a cost and a benefit that vary with information quality. The model assumes that the decision maker first assesses the quality of the information available to him in order to determine the evidence required in the form of a criterion or threshold to be reached. The subject continuously receives a pair of signals from each choice stimulus and converts each pair of signals to representations of the pair to a location on an ordered line segment from which a preference comparison is made. When enough evidence is gathered so that the number of favorable comparisons for one of the choices reaches the threshold, the subject makes the choice.

A critical element in this model is the notion of the certainty equivalent of a lottery. The certainty equivalent is the monetary amount that makes the economic agent indifferent between the lottery and that amount. The certainty equivalent can be used to implement a cutoff policy: when the value of the certain amount is larger than the certainty equivalent, the subject will choose the certain amount, otherwise he will choose the lottery. The choice between two lotteries can similarly be considered as a comparison between the two certainty equivalents. Thus we can think that the decision process consists of determining the certainty equivalents and then comparing them. It is this process that we will model and test with our experiments. Questions related to ours have played a prominent role in economic experiments [1-5]. Recently efforts have emerged in applying neuroscience methods to studying subjects making choices between such gambles [6-10], but the determination and understanding of the process involving certainty equivalent cutoffs has not been part of those inquiries. In particular we will uncover cutoffs associated with risky and ambiguous lotteries as revealed by the choices of our subjects. In previous studies of similar tasks two regularities have emerged.

The first regularity is: the closer the options are in value, the more frequent the error in choice (i.e. near indifference the frequency of error is higher.) The first description of such a finding in choice experiments was the seminal work of Sidney Siegel [11]. A related effect in general decisions is the classical "symbolic distance" effect [12]. Rustichini et al. [10] recently elaborated this phenomenon for subjects in an fMRI experiment. In this case the further from indifference the faster the reaction time. These two findings together present a conundrum. It is often argued that the reason for more errors close to indifference is because the cost of error is less. But this explanation is difficult in light of the longer reaction time necessary to decide when the options are closer to indifference.

A second important regularity has recently surfaced in imaging studies. Rustichini et al. [10] and Huettel et al. [8] find that choices between ambiguous and certain amount require less processing time than choices between risky and certain amounts. Our model leads to the prediction of such differences in reaction time and also predicts that relative brain activation will be more for risky than ambiguous gambles when certainty equivalent cutoffs guide the result.

In summary our model implies the closer to indifference an option is the more the response time, the more errors and the more brain activation. Furthermore we predict more response time and brain activation for risky than ambiguous choices. Our experimental results are consistent with this model.

2. Economic Theory Background

The economic theory of choice considers three different types of options: deterministic, risky, and ambiguous. A *deterministic* option is an outcome that will occur for sure, like a payment of $10. *Risky* options represent known probabilities of particular deterministic outcomes, for example, a 50/50 chance at $10 and $50. An *ambiguous* option is an option for which the probabilities of the outcomes are not known by the subject. Simply being told that the deterministic outcomes of a risky gamble can be $10 and $50, with no statement of the chances of those outcomes, leaves open the possibility that the risks may be anything ranging from a 0% chance at $10 to a 100% chance at $10.

Gilboa and Schmeidler's [13] theory describes how such choices are made absent any costs of making a decision. Their assumptions on individual behavior imply that there is a function *U* defined on deterministic outcomes that permits the representation of all possible options by numerical magnitudes such that if the numerical magnitude associated with any option A is larger than the numerical magnitude associated with another option B, then the subject prefers option A to

B. The process of assigning magnitudes to these options is referred to as the expected utility operator, EU. The economic theory is an as-if theory: it does not say that human beings are aware of forming these numerical magnitudes; rather, they act as if they formed such magnitudes.
Smith and Walker [14] model choice as a productive activity, with returns and costs of making the choice taking the form of an additively separable utility function. A similar approach is outlined in several different approaches to behavioral economics [15–16]. The neuronal theory presented here assumes that there is a cost of neuronal effort associated with making the decisions described by Gilboa and Schmeidler [13]. Subject choice is the choice of the level of effort to expend in evaluating the information generated by the experimental stimulus.

3. Review of the Literature

3.1. Early examinations of economic choice models

The as-if nature of theory economic choice does not imply that predictions on choices cannot be made. Information about the relative preferences of a few options can be used to develop predictions about choices between other options. For example, suppose that option A is preferred to option B; then, an option in which there is a p chance ($0 < p < 1$) at A and a ($1 - p$) chance at B will be preferred to option B, but not to option A. In the field of experimental economics, it is often assumed that subjects come to the experiment with an ordering on dollars (more are preferred to less), and then additional attempts to understand a subject's preferences proceed from there [17]. Also, choices between options can be used to determine distance information between preferred options. Suppose that a 50/50 chance at A and B is preferred to a 50/50 chance at C and D. Then

$$0.5\ U(A) + 0.5\ U(C) > 0.5\ U(B) + 0.5\ U(D)$$

implies that

$$U(A) > U(D),$$

which means it is possible to order some of the numerical distances between the utilities of stimuli and form elements of a numbered line segment.

Such determinations of the implicit numerical properties of these stimuli were part of the constructions of Davidson et al. [18] and were used to derive utility representations of various objects such as books, records, and grades of students as well as cigarettes for prisoners [19]. In these studies, experimenters would have subjects make a collection of choices, and then experimenters would pay off each subject on only one choice, selected randomly from the set of all choices to not let augmentations of wealth affect the choices made. One point of the current paper is to see if there are substrates of the brain that organize an ordered line segment that could be interpreted as representing utility in economic choice studies [20].
Some experiments, such as Allais [1] and Ellsberg [2], document failures of expected utility theory by revealing subject choices that do not allow consistent numerical representations under the theory. Thus economic choice behavior can be thought of as a calculational theory, in which explicit numerical magnitudes form the basis of choice, in which the economic actor behaves as if producing these calculations.

3.2. Findings from early studies of choice

Inconsistency near cutoffs. Early findings in studies of choice involve paired comparisons between deterministic outcomes. Thurstone [21] outlined a methodology in which a person was asked to choose between two-tuples at a time, for example, (2 hats, 4 pairs of shoes) versus (4 hats, x pairs of shoes), where x was varied. Using this approach creates an estimated cutoff point z such that for an x below z, (2 hats, 4 pairs of shoes) will likely be preferred to (4 hats, x pairs of shoes), and for x above z, (4 hats, x pairs of shoes) will likely be preferred to (2 hats, 4 pairs of shoes). Thurstone's data reveal that subjects tend to reveal more noise in choices relative to a cutoff when x is close to the cutoff point (see also Mosteller and Nogee [22] and Edwards [23]).

Increased reaction time near cutoffs. Mosteller and Nogee [22] measured reaction time in choosing between risky gambles during choices and were able to determine that the closer to indifference a subject was, the higher the reaction time tended to be. The net result in these early studies is that when distance comes into play between utilities, two things are affected: consistency and reaction time. Recently, efforts have emerged to apply neuroscientific methods to studying subjects making choices between gambles, but the determination and understanding of the process regarding cutoffs in choices has not been elaborated neuronally. We will uncover cutoffs associated with risky and ambiguous lotteries, as revealed by the choices of our subjects.

Neuronal studies. Neuronal studies have focused on the impact of administered reward and isolated areas of the nucleus accumbens and mesial prefrontal cortex associated with the processing of gains and the probability of gains, respectively [24]. Kable and Glimcher [25] found evidence of sensitivity to estimated subjective value (as opposed to objective rewards) in the striatum, mesial prefrontal cortex, and posterior cingulate cortex in a discounting task.
Using positron-emission tomography (PET), several studies of choice suggested that the type of gamble (ambiguity, risk, and certainty) leads to differential activation in the parietal, frontal, and orbital frontal areas [10]. For responses to gains and losses, Sabrina et al. [26] showed extensive dopaminorgic engagement as well as prefrontal involvement for individual subject analysis.
Two studies focused on the ambiguity, risk, and certainty distinctions we employ in this paper. Hsu et al. [9] inferred differences in activation between risk and ambiguity in the amygdala and lateral orbital frontal cortex, whereas Huettel et al. [27], using stimuli similar in construction to those used in the current study, found that risky stimuli activate the parietal area, while ambiguous stimuli differentially activate the lateral prefrontal cortex.

Studies of numerosity underlying our model of human economic choice. Our model of human economic choice is informed by evidence on the processing of numerical magnitudes (see Dehaene [20]). Consider, for instance, a simple number comparison task: a human subject is shown a number on a screen and is asked to press a left button to indicate that the number is less than 55 and a right button to indicate that the number is greater than 55. An interpretation of the process is as follows. The human brain forms a spatial representation of numerical magnitudes. When a comparison stimulus is presented on a screen, photons are stochastically sent to the retinas, and various characteristics are transferred stochastically to secondary areas of occipital cortex and, finally, to areas distinctly responsible for making comparisons of the spatial representations: the bilateral horizontal segment of the intraparietal sulcus, the left angular gyrus, and the bilateral posterior superior parietal lobe [28]. The projections in terms of neuronal activation of the comparison stimulus are registered on "greater than 55" counters and "less than 55 counters." Which counter hits a barrier first will determine whether the subject indicates that the number is greater than or less than 55. Several of the main modeling results in economic choice in this study are paralleled by results with numerical magnitudes. In studies of numerosity, there are distance and inconsistency effects, just like in the early studies on choice. The distance effect with numbers has been replicated many times with humans and nonhuman primates. The underlying explanation of such phenomena is that number symbols, such as Arabic or Roman numerals, or collections of dots (in the case of monkeys) are transformed to associative cortex, and semantic representation is made downstream [29]. Such representations are noisy. For example, chimpanzees trained to match to sample will most frequently match 3 dots with 3 dots previously shown; 2 and 4 dots are less frequently matched to 3 and 1; and 5 dots are rarely matched, if ever. The explanation for the human judgments is that humans also work with a noisy representation, even of certain amounts, and that they map directly from the noisy representation to motor actions so that such actions are noisy in response to experimental stimuli [30].

Behaviors that require combining numerical operations. A verification task such as answering true or false to the statement 3 + 4 = 12 is generally construed to require more than one operation [31]. In a verification task, addition and comparison are two such operations. With no constraints, reaction time is larger in such tasks and often different from the sum in both tasks, indicating that the subject adjusts reaction time as a consequence of the context of the numerical problem. Thus task differences in using numbers can produce different reaction times, much like tasks using ambiguous and risky stimuli produce different reaction times.

Neuronal substrates implicated in numerical operations. Generally, several forms of study, lesion research, single-cell recording, PET, and functional magnetic resonance imaging (fMRI) tend to implicate the inferior parietal cortex and, secondarily, the frontal lobe in numerical operations [32]. The most frequently implicated locations for neuronal involvement in numerical operations are the left and right inferior parietal sulci. In studies of patients with lesions in the general parietal area, approximation in division and subtraction tasks is severely hampered. Single-cell recordings implicate specific neurons associated with numerosity. For example, detection neurons sensitized to the number 6 have been reported in association cortex in an anaesthetized cat.

Monkeys' superior parietal lobules are tuned to counting the number of muscle movements of monkeys as well as a distance effect [33]. In a match-to-sample test using monkeys, inferior parietal and frontal lobe activation showed a larger proportion of neurons sensitive to numbers, and sensitivity in the inferior parietal area preceded sensitivity in the frontal area [34]. These findings generally provide support for connectivity in nonhuman settings with respect to number assessment. The earliest imaging study of numerical processing supports an important role for the intraparietal sulcus as well as the prefrontal cortex in a subtraction task (xenon was injected into a carotid artery to detect behavior of single photons) [35]. The original result was replicated using PET and, later, was used to specifically isolate responses in a subtraction task to isolate parietal and prefrontal activation.

Random walk model. The results (for nonhuman primates and numerosity) suggest a general model of the decision process based on a random walk. We propose that such a process governs choices among economic options, as those made by subjects in our experiment, who had to choose between a lottery and a certain amount. A lottery specifies a payment made to the subject depending on the outcome of a random device. This random device could be risky or ambiguous. When a subject considers, say, a risky lottery, he or she can assign to it a corresponding certain utility amount that is located on the ordered real line. When he or she has to choose between this lottery and a certain amount, the subject can simply compare the utility of the risky lottery and the utility of the certain amount. The choice between the two lotteries, certain and uncertain, is now determined by the comparison of these two values.
This process resembles a random walk model. The essential elements of the model are a space of evidence, a stochastic process on this space (reflecting accumulated evidence), and a barrier. The process stops when the barrier (high or low) is reached, and a decision corresponding to the barrier is made. The decision process has been reduced to a random walk on a space and a level of evidence described by a barrier.

Endogenous Barriers. We now discuss how this barrier adjusts to the nature of the decision problem. The barrier in the random walk model of choice can be interpreted as expressing the level of satisfactory evidence which is considered necessary to take a decision. If this level is independent of the quality of the information, then prediction on response time is clear: The conditions in which the information is inferior would require a longer time to reach the same level of satisfactory evidence; however the information-processing device (the brain) should consider also that inferior information gives a smaller incentive to wait while observing. For example, if the signal observed provides no information at all, then clearly the optimal policy is to decide immediately, since observing the signal is not going to improve the quality of the decision. In this case the barrier is reached very close to the starting point of the process.

In our experiment the quality of the information is different in the case of risky and ambiguous decisions. When the quality is inferior (as in the case of ambiguous choices) the barrier will be reached closer to the starting point than it is when the information is superior (as in the risky choice). We conclude that a less informative signal has the effect of reducing the level of evidence considered necessary before the information processing stops and decision is reached.

In summary, given a barrier, our model of the decision process has two additional components: first, the process reduces the complex information describing two economic options to a numerical value, the utility of each option. The second performs the comparison between these two quantities and determines, possibly with an error, the larger of the two. The comparison in this second step is well described by a random walk of decision with endogenous barriers as a function of the chosen effort. The model, extends Dichhaut et al. [36] by incorporating endogenous barriers as a function of the chosen effort (determined by the nature of decision problem), is formally presented in the supplementary material, and simplified presentation and linkage with economic variables is presented in the next section.

4. Economic Model and Predictions

4.1. The application of the choice model to the experiment

In the experiment, the subject makes 42 choices (see the task description in section 5). We assume that prior to any choice, subjects' behavior can be specified in terms of an economic model that incorporates a selection of effort to generate internal (neuronal) information about which choice to make (the left or right alternative on a computer screen). By *economic*, we mean a model that formally represents prior information, conditional information on outcomes, preferences on outcomes, and separable preferences on effort. Furthermore, information has a definite impact on beliefs through a mechanism that organizes processing. In an economic model, this is Bayes' theorem. We assume the subject is solving

$$\max_{e\in E}\left(\int_Y c\max_{a\in A}\int_\Theta \mu(\theta)P_\theta(y;e)V(\theta,a)-C(e)\right). \qquad (1)$$

Here μ will be the prior probability of a particular utility pair from the set of possible gambles. $P_\theta(y; e)$ is the probability of an internal message, *y*, given that the pair of choices comprises two particular utilities $\theta=(\theta_1,\theta_2)$ and given the effort, *e*. Variable *c* is the magnitude of the certainty amount in the choice pair. Variable *v* is the utility of the action when the utility pair is θ_1 and θ_2 and action *a* is taken. *C* is a cost of effort function. The problem $\max_{a\in A}\int_\Theta \mu(\theta)P_\theta(y;e)V(\theta,a)$ is the optimization in light of having exerted the processing effort *e*. Thus the decision maker considers choosing each action, *l* and *r*, the left-side and right-side option, respectively, and picks the action that has the highest net expected utility after the effort is exerted. The complete model is provided in the supplementary material.

The source of *y*. The signal, *y*, results from the brain stochastically processing an interim collection of photons generated by the experimental stimulus. These signals form an estimated comparison metric, *A*, of the relative utility of the left and right choices; such a metric might be ($\theta_1 - \theta_2$) or log (θ_1/θ_2). An example of the distribution of a *y* that results from the brain stochastically processing an interim collection of photons generated by the experimental stimuli is shown in Figure 1A. This graph shows the conditional distribution of the comparison metric *A* when the real difference ($\theta_1 - \theta_2$) is located on the positive horizontal axis. Note that *A* yields a +1 if *A* is bigger than 0 and a –1 otherwise. Given that *A* is an estimate of the difference ($\theta_1 - \theta_2$), then a *y* of 1 is said to be a match. Two features of *A* are that it is noisy and that it is only indirectly available for downstream purposes by the brain through the creation of the signal *y*. Both these features are reflections of the basic underlying stochasticity of brain function and the fact that the brain does not store all its computations.

The impact of effort, *e*, on the signal *y*. *A* also reflects some level of effort. Note that if there were no effort, the distribution would not be distinguishable from an improper uniform distribution on {–∞, ∞}, meaning that a *y* of 1 would not distinguish the left- and right-hand options to the subject. The effect of increasing effort is captured by Figure 1B.

The impact of effort is to make *y* more discriminating in the sense that when the effort is high, a *y* (integrate *A* over the positive domain) of 1 is more likely to occur for a positive ($\theta_1 - \theta_2$). Asymmetric construction holds for a negative ($\theta_1 - \theta_2$). It will be the case that as effort increases, the quality of the signal *y* increases in the sense that *y* will have a higher probability of being 1 given that ($\theta_1 - \theta_2$) is any positive magnitude.

The role of the magnitude of the certainty option in the choice. Expression (1) represents updated beliefs by the subject after knowing *y* and having expended effort *e*. The added term, *c*, stands for the magnitude of the certainty option in the decision (out of 42) the subject is currently making. We assume that *c* affects the prior directly in the following way: $\mu(c,\theta) = c * \mu(\theta)$. Higher *c* is related to the initial attention the subject pays to the choice (as opposed to effort to process the choice). There is substantial literature on magnitude of the payoffs and the relationship of the payoffs to performance. Research began with the work of Davidson et al. [18], carries through to the discussion of salience by Smith [37], and finds numerous other examinations by such authors as Harrison [38,39], Binswanger [40], and Khaneman and Tversky [4], although results in this literature are not always in agreement. In the current model, we capture the proposition that a higher magnitude attracts more attention so that a subject, when he or she sees a $15 option, will attend less than when he or she sees a $30 option. Traditionally, attention has been a psychological variable, but we argue that attention has a definite effect on the choice model through its effect on priors. Specifically, the assumption we make is that a higher certainty option means that the subject entertains a higher probability for all the alternative pairs, *O*, under consideration. The magnitude of *c* will, via attention, affect the effort level chosen. For a formal statement of how this process works, see the supplementary material.

The role of experience and quality of information. There are 42 choices in the experiment. We assume that the signal derived from the choices will improve, and hence the expected utility will improve at a lesser rate for the same level of effort as the experience of the subject increases. This is shown in the marginal benefit curve in Figure 1C. On the other hand, the subject can, in a particular choice, face higher- or lower-quality information. The lower-quality information will demand less effort to process (there is less of it) and result in a faster rate of extraction, and hence the optimal level of effort will decrease.

4.2. Predictions

This theoretical construction allows us to discipline our examination of the data. We formally derive the following predictions in the supplementary material:

1. An increase in experience shifts the derivative of the benefit function down and leads to a reduction of effort.
2. An increase in the quality of information similarly shifts the derivative of the cost function down, leading to an increase in effort.
3. An increase in the value of the certain amount has the effect of an increase in effort.
4. An increase in the distance in utility between the two options decreases the time to respond.
5. An increase in the distance in utility between the two options decreases the error rate.

It should be noted that effort in brain scanning data will be assessed through changes in the level of deoxygenated blood flow and through behavioral data by reaction time.

5. Methods

5.1. The choice task

Each experimental session consisted of three successive blocks. Each block consisted, in turn, of 14 periods. In each period, the subject had a choice between two options. One of the options was an uncertain lottery, while the other was a certain amount. An uncertain lottery consisted of two monetary prizes and a probability for each one of the two prizes. Seven of the lotteries were risky, and seven were ambiguous. In a risky lottery, a pie drawing represented the probability of the two outcomes. In the ambiguous choice, no information was given about the probability. The monetary prizes for the uncertain lotteries were, in every case, $50 and $10; the values of the certain lotteries were $15, $17, $20, $25, $25, $27, and $30.

In the first and third blocks, the subject had no feedback after his or her choice

on the outcome of the random draw; in the second block, the subject was informed of the outcome after making a choice. In the first and third blocks, the period began with the display of the choice for 5 s. During this time, the subject could only observe the available options. At the end of the 5-s interval, a cue on the screen signaled to the subject that he or she could choose one of the two options (GO signal). The selection of the choice was made by pressing one of two buttons on a box. The subject had no time restriction. After the subject's choice, a period of 13 s of rest followed, in which the subject had no task to perform (see Figure 2 for the time line of blocks 1 and 3).
The design of the second block was identical up to the point at which the subject made his or her choice. After that, the pie chart displayed the probability of the two choices in the ambiguous case. Then, in both the risky and the ambiguous case, a spinner rotated and then stopped in one of the two pie sectors, indicating the outcome for the subject. The spinner had the shape of an arrow, rotating around the center of the pie, displaying the probability of the two outcomes. This entire step lasted 23 s. After the spinner step, the subject waited for a period of 23 s, after which the next period began. The sequence of events that we have described after the choice was the same, independently of the choice made by the subject, so the subject would know the outcome of the uncertain choice even if he or she had chosen the certain amount (see Figure 2 for the time line of block 2).

5.2. Subjects

Fourteen subjects were scanned while performing the task. During the entire experiment, they were placed in supine position in the bore of the scanner. The subject communicated his or her choices through a MR-compatible button box. The choices were presented on a screen located behind the subject, who could see them through a mirror attached to the head coil.
The study had the approval of the Institutional Review Board at the University of Minnesota. The subjects were students recruited at the university (average age 22.5 years, standard deviation 2.95). Seven of the subjects were female. The average payment to the subjects, including the participation payment and the money gained in the task, was $83.

5.3. Statistical analysis of behavioral data

Subjects in the experiment had to choose in every period between a lottery and a certain amount. The lottery could be risky or ambiguous, depending on whether the probability over outcomes was clearly specified. We recorded participants' response times measured in seconds from the moment the GO signal appeared on the screen until a choice was made. For each subject, we computed a cutoff value as the empirical estimate of the indifference point. This was defined to be a value that would give the observed number of choices of the certain amount made by the subject. The empirical cutoff may be different in different blocks, and for ambiguous and risky choices, so it was computed separately, conditional on the block and the quality of information. On the basis of this empirical cutoff, we assessed the variable error. More precisely, the error was defined to be 0 for a given trial if the value of the certain amount was less than the empirical cutoff (so that the subject should choose the lottery) and the subject chose the lottery; it was defined to be 1 if the subject chose the certain amount. The error was defined similarly for the case in which the certain amount was larger than the cutoff.

5.4. Imaging parameters

We used the 3 Tesla Whole Body MR System (Magnetom Trio, Siemens Medical Center, Erlangen, Germany) at the Center for Magnetic Resonance Research at the University of Minnesota.

Before the functional run, 144 or 160 (depending on the subject's head size) FLASH images were acquired in slices of 1-mm thickness in sagittal plane, with a field of view (FOV) of 256 × 256 mm and a matrix size of 256 × 256 lines, giving a spatial resolution of 1 mm^3 for the anatomical volume. The TR was 20 ms, the TE was 4.7 ms, and the flip angle was 22°.

Afterward, a whole-brain fMRI was performed while the subject executed the task. For the functional acquisition, we used an echo-planar imaging sequence to measure the blood oxygen level–dependent (BOLD) signal. A total of 30–38 functional slices per volume were acquired for each subject: the number of slices needed to cover the whole brain, from vertex to the most inferior part of the cerebellum, varied by subject. The slices had a thickness of 3 mm and were acquired in transversal plane with a FOV of 192 × 192 mm. No gap separated the slices. The TR was 2,000 ms, the TE was 23 ms, and the flip angle was 90°. The matrix size was 64 × 64, giving a voxel resolution of 3 mm^3. The total number of acquired volumes was variable, depending on the length of time necessary to complete the task for each subject.

5.5. Preprocessing of fMRI data

Brain Voyager QX, version 1.7 (Brain Innovation, Maastricht, Netherlands) software was used for fMRI data preprocessing and analysis. First, we employed the preprocessing algorithms embedded in the software to remove the linear trends and to correct for the differences in slice scan time acquisition. Then, the functional images of every subject were preprocessed to correct for motion artifacts to ensure that subjects did not move more than 3 mm in any direction. These motion-corrected functional images were then used to construct a three-dimensional (3-D) functional volume for every subject and every run. At this stage, spatial smoothing was performed using a Gaussian fixed width at half maximum kernel of 7 mm. For each subject, the 3-D functional volume was aligned with the corresponding 3-D anatomical volume. Both were then normalized to standard Talairach space [41].

5.6. Statistical analysis of imaging data

We used a rapid event-related design to analyze the data in our experiment. In each block, we defined various temporal events, based on the task and subjects' reaction to the stimuli. For instance, in each block, we had predictors describing the presentation of the choices, the decision interval, and the feedback interval (where available). When needed, these predictors were further categorized depending on the block in which they occurred, the type of trial (risky vs. ambiguous), and the subject's choice (certain vs. uncertain). These predictors were entered as fixed factors in a single-subject general linear model (GLM), and then the parameters of this GLM model were subsequently entered into a second level of analysis corresponding to a random-effect GLM model, which was used for group analysis [42]. The statistical parameters of this latter model were estimated voxelwise for the entire brain, and activation maps were computed for various contrasts between the predictors. The criteria used to display the activation maps for the main GLM model were as follows: a cluster size of 100 adjacent significant voxels and a statistical threshold for each voxel in the cluster of $p < 0.00005$, corresponding to a minimum t-value of 5.93. A second series of GLM models was constructed using the categorical predictors from the previous GLM models and by adding parametrical covariates based on subjects' performance (e.g., distance from cutoff). The criteria used to display the activation maps for these secondary GLM models were as follows: a cluster size of 100 adjacent significant voxels and a statistical threshold for each voxel in the cluster of $p < 0.005$, corresponding to a minimum t-value of 3.38. In the second stage of the analysis, the clusters obtained as a result of various contrasts in the first stage were defined as regions of interest (ROIs), and further GLM analyses were performed separately for each ROI to compute the percentage of signal change or to correlate imaging data with the behavioral measures. These latter analyses were performed taking into account the time course of the signal averaged over all voxels in the ROI for each subject, and they were performed using SPSS (SPSS Inc., Chicago, IL) and Stata (StataCorp LP, College Station, TX).

6. Results

6.1. The choices

Subjects were less willing to choose the ambiguous lottery than the risky one. The logit regression of the choice of the certain amount supports this conclusion: the coefficient of the variable ambiguous (equal to 1 if the lottery was ambiguous) is positive and significant. Similarly, the value of the coefficient of the certain amount itself is positive and significant.

6.2. The error rate in choices

A first prediction of our model is that the probability of an error will increase as the difference in value between the two options decreases; that is, the difference between the estimated cutoff and the certain amount decreases. Figure 3 plots the distance from the cutoff against the error rate by simulating a distribution of error rates based on the data. The number of errors approaches zero as the distance from the cutoff reaches 14. Details of the calculation of the confidence bands and path estimation are provided in the supplementary material.

6.3. The response time in choices

Our model leads us to predict two additional important regularities concerning time to response. The first is that the response time decreases as the distance between the value of the certain amount and the cutoff increases. We measure the difference between the certain amount and the cutoff value of that subject and graph such differences against the response time for that choice. There is a strong tendency for the response time to decline with the distance from the cutoff point of the certainty amount, for values both larger and smaller than the cutoff (see the supplementary material).

The second prediction is that risky choices require longer response times than ambiguous choices. We tested this hypothesis with a panel data analysis of the response time for each choice against three independent variables: the experience of the subject (measured by the total number of choices made up to that point, cumulated choices), ambiguity (equal to 1 if the choice was an ambiguous choice, and 0 if the choice was risky), and distance (equal to the absolute value of the difference between the certain amount in the choice and the estimated cutoff for that subject). The estimated coefficients are all significantly different from zero. An ambiguous choice reduced the time by 9.8% ($p = 0.034$). One additional dollar of distance from the cutoff reduced the time by 2.7% ($p < 0.0001$), and one additional choice made reduced the time by 1% ($p < 0.0001$).

6.4. Brain imaging

We examine the overall decision process by contrasting the brain activity leading to the decision with brain activity after the choice has been made, in all trials, in all blocks. Figure 4 (top) shows the areas that were significantly more active during the display period leading to the decision than during the after-choice period (orange-yellow clusters). Two main areas of interest appear: the intraparietal sulci (IPS), both in the left and right hemispheres, and the medial frontal gyrus (MFG), extending to the supplementary motor area and the anterior cingulate cortex. Figure 4 also shows areas more active in the after-choice period than during the display period (blue clusters). The strong activation in this contrast was observed in the orbitofrontal cortex (OFC), which is part of the system responsible for evaluation. We believe that in this task, there are evaluative processes that occur after the choice has been committed. Padoa-Schioppa and Assad [43] have shown activation in after-choice in some neurons in area 13 of the OFC of the monkey that is larger than the activation occurring at the choice stage. The event-related average of the activation around the onset of the event confirms the results of the analysis we have presented.

The MFG and IPS clusters identified previously covary in a similar manner, increasing at display and decline in after-choice, while the OFC cortex varies in the opposite way (see Figure 4, bottom).

6.5. Tests of the activations predicted by the theory

Our model predicts a differential effect of the nature of the choice (ambiguous versus risky) on brain activations: the relative lack of information in the ambiguous choices will result in a lower barrier to cross at the moment of the comparison so that a given certainty option will require less activation before the barrier is crossed in the ambiguous case than the risky case. To test this hypothesis we determined the relatively most active regions at display when the subject is facing ambiguous choices and compared them with the relatively most active regions when he is facing risky choices. The results are reported in the top panel of Figure 5. The pattern of activation is similar in the two conditions, but for both clusters of activation (IPS and MFG), the risky choices generate more activation than the ambiguous choices.

In the bar graphs in Figure 5 (bottom), we assess the impact of experience, distance of the choices, quality of information, and the level of the certainty amount on the percentage change of the BOLD signal during a trial. The graphs display the average value of the coefficient of the four variables (distance of the certain amount from the cutoff, certain amount, number of choices made by the subject, and ambiguous choice), with one standard error superimposed. We assess coefficients for the four clusters identified in Figure 4 (top): right and left IPS, MFG, and OFC. The coefficients were obtained by first estimating the percentage BOLD change (PBC) for each of the 42 trials (the experiment had three blocks of 14 trials each), obtained by taking the event display in each trial as a distinct predictor. We then ran a regression on PBC with respect to the following variables: distance of the certain amount from the cutoff, certain amount, ambiguity, and number of choices made by the subject. The four variables are normalized to have range between 0 and 1 so that the coefficients are directly comparable. The signs of the coefficients are as predicted by the model. Ambiguity has a negative effect. In addition, experience reduces the activation, as indicated by the negative value of the coefficient for the number of choices made.

The effect of the distance from the cutoff is negative: when the value is higher than the cutoff, the activation is reduced. This occurs in all four regions, including the OFC, in a significant way. The effect of the certain amount is positive. Both coefficients are strongly significant for the two IPS regions (p = 0.0001 for the left IPS).

7. Conclusions

Theories of economic choice have long ignored the fundamental role that effort can play in the resolution of the choice process. Smith and Walker [14] explicitly model effort in the choice process, and their perspective is also reflected in the work of Camerer and Hogarth [15] and Johnson and Payne [16]; it has been hypothesized that choice itself requires some form of effort allocation to resolve options in choice. The current study hypothesizes that effort can be represented as neuronal activation that has a cost and a benefit that vary with experience, information quality, and similarity of the stimuli. These effects are confirmed in measurements of blood flow, reaction time, and noise in choices in a way that is consistent with economic theory.

The theory uses as underpinnings work on numerical stimuli and earlier neuroeconomic work [44]. We have assessed if the numerical properties associated with the intraparietal sulcus provide a locus for representing objects of choice as presented in the classical choice task. We find evidence for reduced reaction time and inconsistency in choice and activation the further the inferred distance on the ordered line segment choices are from each other. Furthermore, we postulate that it is possible, and demonstrate the possibility, to reduce reaction time and activation for ambiguous choices.

A number of neuroscientific studies are consistent with the calculational properties of the horizontal intraparietal sulcus playing an integral role in choice. Shadlen and Newsome [45] found that areas in LIP encode differential counts of representations of movement in MT. Yang and Shadlen [46] showed monkeys different shapes that implied a likelihood that a gaze right would result in a reward. They found evidence of the ability to add and subtract log likelihood information in the intraparietal sulcus.

We postulate that the numerical representations of choices are coded noisily in the intraparietal sulcus; we do not find that all processing occurs there. A parallel study with single-cell recording in monkeys suggests that numerical tasks can be distributed to other brain areas for analysis. For example, it has been found that when monkeys are matched to sample in a number-of-dots task, activation first occurs in the parietal lobe, but at a later time, the dorsolateral prefrontal cells are activated. Thus it is reasonable that this type of behavior is going on in our task during the display stage and that signals are being distributed from parietal to dorsolateral prefrontal. This difference could be producing the differential activation for ambiguous stimuli isolated by Huettel et al. [27], although their level of time stamping is not achievable in our task. Another aspect of the Huettel et al. task that differs from the current task is that an explicit question mark symbol is placed on the ambiguous stimulus; in the current task, there is no such process of indicating ambiguity (see Figure 2). A more thorough understanding of time sequencing of tasks may lead to an equivalent representation of the utility of choices in distinct areas such as those uncovered by Kable and Glimcher [25].

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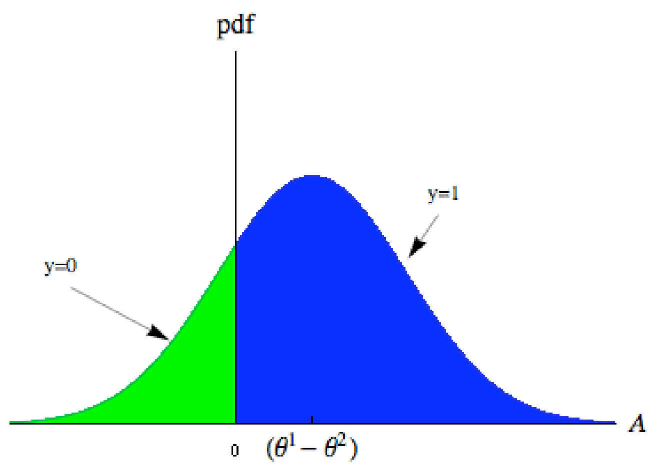
Figure 1: Information processing and choice determination

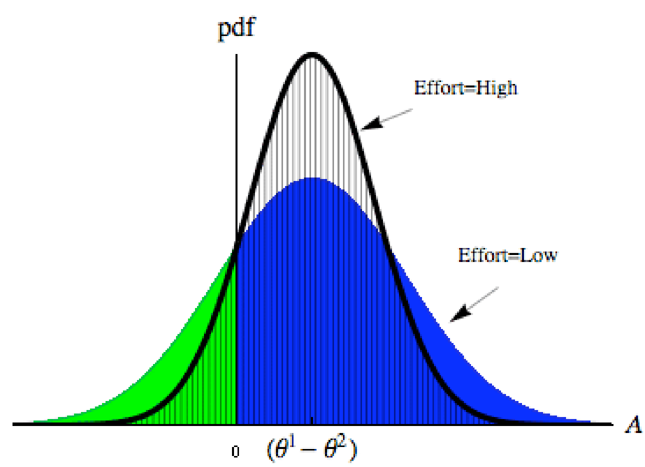
Figure 1.A is a description of an assumed theoretical distribution of the comparison metric, A, theorized to be internally generated by the subject (its exact form is not known). The metric when is converted to a signal, y, which either matches the statement that ($\theta_1 - \theta_2$) is greater than 0, resulting in a signal 1 or the statement ($\theta_1 - \theta_2$) is not greater than 0 resulting in the signal -1. Figure 1.B indicates what happens to the distribution of the signal with increased effort. It shrinks the distribution leading to the conclusion that it is more likely a match will be detected when in fact there is a match. Figure 1.C the marginal costs of effort moving from southeast to northwest and the marginal benefits of effort moving from northeast to southwest. When a subject has greater experience the marginal benefit from effort shifts to the left because there are higher benefits to making efforts elsewhere. With lower quality information the cost of processing increases and reduces the efforts expended in the task.

A.



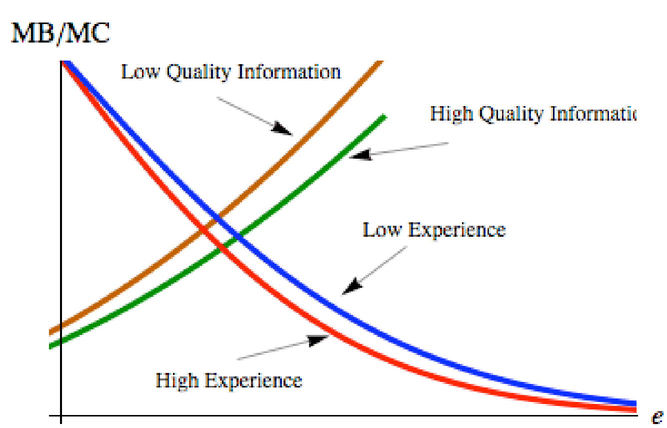
pdf
y=1
y=0
A
0
$(\theta^1 - \theta^2)$

B.



pdf
Effort=High
Effort=Low
A
0
$(\theta^1 - \theta^2)$

C.



MB/MC
Low Quality Information
High Quality Informatic
Low Experience
High Experience
e

Figure 2: Experimental design.

The tasks in the behavioral and imaging experiment were identical. The experiment consisted of three blocks. Each block consisted in turn of fourteen periods. In each period, the subject had a choice between two options. One of the options was a lottery while the other was a certain amount. A lottery consists of two monetary prizes. Seven of the lotteries were risky, and seven ambiguous.

Panel A. The certain amount was one out of the set of seven elements 15, 17, 20, 23, 25, 27 and 30. The risky lottery represented a 50 per cent probability of 10 dollars (red area of the spinner) and a 50 percent probability of 50 dollars (green area of the spinner). The ambiguous lottery presented the payoffs for red and green, but the subject was not told the exact proportion of the two areas.

Panel B. A risky (ambiguous) choice is the choice between the risky (ambiguous) lottery and a certain amount. The risky lottery and the seven certain amounts lead to seven risky choices; similarly there are seven ambiguous choices. The right or left position of the lottery was randomly determined in each trial.

Panel C. The 14 stimuli were presented in three separate blocks.
Blocks 1 and 3 were identical. Each trial in these blocks had a display period that lasted 5 seconds, a choice period in which the subject could indicate a preferred option, with no time constraint, and an after-choice period, which lasted 13 seconds. In these blocks, the subject received no intermediate feedback.
In Block 2 the order of stimulus presentation was similar to the one in Blocks 1 and 3: in this block however the subject received information on the proportion of green and red in the ambiguous lotteries and then on the outcome of the uncertain lotteries. For ambiguous choices the proportions of green and red was revealed by the display of a pie chart identical to the one used to present risky lotteries. So in Block 2, after the subject had made the choice, the proportion of green and red areas was presented when the uncertain lottery was ambiguous, and then a spinner was spun superimposed on the pie chart representing the uncertain lottery: the rest position of the spinner indicated the outcome for that lottery. The overall spin time was 10 seconds. Following the spin there was a 13 second after-choice period as in Block 1 and 3.

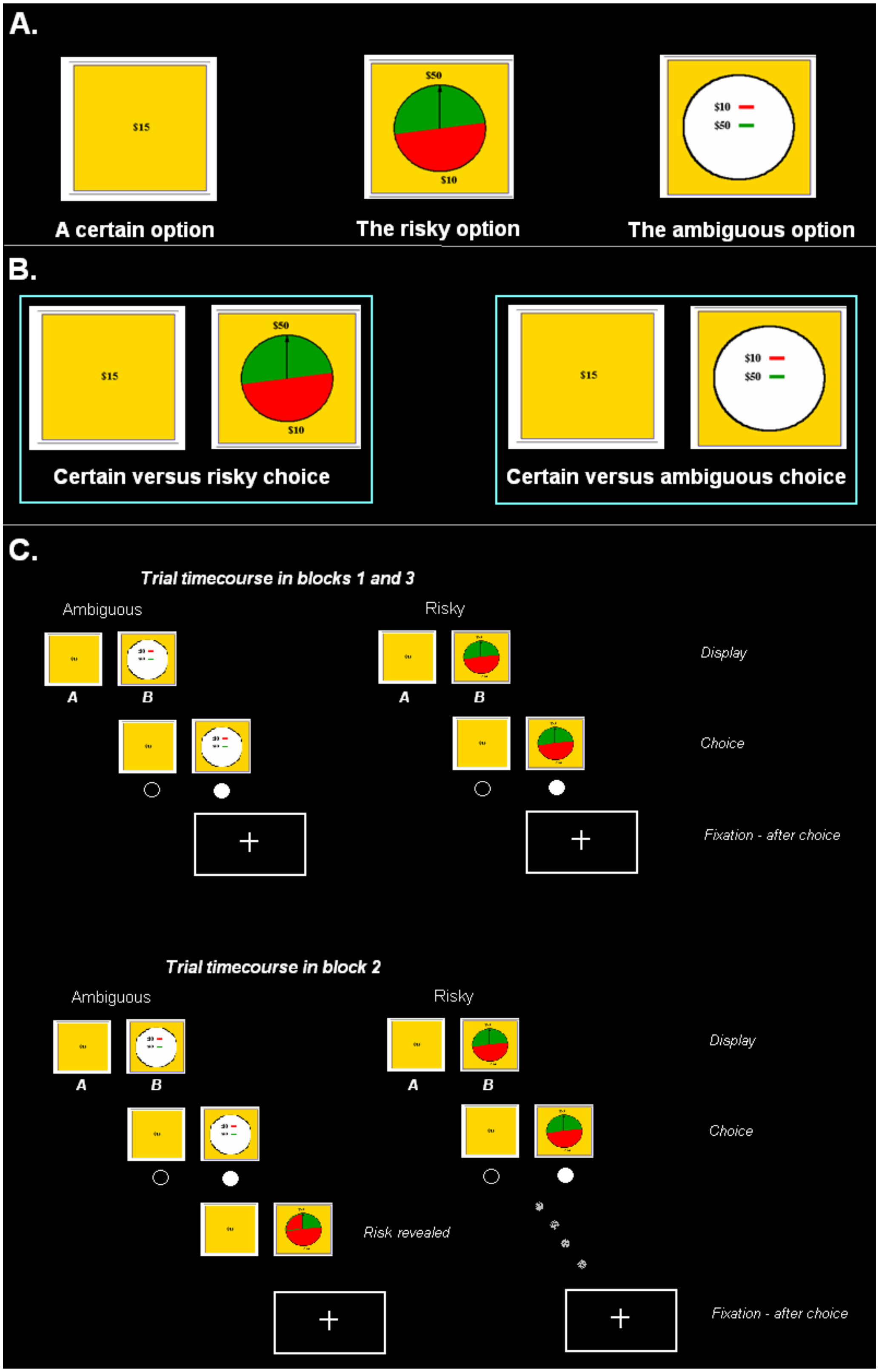

A.
$15
$50
$10
$10
$50
A certain option
The risky option
The ambiguous option
B.
$15
$50
$10
Certain versus risky choice
$15
$10
$50
Certain versus ambiguous choice
C.
Trial timecourse in blocks 1 and 3
Ambiguous
Risky
Display
A
B
A
B
Choice
Fixation - after choice
Trial timecourse in block 2
Ambiguous
Risky
Display
A
B
A
B
Choice
Risk revealed
Fixation - after choice

Figure 3: Effect of the distance from the certainty equivalent on Error rate

The figure displays the smoothed Error Rate. Values are calculated by running a regression of the Error Rate on dummy variables indicating the value of the difference, values varying from 1 to 15. The predicted values are calculated using a normally distributed density function *f*, and the corresponding solutions for error rate (see the Supplementary material for details).

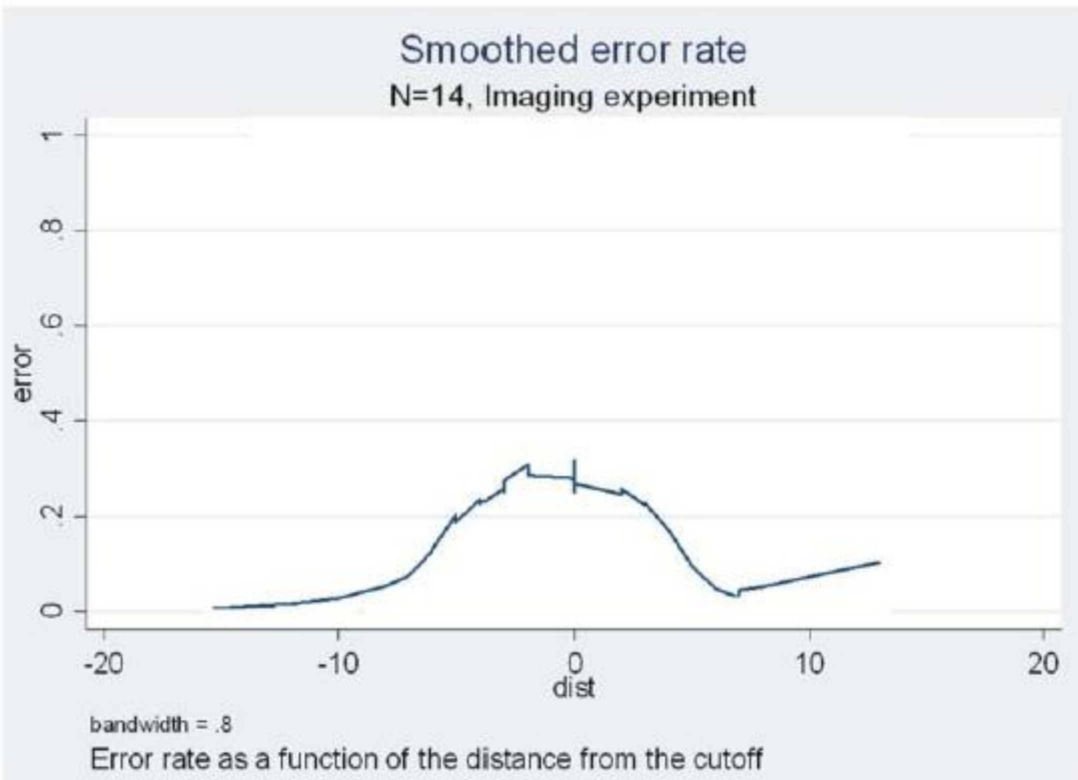


Figure 4: Activation at Display and After Choice.

Top panel. Areas that yielded a higher or lower activation during the display period than after the choice has been made. Data include all blocks. The color codes are reported on the right side of the figure. Higher activation was observed in two main areas of interest: bilaterally in the intraparietal sulcus, IPS, in the left hemisphere, [-39, -38, +44], shown in the figure, and [+32, -49, +40], not shown in the figure, and medial frontal gyrus, (MFG), encompassing supplementary motor area (SMA) and anterior cingulate cortex [+8, 0, +50]. The IPS houses the neuronal representation of the ordered line segment. The cingulate and SMA are often found to be responsible for integrating various aspects of the choice process. Higher activation was observed in the orbito-frontal cortex (OFC), [-3, +42, -8] in after choice than in display, which suggests a predominant evaluative role associated with the after-choice step.
Bottom panel. Event Related Averaging (ERA) of the time course of the BOLD signal time-locked to the onset of the stimulus onset, for the two stimuli, Display and After Choice. The bars report the 95% confidence interval. The ERA's are displayed for the clusters in IPS, MFG and OFC described above.

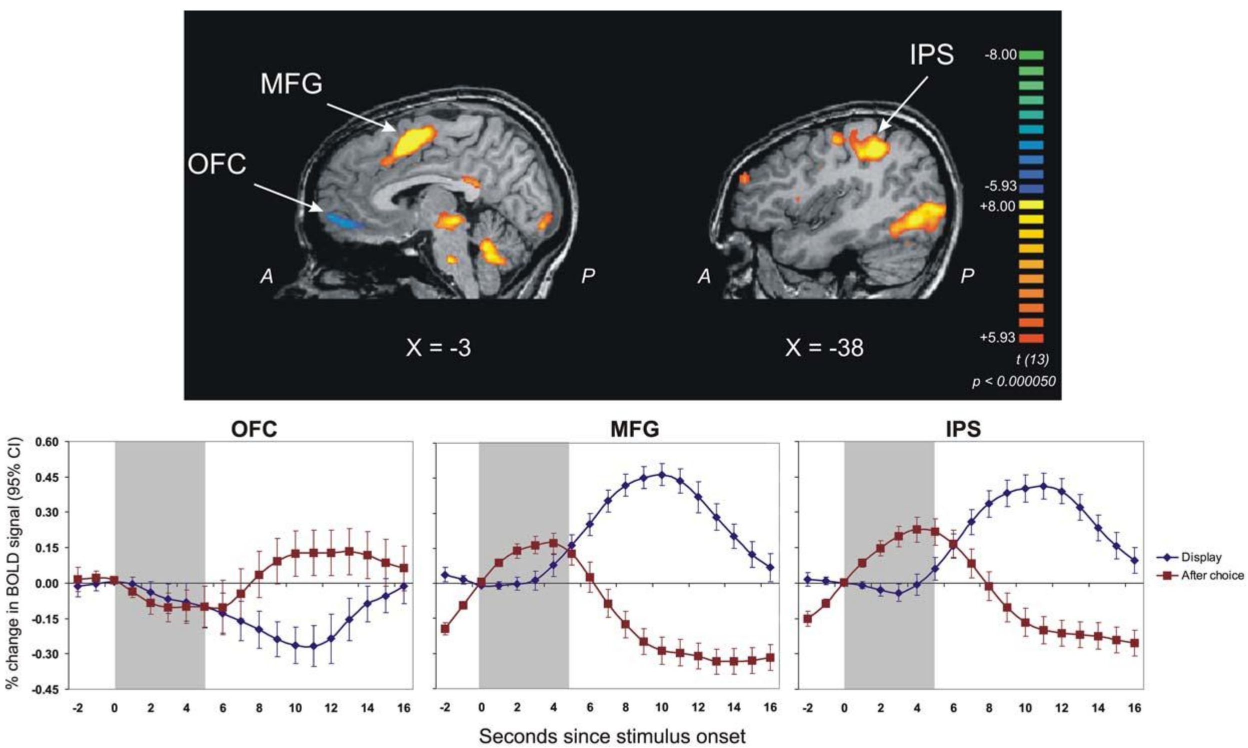
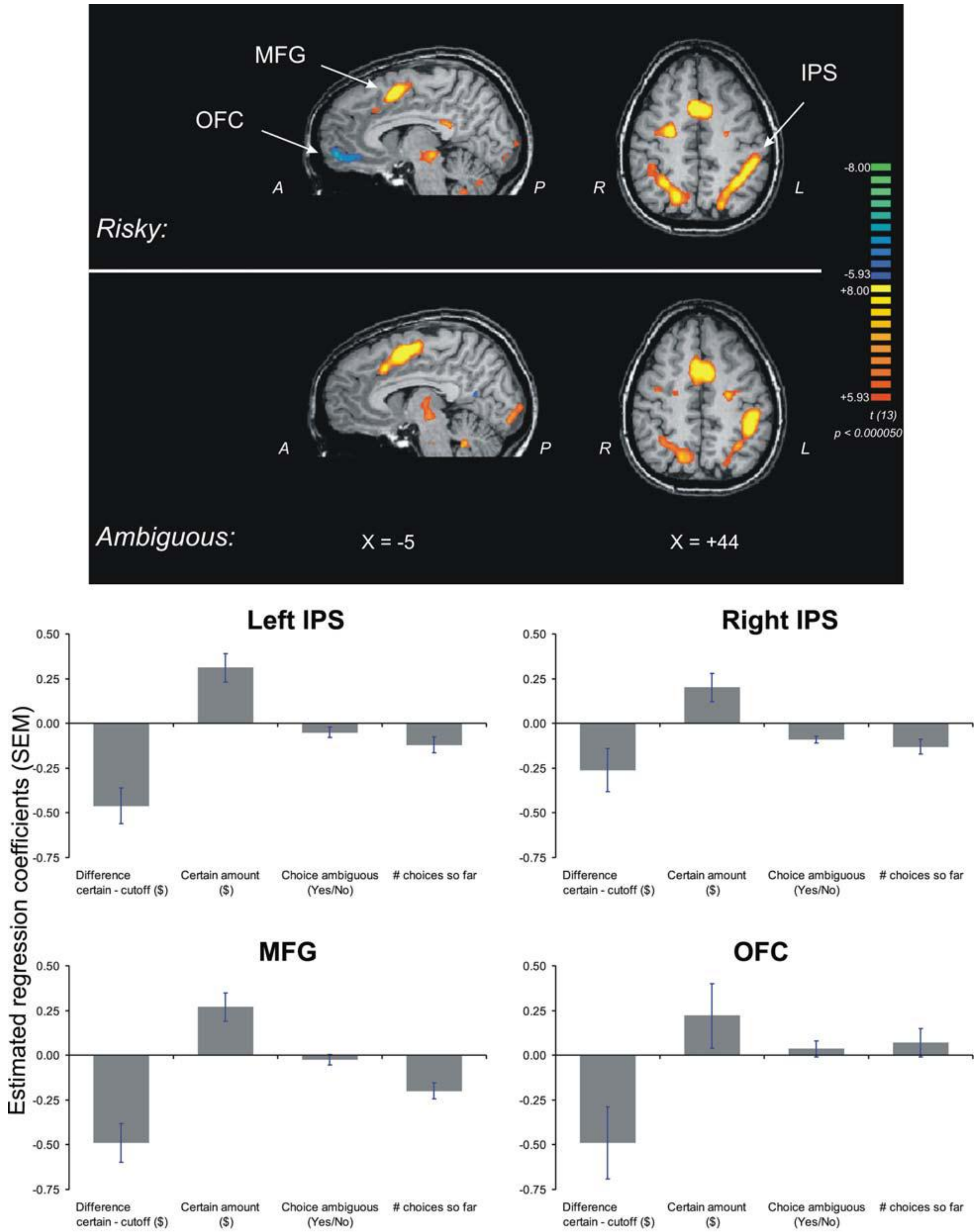


Figure 5: Risk, Ambiguity and distance from the cutoff.

Top panel: Clusters of activation in the contrasts Display versus AfterChoice, in the two conditions of risky and ambiguous choices. The model is a parametric model with independent variables the distance of the certain amount from the certainty equivalent, the values of the certain amount, the number of choices made by the subject and the ambiguous choice. Data include all blocks. The color codes are reported on the right side of the figure. The pattern of activation is similar in the two conditions, with the size of the clusters and the maximum value of the t-statistic both larger in the condition of risky choice.
Bottom panel: The bar graphs display the average value of the coefficient of the four variables (distance of the certain amount from the cutoff, certain amount, number of choices made by the subject, and ambiguous choice), with one standard error super-imposed, for the four clusters identified in the Panel A: right and left IPS, Middle Frontal Gyrus (MFG) and Orbito-frontal (OFC). The coefficients were obtained by first estimating the percentage BOLD change (PBC) for the each of the 42 trials (the experiment had 3 blocks of 14 trials each), obtained by taking the event display in each trial as a distinct predictor. We then ran a regression on PBC with respect to the following variables: distance of the certain amount from the cutoff, certain amount, ambiguous and number of choices made by the subject. The four variables are normalized to have range between 0 and 1, so the coefficients are directly comparable. The sign of the coefficients are as predicted by the model. Ambiguity has a negative effect. In addition, experience reduces the activation, as indicated by the negative value of the coefficient for the number of choices made.
The effect of the distance from the cutoff is negative: when the value is higher than the cutoff, the activation is reduced. This occurs in all four regions, including the OFC, in a significant way. The effect of the certain amount is positive. Both coefficients are strongly significant for the two IPS regions (p =0.0001 for the Left IPS).


MFG
OFC
IPS
A
P
R
L
Risky:
Ambiguous:
X = -5
X = +44
-8.00
-5.93
+8.00
+5.93
t (13)
p < 0.000050
Left IPS
Right IPS
MFG
OFC
Estimated regression coefficients (SEM)
0.50
0.25
0.00
-0.25
-0.50
-0.75
Difference certain - cutoff ($)
Certain amount ($)
Choice ambiguous (Yes/No)
choices so far

Supplementary Materials

1. Theoretic Model

The model is a direct extension of Dichhaut et al. [36], which combines the main ideas from the theory of signal detection and of the random walk models of decision and extends to the realm of economic decisions. We explicitly introduce two additional elements. First, since we are interested in what level of brain activity is associated with risky and ambiguous tasks, we introduce a variable, the quality of the information environment in which subjects are making decisions. Second, we introduce endogenous barriers as a function of effort that varies with information quality into the random walk model of decision process.

Optimal signal detection model

The subject has to choose one action out of a set A. His payoff V will depend on the action and a state of nature that is unknown, $\theta \in \Theta$. Before the choice of the action he can observe a signal $x \in X$, according to an experiment (that is, a map from Θ to probabilities on X) that depends on the level of effort $e \in E$ that the subject chooses. The probability of the signal x given the true state θ and the effort e is denoted by P_θ^e. He has an initial belief on the states of nature, denoted $\mu \in \Delta(\Theta)$. We denote

$$P_\mu^e(x) \equiv \int_\Theta P_\theta^e(x)\mu(d\theta)$$

The subject is choosing the optimal level of effort and, conditional on the signal observed, the best action:

$$\max_{e \in E} \int_X P_\mu^e(x) \max_{a \in A} \int_\Theta \frac{\mu(\theta) P_\theta^e(x)}{P_\mu^e(x)} V(\theta, a)$$

In our experiment the subject has to decide which option in feasible set is optimal for him, so the set of options is his set of actions. More specifically, if the subject is choosing between two lotteries, the lottery on the left and the lottery on the right, then we can describe the action set as {L, R}. He is trying to evaluate which of the two has a larger utility.

He does not know the utility of the two options, so the unknown state is the value of the two utilities, $\theta = (\theta_1, \theta_2)$. If the value θ was known, then the optimal choice would be clear: choose the lottery with the largest value. He does not know the value, however, so his choice has to rely on a noisy signal on the comparative value of the two options.

Comparative value We consider this comparative value to be represented by a function denoted by $\theta \to A(\theta_1, \theta_2)$. This function is assumed to have two properties: It is strictly increasing in the first component and strictly decreasing in the second, and for every θ_1 and θ_2: $A(\theta_2, \theta_1) = -A(\theta_1, \theta_2)$. Examples of comparative value functions are: $A(\theta_1, \theta_2) \equiv \theta_1 - \theta_2$; $A(\theta_1, \theta_2) \equiv \log(\frac{\theta_1}{\theta_2})$.

Signal The signal set is Y = {−1, 1}: a 1 (respectively a −1) is a "match" (respectively "mismatch") in Ratcliff's [47] terminology. We assume that this probability is only dependent on the comparative value $A(\theta)$, and that the signal provided by P is a location signal. Specifically the probability of a match is given by:

$$P_\theta(1:e) = \int_0^{+\infty} \pi(n) \cdot e \cdot f(\pi(n)e(x - A(\theta))dx = \int_{-\pi(n)eA(\theta)}^{+\infty} f(x)dx$$

for a given density function *f*. The effectiveness of the effort is increased by a function π of the variable *n*, representing the number of choices made by the subject (a measure of his experience in the task). Note that as the effort tends to infinity the probability of a match tends to 1 when $A(\theta) > 0$, that is when θ_1, which is the value of the left option is larger than θ_2. For example, if $A(\theta_2, \theta_1) = -A(\theta_1, \theta_2)$ then a mismatch occurs for sure in the limit if and only if $\theta_1 < \theta_2$.

The density function f We make two assumptions on the function *f*. First, there is no differential treatment of options on the right and on the left, that is *f(x) = f(−x)*. Second, the probability decreases as we move away from 0: if *x > y > 0* then *f(y) >f(x)*. The properties of the comparison function *A* and the density function *f* imply on the experiment *P* that $P_{(\theta_1,\theta_2)}(1;e) = 1 - P_{(\theta_2,\theta_1)}(1;e)$; $P_{(\theta_1,\theta_2)}(1;e) = P_{(\theta_2,\theta_1)}(-1;e)$. Also if $\theta_2 > \theta_1$ then $P_{(\theta_1,\theta_2)}(1;e) > P_{(\theta_2,\theta_1)}(1;e)$. Finally, if $A(\theta_1, \theta_2) \equiv \log(\frac{\theta_1}{\theta_2})$ then for every $c > 0$, $P_{\frac{\theta}{c}}(1;e) = P_{(\theta)}(1;e)$. The subject has a prior μ on the set Θ, which is symmetric: $\mu(d\theta_1, d\theta_2) = \mu(d\theta_2, d\theta_1)$.

Cost of Effort The provision of effort in the observation and processing of the signal is costly. The cost depends directly on the effort, but is also influenced by the quality of information that he has available. In the experiment we report below the subjects had to choose the quality of information can be high (when the lottery is risky, and the probability of the two outcomes is known) or low (when the lottery is ambiguous). We summarize the quality of information with a variable q which can be 0 (ambiguous lottery) or 1 (risky lottery); so a higher value of i corresponds to better information. Putting these elements together, the cost is

$$C(e;q) = \gamma(q)c(e).$$

We assume that the function is decreasing in the quality of information q, and that the function c is convex.

Optimal Effort We now present the implications one can derive on the optimal effort as the parameters change. This is going to give us the predictions on the pattern of brain activation that we observe. We do this in the case of the model with a single observation. In the experiment subjects were choosing between a lottery and a certain amount. The perception of the certain amount c induces a revision of the prior: the higher the c, the more likely the subject considers a high value at stake. The revision has the form: $\mu(O\,|\,c) = \mu(\frac{O}{c})$ that is the posterior is equal to the linear translation of the prior: the higher the c the higher are the values of θ, in the same proportion. Note that the second is really a property of the updating function of the prior given the information on the certain amount c. The optimization problem has the form:

$$\max_{e\in E}(\int_Y c \max_{a\in A} \int_\Theta \mu(\theta)P_\theta(y;e)V(\theta,a) - C(e;q).$$

One can prove, given the symmetry assumptions on the beliefs and the signal, that the optimal action given the signal y = 1 is l. Let $\hat{a}(y)$ be the optimal action with signal y, $W(e\,|\,y) = \int_\Theta \mu(\theta;y,e)V(\theta,\hat{a}(y))$ and $W(e) = \int_Y P_\mu(y)W(e\,|\,y)$. The functions $W(\cdot\,|\,y)$ and W are increasing in e, and the derivative is decreasing, for fixed n. Also, for every e, $W(e) = \frac{1}{2}W(e\,|\,1) + \frac{1}{2}W(e\,|\,-1)$ and $W(e\,|\,1) = W(e\,|\,-1) = W(e)$.

Optimal effort is determined equating marginal returns and marginal cost of effort.

Random Walk Model

The general case where the subject can make several observations allows predictions on the time to respond. Note that this is the prediction made for fixed beliefs and effort cost; we are comparing the effect of different realizations of the true utility to the subject. These predictions can be obtained by the analysis of the random walk model induced by the optimal solution. The subject can observe not one but several signals. We denote $p = P(1|\theta, e)$, and $t = 1 - p$. As in Ratcliff, the decision is taken when the sum of the values of the signals reaches a pre-assigned boundary. As in the simple model, $p > 1/2$ is the probability of a correct match.

This model can offer predictions on both the error rate (just like the simple model presented in the previous section) and the response time. The expected values of these two variables are well known from the analysis of the gambler's ruin problem [48]]. The random walk is defined on a subset of the non-negative integers, and it transits from z to $z+1$ with probability p, and to $z-1$ with probability $1-p$. The walk ends when either the upper barrier $a > 0$ or the lower barrier (ruin) equal to 0. We consider the symmetric case in which a is even and the process starts in the midpoint $z = a/2$.

The two relevant formulas are the one for the probability of a mismatch (corresponding in the Gambler's ruin problem to the ruin of the gambler) and the expected duration of the game.

The first is $q_z = \frac{(t/p)^{2z} - (t/p)^z}{(t/p)^{2z} - 1}$ while the second is $D_z = \frac{z}{t-p} - \frac{2z}{t-p}\frac{1-(t/p)^z}{1-(t/p)^{2z}}$.

These are the formulas used in the simulation of the error rate and response time for the data in our experiment.

Predictions

We summarize the main predictions of our model. We define error the event in which the subject chooses the option with smaller utility.

1. An increase in ***experience*** (the variable n) has an increasing returns and a substitution effect: the first increases optimal effort, the second decreases it. For experienced subjects, the second effect is dominant, so the effect is a reduction of the effort;
2. An increase in the ***quality of information*** (the variable q) also has the effect of an increase in the effort, because it shifts the marginal cost function down;
3. An increase in the ***value of the certain amount*** (the variable c) has the effect of an increase in the effort, since it shifts upwards the derivative with respect to the effort.

4. An increase in the ***distance*** (the variable $|\mu(x_1)-\mu(x_2)|$, equal to the absolute value of the difference between the utility of the two options) decreases the time to respond.

5. An increase in the ***distance*** between utility of the two options decreases the error rate.

2. Regressions

Table 1: Panel data logit regression of the Choice of the certain amount. *ambiguous* = 1 if the choice was ambiguous; *certain amount* = value of the certain amount, in dollars.

Choice of the certain amount coefficient/standard error	
ambiguous	0.464**
	(0.229)
certain amount	0.387***
	(0.032)
constant	–9.409***
	(0.841)
N	588

Table 2: Panel data logit regression of the Error rate. *distance* = absolute value of the difference between certain amount and cutoff, in dollars; *cumulated choices*: number of total choices made up to the current trial by the subject.

Error Rate coefficient/standard error	
distance	–0.586***
	(0.065)
cumulated choices	–0.027**
	(0.011)
constant	0.924***
	(0.309)
N	588

Table 3: Panel data regression of the Response Time. *ambiguous* = 1 if the choice was ambiguous; *distance* = absolute value of the difference between certain amount and cutoff; *cumulated choices*: number of total choices made up to the current trial by the subject.

Response Time coefficient/standard error	
ambiguous	–0.216***
	(0.102)
distance	–0.061***
	(0.013)
cumulated choices	–0.021***
	(0.004)
constant	2.195***
	(0.214)
N	588